PRACTICAL FILE

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PRACTICAL-1

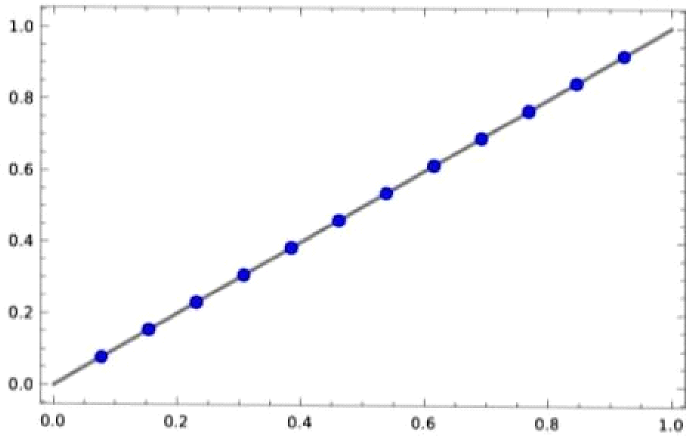
Plotting of Legendre polynomial for n=1 to 5 in the interval [0,1].Verifying graphically that all the roots of P(x) lie in the interval[0,1].

In[4]:=
```
n = 1;
```

In[5]:=
```
Subscript [P, n][x_] := Sum[(- 1)^ r * (2 * n - 2 * r) ! *
    x ^(n - 2 * r) / (2 ^ n * r!(n - r) !(n - 2 * r) !), {r, 0, (n - 1) / 2}]
```

In[6]:=
```
s1 = Plot[Subscript[P, 1][x], {x, 0, 1}, PlotStyle → {Gray, Thick}, Frame → True, Mesh → 12,
  MeshStyle → Directive[PointSize[Large], Blue], PlotLegends → "Expressions "]
```

Out[6]=



In[7]:=
```
NRoots[Subscript [P, n][x], x]
```

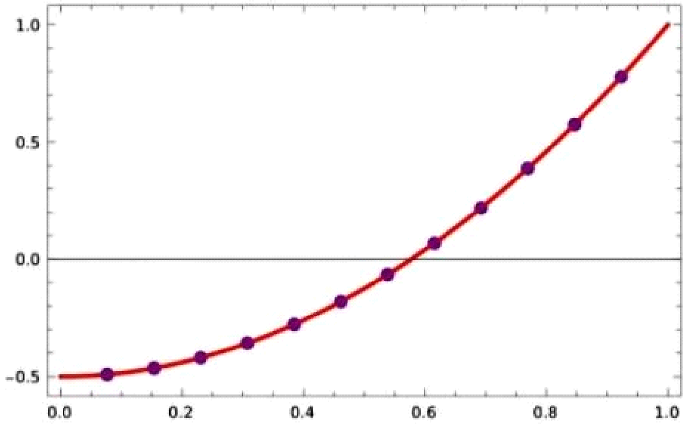
Out[7]=
```
x == 0.
```

In[8]:=
```
n = 2;
```

In[9]:=
```
Subscript [P, n][x_] := Sum[
  (- 1)^ r * (2 * n - 2 * r) ! * x ^(n - 2 * r) / (2 ^ n * r!(n - r) !(n - 2 * r) !), {r, 0, (n) / 2}]
```

In[13]:=
```
s2 = Plot[Subscript[P, 2][x], {x, 0, 1}, PlotStyle → {Red, Thick}, Frame → True, Mesh → 12,
  MeshStyle → Directive[PointSize[Large], Purple], PlotLegends → "Expressions"]
```

Out[13]=



In[14]:=
```
NRoots[Subscript[P, n][x], x]
```

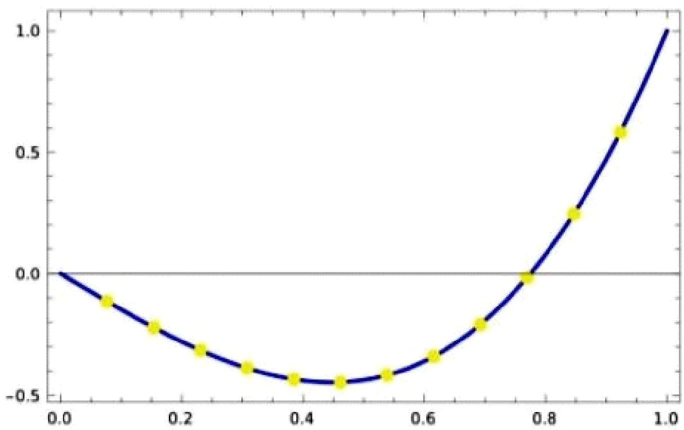
Out[14]=
```
x == -0.57735 || x == 0.57735
```

In[16]:=
```
n = 3;
```

In[17]:=
```
Subscript[P, n][x_] := Sum[(- 1)^ r * (2 * n - 2 * r) ! *
   x ^(n - 2 * r) / (2 ^ n * r!(n - r) !(n - 2 * r) !), {r, 0, (n - 1) / 2}]
```

In[18]:=
```
s3 = Plot[Subscript[P, 3][x], {x, 0, 1}, PlotStyle → {Blue, Thick}, Frame → True, Mesh → 12,
  MeshStyle → Directive[PointSize[Large], Yellow], PlotLegends → "Expressions"]
```

Out[18]=



In[19]:=
```
NRoots[Subscript[P, n][x], x]
```

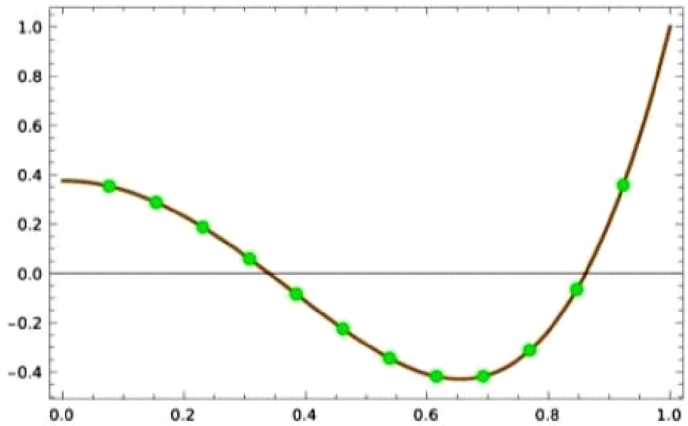
Out[19]=
```
x == -0.774597 || x == 0. || x == 0.774597
```

In[20]:=
```
n = 4;
```

In[21]:=
```
Subscript[P, n][x_] :=
 Sum[(- 1)^ r * (2 * n - 2 * r) ! * x ^(n - 2 * r) / (2 ^ n * r!(n - r) !(n - 2 * r) !), {r, 0, n / 2}]
```

In[22]:=
```
s4 = Plot[Subscript[P, 4][x], {x, 0, 1}, PlotStyle → {Brown, Thick}, Frame → True, Mesh → 12,
  MeshStyle → Directive[PointSize[Large], Green], PlotLegends → "Expressions"]
```

Out[22]=



In[23]:=
```
NRoots[Subscript[P, n][x], x]
```

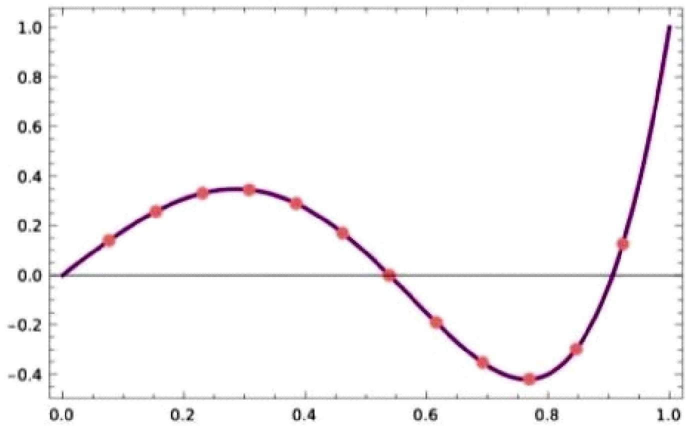
Out[23]=
```
x == -0.861136 || x == -0.339981 || x == 0.339981 || x == 0.861136
```

In[24]:=
```
n = 5;
```

In[25]:=
```
Subscript[P, n][x_] := Sum[(- 1)^ r * (2 * n - 2 * r) ! *
   x ^(n - 2 * r) / (2 ^ n * r!(n - r)!(n - 2 * r)!), {r, 0, (n - 1) / 2}]
```

In[26]:=
```
s5 = Plot[Subscript[P, 5][x], {x, 0, 1},
  PlotStyle → {Purple, Thick}, Frame → True, Mesh → 12,
  MeshStyle → Directive[PointSize[Large], Pink], PlotLegends → "Expressions"]
```

Out[26]=

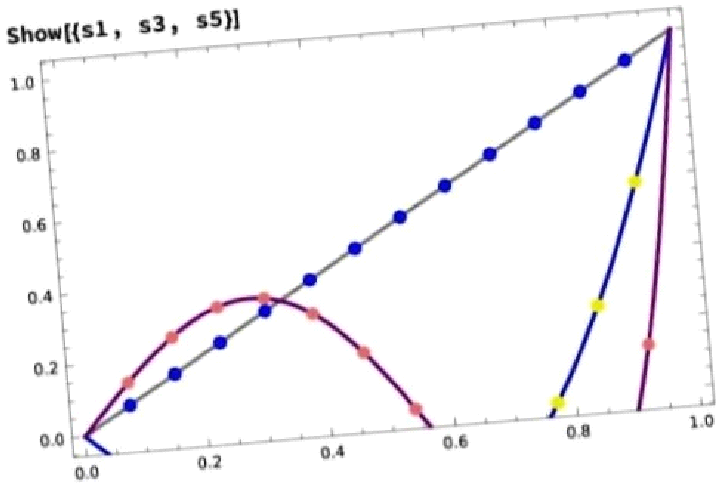


In[27]:=
```
NRoots[Subscript[P, n][x], x]
```

Out[27]=
```
x == -0.90618 || x == -0.538469 || x == 0. || x == 0.538469 || x == 0.90618
```

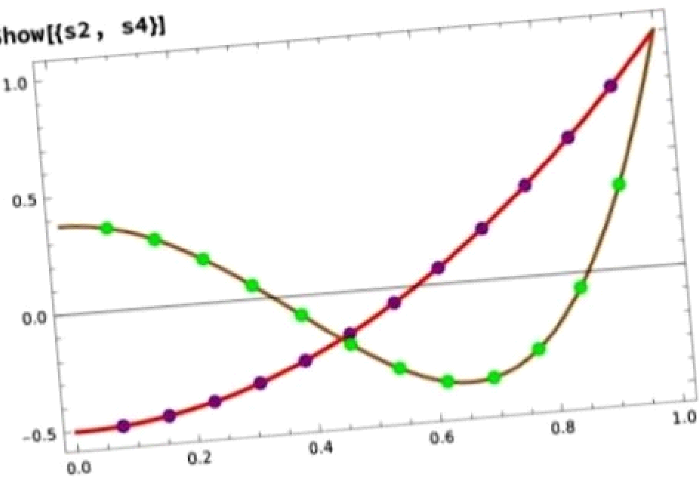
In[28]:= **Show[{s1, s3, s5}]**

Out[28]=



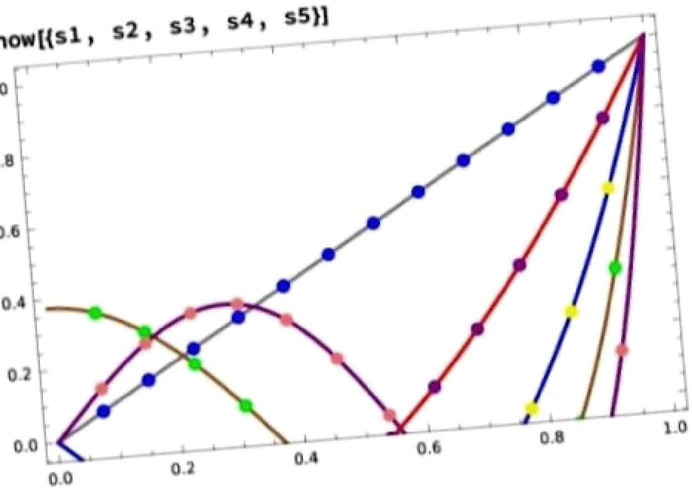
In[29]:= **Show[{s2, s4}]**

Out[29]=



In[30]:= **Show[{s1, s2, s3, s4, s5}]**

Out[30]=



PRACTICAL-2

Automatic computation of coefficients in the series solution near ordinary point.

```
In[503]:= ClearAll ;

In[504]:= x0 = 1

Out[504]= 1

In[505]:= A[x_] = 2 x;
          B[x_] = 3;
          C1[x_] = - 1;
          P[x_] = B[x] / A[x];
          Q[x_] = C1[x] / A[x];

In[510]:= de = A[x] × y ''[x] + B[x] × y '[x] + C1[x] × y[x] == 0;
          Print[de];

In[512]:= If[A[x0] == 0,
          Print["The point x0 = ", x0,
            " is a singular point of the Differential Equation "];
          Print["Proceed with a Frobenius solution ."];
          Print["The point x0 = ", x0,
            "is NOT a singular point of the Differential Equation "];
          Print["y''[x]+f1[x] y'[x]+f2[x] y[x]=0"];
          Print["P[x]=B[x]/A[x]= ", Together [B[x] / A[x]], " =", Series[P[x], {x, x0, 3}]];
          Print["Q[x]=C1[x]/A[x] = " Together [C1[x] / A[x]], " = ", Series[Q[x], {x, x0, 3}]];
          If[Subscript [x, 0] == 0,
          Print[
             "\"y[x]=\!\(\*UnderoverscriptBox [\(Σ\), \(n = 0\), \(∞\)]\)\!\(\*SubscriptBox [\(a\),
               \(n\)]\) \!\(\*TemplateBox [{\"x\", \"n\"},\"Superscript \"]\),etc.\""]];
          If[Subscript [x, 0] > 0,
          Print["\"y[x]=\!\(\*UnderoverscriptBox [\(Σ\), \(∞\), \(n =
               0\)]\)\!\(\*SubscriptBox [\(a\), \(n\)]\) (x-\"" , x0 , ")^n,etc."]];
          If[Subscript [x, 0] < 0, Print["\"y[x]=\!\(\*UnderoverscriptBox [\(Σ\), \(∞\),
               \(n = 0\)]\)\!\(\*SubscriptBox [\(a\), \(n\)]\) (x+\"" , - x0, ")^n,etc."]];]

In[513]:= Clear[c, k, n, r, s, x];
```

```
In[514]:= c = 0;

In[515]:= Remove[c];

In[516]:= n = 9;

In[1]:= s[x_] = Expand[(Sum[Subscript [c, k] * (x - x0)^k, {k, 0, n}] + o[x]^(n + 1))];
 coeff = Table[Subscript [c, k], {k, 1, n}];(*Print[" s[x] = "s[x]];
Print["s'[x]=",s'[x]]; 5 Print["s''[x]="s''[x]]; Print["Substitute it into "];
Print["2 x y''[x]+3 y'[x]-y[x]==0"]; Print["to get"];*)
 deqn = Expand[2 x s ''[x] + 3 s '[x] - s[x]] == 0; (*Print[deqn];*)
 eqns = LogicalExpand [deqn];
(*for coefft comparison of like powers of x on both sides*)
Print[TableForm[ReplaceAll[eqns, And → List]]];
(*Replaces all "&&" and shifts every new expression to the next line*)
solution = Solve[eqns, coeff];
  Print[TableForm [Sort[Solution [[1]]]]];
(*sort is used to sort the list in increasing order c0,
```

```
c1,c2 and so on and tableform is used to display
 the list of coeffts in a column one below the other*)
s[x_] = ReplaceAll[s[x], solution[[1]]];
(*will replace all the coeffts in s(x) with ther actual value*)
Print["y = ", Normal[s[x]], " +..."];
s₁[x_] = ReplaceAll[s[x], c₀ → 1];
s1[x_] = Normal[s₁[x]];
Print["y₁[x] = ", s1[x], " +..."];
Plot[s1[x], {x, 0, 2 π}]
```

$$-y[x] + 3\,y'[x] + 2\,x\,y''[x] == 0$$

The point $x_0 = 1$ is NOT a singular point of the DE

$$y''[x]+f_1[x]y'[x]+f_2[x]y[x]=0$$

$$P[x]=\frac{B[x]}{A[x]} = \frac{3}{2\,x} = \frac{3}{2} - \frac{3\,(x-1)}{2} + \frac{3}{2}\,(x-1)^2 - \frac{3}{2}\,(x-1)^3 + O[x-1]^4$$

$$Q[x]=\frac{C1[x]}{A[x]} = -\frac{1}{2\,x} = -\frac{1}{2} + \frac{x-1}{2} - \frac{1}{2}\,(x-1)^2 + \frac{1}{2}\,(x-1)^3 + O[x-1]^4$$

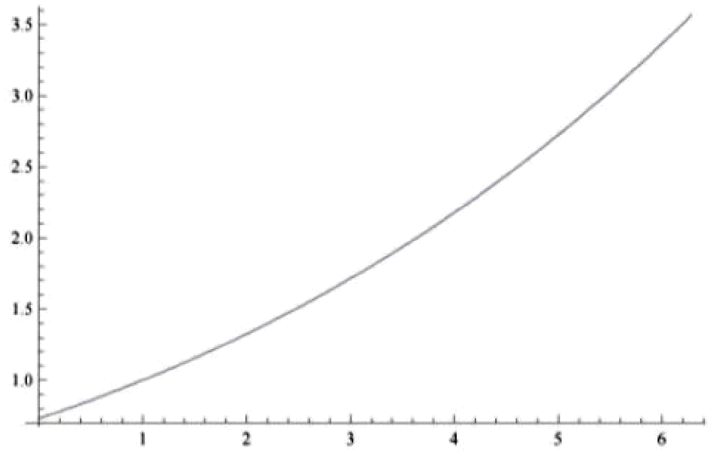
$$y[x]=\sum_{n=0}^{\infty} a_n\,(x-1)^n, \text{ etc.}$$

$$-c_5 + 6\,c_6 - 21\,c_7 + 56\,c_8 + 78\,(c_6 - 7\,c_7 + 28\,c_8 - 84\,c_9) - 126\,c_9 == 0$$
$$-c_3 + 4\,c_4 - 10\,c_5 + 20\,c_6 - 35\,c_7 + 56\,c_8 + 36\,(c_4 - 5\,c_5 + 15\,c_6 - 35\,c_7 + 70\,c_8 - 126\,c_9) - 84\,c_9 == 0$$
$$-c_7 + 8\,c_8 + 136\,(c_8 - 9\,c_9) - 36\,c_9 == 0$$
$$-c_1 + 2\,c_2 - 3\,c_3 + 4\,c_4 - 5\,c_5 + 6\,c_6 - 7\,c_7 + 8\,c_8 + 10\,(c_2 - 3\,c_3 + 6\,c_4 - 10\,c_5 + 15\,c_6 - 21\,c_7 + 28\,c_8 - 36\,c_9) -$$
$$-c_8 + 180\,c_9 == 0$$
$$-c_0 + c_1 - c_2 + c_3 - c_4 + c_5 - c_6 + c_7 - c_8 + c_9 + 3\,(c_1 - 2\,c_2 + 3\,c_3 - 4\,c_4 + 5\,c_5 - 6\,c_6 + 7\,c_7 - 8\,c_8 + 9\,c_9) == 0$$
$$-c_6 + 7\,c_7 - 28\,c_8 + 84\,c_9 + 105\,(c_7 - 8\,c_8 + 36\,c_9) == 0$$
$$-c_2 + 3\,c_3 - 6\,c_4 + 10\,c_5 - 15\,c_6 + 21\,c_7 - 28\,c_8 + 36\,c_9 + 21\,(c_3 - 4\,c_4 + 10\,c_5 - 20\,c_6 + 35\,c_7 - 56\,c_8 + 84\,c_9) =$$
$$-c_4 + 5\,c_5 - 15\,c_6 + 35\,c_7 - 70\,c_8 + 126\,c_9 + 55\,(c_5 - 6\,c_6 + 21\,c_7 - 56\,c_8 + 126\,c_9) == 0$$

$$c_1 \to \frac{96\,209\,476\,345\,449\,c_0}{325\,091\,511\,083\,548}$$
$$c_2 \to \frac{2\,278\,942\,627\,950\,c_0}{81\,272\,877\,770\,887}$$
$$c_3 \to \frac{105\,245\,233\,905\,c_0}{81\,272\,877\,770\,887}$$
$$c_4 \to \frac{2\,866\,363\,164\,c_0}{81\,272\,877\,770\,887}$$
$$c_5 \to \frac{102\,807\,999\,c_0}{162\,545\,755\,541\,774}$$
$$c_6 \to \frac{652\,386\,c_0}{81\,272\,877\,770\,887}$$
$$c_7 \to \frac{6165\,c_0}{81\,272\,877\,770\,887}$$
$$c_8 \to \frac{45\,c_0}{81\,272\,877\,770\,887}$$
$$c_9 \to \frac{c_0}{325\,091\,511\,083\,548}$$

$$y = \frac{5\,399\,729\,244\,000\,c_0}{7\,388\,443\,433\,717} + \frac{1\,799\,909\,748\,000\,x\,c_0}{7\,388\,443\,433\,717} + \frac{179\,990\,974\,800\,x^2\,c_0}{7\,388\,443\,433\,717} + \frac{8\,570\,998\,800\,x^3\,c_0}{7\,388\,443\,433\,717} + \frac{238\,083\,300\,x^4\,c_0}{7\,388\,443\,433\,717} + \frac{47\,616\,660\,x^5\,c_0}{81\,272\,877\,770\,887} + \frac{610\,470\,x^6\,c_0}{81\,272\,877\,770\,887} + \frac{5814\,x^7\,c_0}{81\,272\,877\,770\,887} + \frac{171\,x^8\,c_0}{325\,091\,511\,083\,548} + \frac{x^9\,c_0}{325\,091\,511\,083\,548} + \ldots$$

$$y_1[x] = \frac{5399729244000}{7388443433717} + \frac{1799909748000\,x}{7388443433717} + \frac{179990974800\,x^2}{7388443433717} + \frac{8570998800\,x^3}{7388443433717} + \frac{238083300\,x^4}{7388443433717} + \frac{47616660\,x^5}{81272877770887} + \frac{610470\,x^6}{81272877770887} + \frac{5814\,x^7}{81272877770887} + \frac{171\,x^8}{325091511083548} + \frac{x^9}{325091511083548} + \ldots$$



PRACTICAL-3

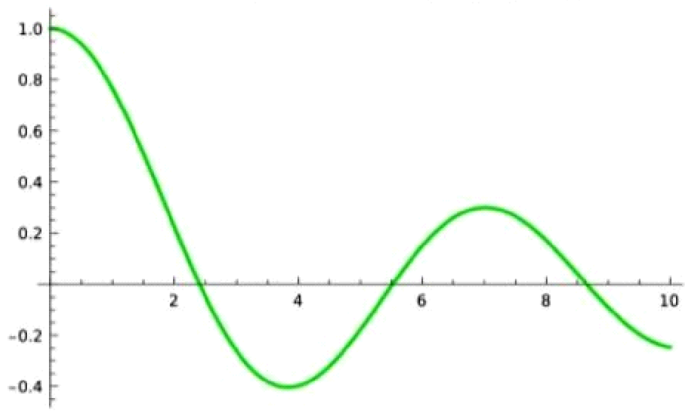
Plotting of the Bessel's function of first kind of order 0 to 3.

In[33]:=
```
Subscript [J, n][x] =
  Sum[(((- 1)^ r) ((x / 2)^(2 r + n))) / (Factorial [r] × Factorial [n + r]), {r, 0, ∞}];
```

In[34]:=
```
Series[BesselJ [0, x], {x, 0, 10}]
```

Out[34]=
$$1 - \frac{x^2}{4} + \frac{x^4}{64} - \frac{x^6}{2304} + \frac{x^8}{147\,456} - \frac{x^{10}}{14\,745\,600} + O[x]^{11}$$

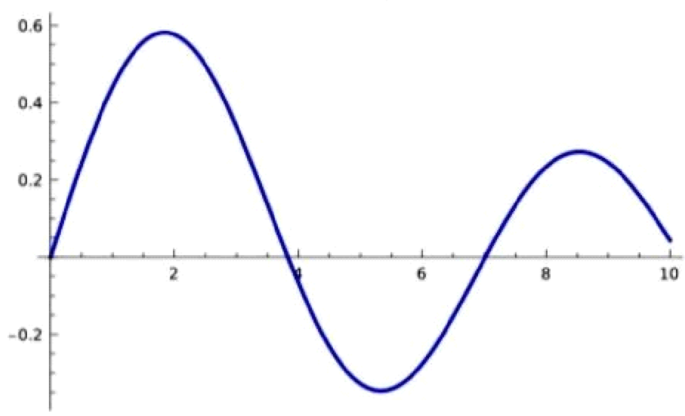
In[51]:=
```
a0 = Plot[Evaluate[BesselJ[0, x], {x, 0, 10}], PlotStyle → {Green, Thick}]
```

Out[51]=


In[40]:=
```
Series[BesselJ [1, x], {x, 0, 10}]
```

Out[40]=
$$\frac{x}{2} - \frac{x^3}{16} + \frac{x^5}{384} - \frac{x^7}{18\,432} + \frac{x^9}{1\,474\,560} + O[x]^{11}$$

In[42]:=
```
a1 = Plot[Evaluate[BesselJ[1, x], {x, 0, 10}], PlotStyle → {Blue, Thick}]
```

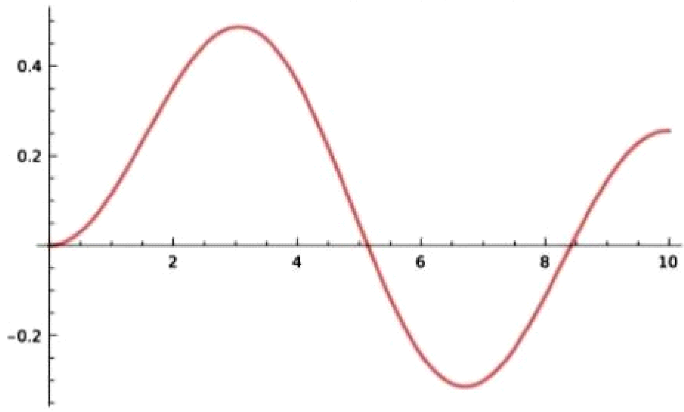
Out[42]=


In[43]:= `Series[BesselJ [2, x], {x, 0, 10}]`

Out[43]= $\frac{x^2}{8} - \frac{x^4}{96} + \frac{x^6}{3072} - \frac{x^8}{184\,320} + \frac{x^{10}}{17\,694\,720} + O[x]^{11}$

In[48]:= `a2 = Plot[Evaluate[BesselJ[2, x], {x, 0, 10}], PlotStyle → {Pink, Thick}]`

Out[48]=

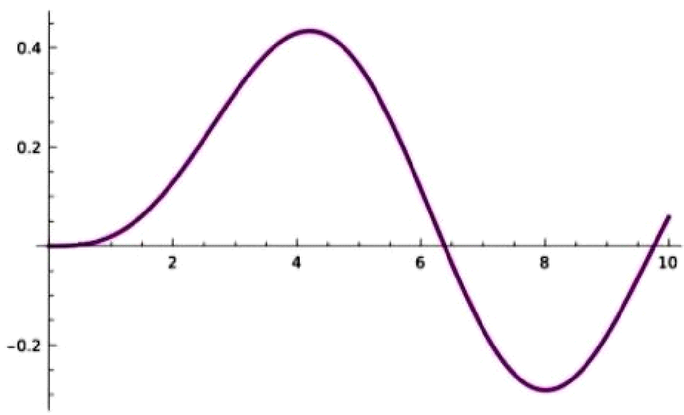


In[45]:= `Series[BesselJ [3, x], {x, 0, 10}]`

Out[45]= $\frac{x^3}{48} - \frac{x^5}{768} + \frac{x^7}{30\,720} - \frac{x^9}{2\,211\,840} + O[x]^{11}$

In[47]:= `a3 = Plot[Evaluate[BesselJ[3, x], {x, 0, 10}], PlotStyle → {Purple, Thick}]`

Out[47]=



Practical 4

The Differential equation is as follows :

$$2 x y''[x] + 3 y'[x] - y[x] = 0.$$

```
x0 = 0;
P[x_] = 2 x;
Q[x_] = 3;
R[x_] = -1;
f1[x_] = Q[x]/P[x];
f2[x_] = R[x]/P[x];
de = P[x] y''[x] + Q[x] y'[x] + R[x] y[x] == 0;
Print[de];
If[P[x0] == 0,
  Print["P[x]= ", P[x]];
  Print["P[", x0, "]=", P[x0]];
  Print["The point x0 = ", x0, " is a singular point of the DE"];
  Print["Proceed with a Frobenius solution."];,
  Print["The point x0 = ", x0, " is NOT a singular point of the DE"];
  Print["y''[x]+f1[x]y'[x]+f2[x]y[x]=0"];
  Print["f1[x]=Q[x]/P[x] = ", Together[Q[x]/P[x]], " = ", Series[f1[x], {x, x0, 3}]];
  Print["f2[x]=R[x]/P[x] = ", Together[R[x]/P[x]], " = ", Series[f2[x], {x, x0, 3}]];
  If[x0 == 0, Print["y[x]=Sum[an x^n, {n, 0, ∞}], etc."]];

  If[x0 > 0, Print["y[x]=Sum[an (x-", x0, ")^n, {n, 0, ∞}], etc."]];
  If[x0 < 0, Print["y[x]=Sum[an (x+", -x0, ")^n, {n, 0, ∞}], etc."]];];
ϕ1[x_] = x Q[x]/P[x];
ϕ2[x_] = x^2 R[x]/P[x];
p0 = Limit[x Q[x]/P[x], x → x0];
q0 = Limit[x^2 R[x]/P[x], x → x0];
Clear[r];
ieqn = r (r - 1) + p0 r + q0 == 0;
Print["P[x]  = ", P[x]];
Print["Q[x]  = ", Q[x]];
Print["R[x]  = ", R[x]];
Print["ϕ1[x]  = x Q[x]/P[x]  = ", x Q[x]/P[x]];
```

```
Print["p0 = lim x→x0 x Q[x]/P[x] = ", p0];
Print["φ2[x] = x^2 R[x]/P[x] = ", x^2 R[x]/P[x]];
Print["q0 = lim x→x0 x^2 R[x]/P[x] = ", q0];
Print["The indicial equation is : r(r-1)+p0r+q0==0"];
Print[ieqn];
ieqn = MapAll[Expand, ieqn];
Print[ieqn];
Print["The indicial equation is "];
Print[ieqn];
Print["The roots are "];
solution = Solve[ieqn, r];
r1 = solution[[2, 1, 2]];
r2 = solution[[1, 1, 2]];
Print["r1 = ", r1];
Print["r2 = ", r2];
Clear[c, k, n, r, s, x];
c = 0; Remove[c];
r = 0;
n = 9;
s[x_] = Expand[x^r (Sum[c_k x^k, {k, 0, n}]) + x^r O[x]^(n+1)];
coeff = Table[c_k, {k, 1, n}];
Print[" s[x] = ", s[x]];
Print[" s'[x] = ", s'[x]];
Print[" s''[x] = ", s''[x]];
Print["Substitute it into "];
Print["2 x y''[x]+3 y'[x]-y[x] == 0"];
Print[" to get "];
deqn = Expand[2 x s''[x] + 3 s'[x] - s[x] ] == 0;
Print[deqn];
eqns = LogicalExpand[deqn];
Print[deqn];
Print[TableForm[ReplaceAll[eqns, And → List]]];
solution = Solve[eqns, coeff];
Print[TableForm[Sort[solution[[1]]]]];
s[x_] = ReplaceAll[s[x], solution[[1]]];
Print["y = ", s[x]];
Print["y = ", Normal[s[x]], " +..."];
s1[x_] = ReplaceAll[s[x], c0 → 1];
Print["f1[x] = ", s1[x]];
Print["f1[x] = ", Normal[s1[x]], " +..."];
Clear[c, k, n, r, s, x];
c = 0; Remove[c];
r = -1/2;
n = 7;
```

```
s[x_] = Expand[x^r (Sum[c_k x^k, {k, 0, n}]) + x^r O[x]^(n+1)];
coeff = Table[c_k, {k, 1, n}];
Print[" s[x]  = ", s[x]];
Print[" s'[x]  = ", s'[x]];
Print[" s''[x]  = ", s''[x]];
Print["Substitute it into "];
Print["2 x s''[x]+3 s'[x]-s[x]  == 0"];
Print[" to get "];
deqn = Expand[2 x s''[x] + 3 s'[x] - s[x] ] == 0;
Print[deqn];
eqns = LogicalExpand[deqn];
Print[deqn];
Print[TableForm[ReplaceAll[eqns, And → List]]];
solution = Solve[eqns, coeff];
Print[TableForm[Sort[solution[[1]]]]];
s[x_] = ReplaceAll[s[x], solution[[1]]];
Print["y = ", s[x]];
Print["y = ", Normal[s[x]], " +..."];
Print["y = c0", x^r, "(", Normal[1/c0 x^-r s[x]], " +..."];
s2[x_] = ReplaceAll[s[x], c0 → 1];
Print["f2[x]  = ", s2[x]];
Print["f2[x]  = ", Normal[s2[x]], " +..."];
s2[x_] = ReplaceAll[s2[x], c1 → 0];
Print["f2[x]  = ", s2[x]];
Print["f2[x]  = ", Normal[s2[x]], " +..."];
Print["f2[x]  = ", x^r, "(", Normal[x^-r s2[x]], " +...)"];
s1[x_] = Normal[s1[x]];
Plot[s1[x], {x, 0, 2 π}]
Print["s1[x]  = ", s1[x]];
s2[x_] = Normal[s2[x]];
Plot[s2[x], {x, 0, 2 π}, PlotRange → {{0, π}, {1, 5}}]
Print["s2[x]  = ", s2[x]];
```

$-y[x] + 3\,y'[x] + 2\,x\,y''[x] == 0$

P[x]= 2 x

P[0]=0

The point x_0 = 0 is a singular point of the DE

Proceed with a Frobenius solution.

P[x] = 2 x

Q[x] = 3

R[x] = -1

$\phi_1[x] = x \dfrac{Q[x]}{P[x]} = \dfrac{3}{2}$

$$p_0 = \lim_{x \to x_0} x \frac{Q[x]}{P[x]} = \frac{3}{2}$$

$$\phi_2[x] = x^2 \frac{R[x]}{P[x]} = -\frac{x}{2}$$

$$q_0 = \lim_{x \to x_0} x^2 \frac{R[x]}{P[x]} = 0$$

The indicial equation is : $r(r-1)+p_0 r+q_0 == 0$

$$\frac{3r}{2} + (-1+r)\, r == 0$$

$$\frac{r}{2} + r^2 == 0$$

The indicial equation is

$$\frac{r}{2} + r^2 == 0$$

The roots are

$$r_1 = 0$$

$$r_2 = -\frac{1}{2}$$

$$s[x] = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 + c_6 x^6 + c_7 x^7 + c_8 x^8 + c_9 x^9 + O[x]^{10}$$

$$s'[x] = c_1 + 2 c_2 x + 3 c_3 x^2 + 4 c_4 x^3 + 5 c_5 x^4 + 6 c_6 x^5 + 7 c_7 x^6 + 8 c_8 x^7 + 9 c_9 x^8 + O[x]^9$$

$$s''[x] = 2 c_2 + 6 c_3 x + 12 c_4 x^2 + 20 c_5 x^3 + 30 c_6 x^4 + 42 c_7 x^5 + 56 c_8 x^6 + 72 c_9 x^7 + O[x]^8$$

Substitute it into

$$2\, x\, y''[x]+3\, y'[x]-y[x] == 0$$

to get

$$(-c_0 + 3 c_1) + (-c_1 + 10 c_2) x + (-c_2 + 21 c_3) x^2 + (-c_3 + 36 c_4) x^3 + (-c_4 + 55 c_5) x^4 + (-c_5 + 78 c_6) x^5 + (-c_6 + 105 c_7) x^6 + (-c_7 + 136 c_8) x^7 + (-c_8 + 171 c_9) x^8 + O[x]^9 == 0$$

$$(-c_0 + 3 c_1) + (-c_1 + 10 c_2) x + (-c_2 + 21 c_3) x^2 + (-c_3 + 36 c_4) x^3 + (-c_4 + 55 c_5) x^4 + (-c_5 + 78 c_6) x^5 + (-c_6 + 105 c_7) x^6 + (-c_7 + 136 c_8) x^7 + (-c_8 + 171 c_9) x^8 + O[x]^9 == 0$$

$-c_0 + 3 c_1 == 0$
$-c_1 + 10 c_2 == 0$
$-c_2 + 21 c_3 == 0$
$-c_3 + 36 c_4 == 0$
$-c_4 + 55 c_5 == 0$
$-c_5 + 78 c_6 == 0$
$-c_6 + 105 c_7 == 0$
$-c_7 + 136 c_8 == 0$
$-c_8 + 171 c_9 == 0$

$c_1 \to \frac{c_0}{3}$
$c_2 \to \frac{c_0}{30}$
$c_3 \to \frac{c_0}{630}$
$c_4 \to \frac{c_0}{22\,680}$
$c_5 \to \frac{c_0}{1\,247\,400}$
$c_6 \to \frac{c_0}{97\,297\,200}$
$c_7 \to \frac{c_0}{10\,216\,206\,000}$
$c_8 \to \frac{c_0}{1\,389\,404\,016\,000}$
$c_9 \to \frac{c_0}{237\,588\,086\,736\,000}$

$$y = c_0 + \frac{c_0 x}{3} + \frac{c_0 x^2}{30} + \frac{c_0 x^3}{630} + \frac{c_0 x^4}{22\,680} + \frac{c_0 x^5}{1\,247\,400} + \frac{c_0 x^6}{97\,297\,200} + \frac{c_0 x^7}{10\,216\,206\,000} + \frac{c_0 x^8}{1\,389\,404\,016\,000} + \frac{c_0 x^9}{237\,588\,086\,736\,000} + O[x]^{10}$$

$$y = c_0 + \frac{x c_0}{3} + \frac{x^2 c_0}{30} + \frac{x^3 c_0}{630} + \frac{x^4 c_0}{22\,680} + \frac{x^5 c_0}{1\,247\,400} + \frac{x^6 c_0}{97\,297\,200} + \frac{x^7 c_0}{10\,216\,206\,000} + \frac{x^8 c_0}{1\,389\,404\,016\,000} + \frac{x^9 c_0}{237\,588\,086\,736\,000} + \ldots$$

$$f_1[x] = 1 + \frac{x}{3} + \frac{x^2}{30} + \frac{x^3}{630} + \frac{x^4}{22\,680} + \frac{x^5}{1\,247\,400} + \frac{x^6}{97\,297\,200} + \frac{x^7}{10\,216\,206\,000} + \frac{x^8}{1\,389\,404\,016\,000} + \frac{x^9}{237\,588\,086\,736\,000} + O[x]^{10}$$

$$f_1[x] = 1 + \frac{x}{3} + \frac{x^2}{30} + \frac{x^3}{630} + \frac{x^4}{22\,680} + \frac{x^5}{1\,247\,400} + \frac{x^6}{97\,297\,200} + \frac{x^7}{10\,216\,206\,000} + \frac{x^8}{1\,389\,404\,016\,000} + \frac{x^9}{237\,588\,086\,736\,000} + \ldots$$

$$s[x] = \frac{c_0}{\sqrt{x}} + c_1 \sqrt{x} + c_2 x^{3/2} + c_3 x^{5/2} + c_4 x^{7/2} + c_5 x^{9/2} + c_6 x^{11/2} + c_7 x^{13/2} + O[x]^{15/2}$$

$$s'[x] = -\frac{c_0}{2 x^{3/2}} + \frac{c_1}{2\sqrt{x}} + \frac{3 c_2 \sqrt{x}}{2} + \frac{5}{2} c_3 x^{3/2} + \frac{7}{2} c_4 x^{5/2} + \frac{9}{2} c_5 x^{7/2} + \frac{11}{2} c_6 x^{9/2} + \frac{13}{2} c_7 x^{11/2} + O[x]^{13/2}$$

$$s''[x] = \frac{3 c_0}{4 x^{5/2}} - \frac{c_1}{4 x^{3/2}} + \frac{3 c_2}{4\sqrt{x}} + \frac{15 c_3 \sqrt{x}}{4} + \frac{35}{4} c_4 x^{3/2} + \frac{63}{4} c_5 x^{5/2} + \frac{99}{4} c_6 x^{7/2} + \frac{143}{4} c_7 x^{9/2} + O[x]^{11/2}$$

Substitute it into

2 x s''[x]+3 s'[x]-s[x] == 0

to get

$$\frac{-c_0 + c_1}{\sqrt{x}} + (-c_1 + 6 c_2)\sqrt{x} + (-c_2 + 15 c_3) x^{3/2} + (-c_3 + 28 c_4) x^{5/2} + (-c_4 + 45 c_5) x^{7/2} + (-c_5 + 66 c_6) x^{9/2} + (-c_6 + 91 c_7) x^{11/2} + O[x]^{13/2} == 0$$

$$\frac{-c_0 + c_1}{\sqrt{x}} + (-c_1 + 6 c_2)\sqrt{x} + (-c_2 + 15 c_3) x^{3/2} + (-c_3 + 28 c_4) x^{5/2} + (-c_4 + 45 c_5) x^{7/2} + (-c_5 + 66 c_6) x^{9/2} + (-c_6 + 91 c_7) x^{11/2} + O[x]^{13/2} == 0$$

$-c_0 + c_1 == 0$
$-c_1 + 6 c_2 == 0$
$-c_2 + 15 c_3 == 0$
$-c_3 + 28 c_4 == 0$
$-c_4 + 45 c_5 == 0$
$-c_5 + 66 c_6 == 0$
$-c_6 + 91 c_7 == 0$

$c_1 \to c_0$
$c_2 \to \frac{c_0}{6}$
$c_3 \to \frac{c_0}{90}$
$c_4 \to \frac{c_0}{2520}$
$c_5 \to \frac{c_0}{113\,400}$
$c_6 \to \frac{c_0}{7\,484\,400}$
$c_7 \to \frac{c_0}{681\,080\,400}$

$$y = \frac{c_0}{\sqrt{x}} + c_0\sqrt{x} + \frac{1}{6}c_0 x^{3/2} + \frac{1}{90}c_0 x^{5/2} + \frac{c_0 x^{7/2}}{2520} + \frac{c_0 x^{9/2}}{113\,400} + \frac{c_0 x^{11/2}}{7\,484\,400} + \frac{c_0 x^{13/2}}{681\,080\,400} + O[x]^{15/2}$$

$$y = \frac{c_0}{\sqrt{x}} + \sqrt{x}\,c_0 + \frac{1}{6}x^{3/2}c_0 + \frac{1}{90}x^{5/2}c_0 + \frac{x^{7/2}c_0}{2520} + \frac{x^{9/2}c_0}{113\,400} + \frac{x^{11/2}c_0}{7\,484\,400} + \frac{x^{13/2}c_0}{681\,080\,400} + \ldots$$

$$y = c_0\frac{1}{\sqrt{x}}\left(1 + x + \frac{x^2}{6} + \frac{x^3}{90} + \frac{x^4}{2520} + \frac{x^5}{113\,400} + \frac{x^6}{7\,484\,400} + \frac{x^7}{681\,080\,400} + \ldots\right.$$

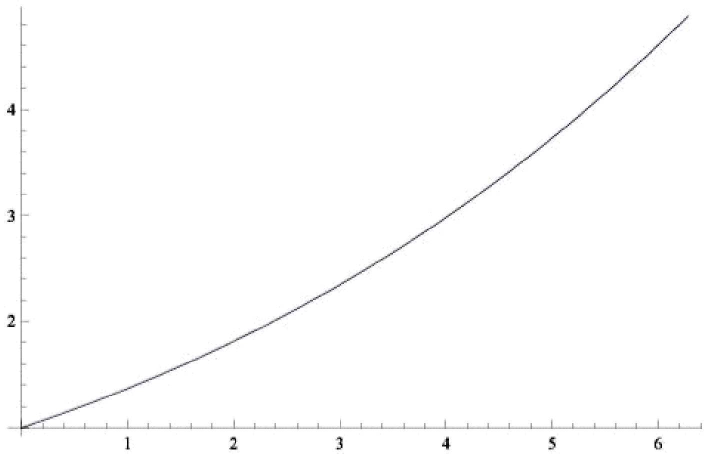
$$f_2[x] = \frac{1}{\sqrt{x}} + \sqrt{x} + \frac{x^{3/2}}{6} + \frac{x^{5/2}}{90} + \frac{x^{7/2}}{2520} + \frac{x^{9/2}}{113\,400} + \frac{x^{11/2}}{7\,484\,400} + \frac{x^{13/2}}{681\,080\,400} + O[x]^{15/2}$$

$$f_2[x] = \frac{1}{\sqrt{x}} + \sqrt{x} + \frac{x^{3/2}}{6} + \frac{x^{5/2}}{90} + \frac{x^{7/2}}{2520} + \frac{x^{9/2}}{113\,400} + \frac{x^{11/2}}{7\,484\,400} + \frac{x^{13/2}}{681\,080\,400} + \ldots$$

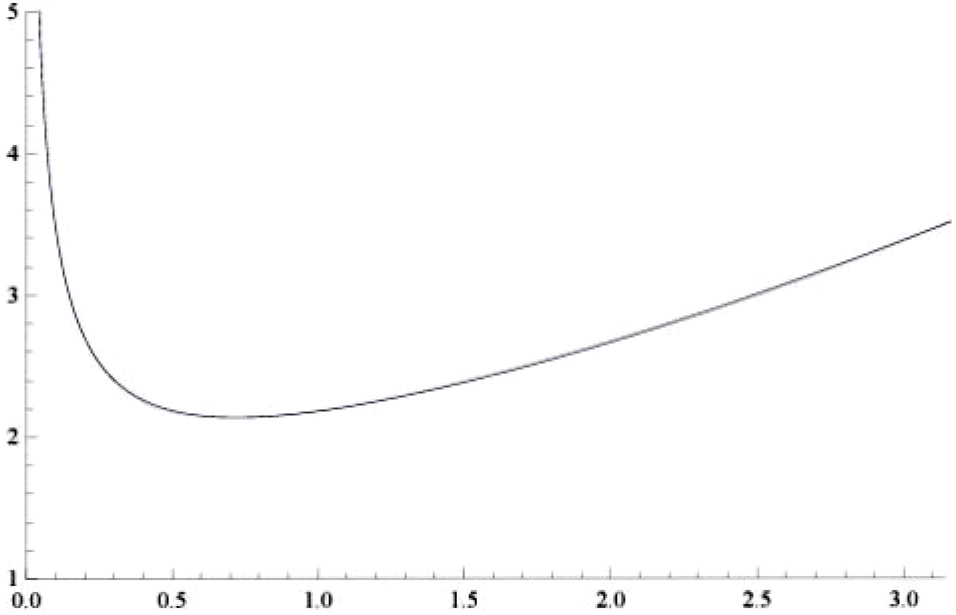
$$f_2[x] = \frac{1}{\sqrt{x}} + \sqrt{x} + \frac{x^{3/2}}{6} + \frac{x^{5/2}}{90} + \frac{x^{7/2}}{2520} + \frac{x^{9/2}}{113\,400} + \frac{x^{11/2}}{7\,484\,400} + \frac{x^{13/2}}{681\,080\,400} + O[x]^{15/2}$$

$$f_2[x] = \frac{1}{\sqrt{x}} + \sqrt{x} + \frac{x^{3/2}}{6} + \frac{x^{5/2}}{90} + \frac{x^{7/2}}{2520} + \frac{x^{9/2}}{113\,400} + \frac{x^{11/2}}{7\,484\,400} + \frac{x^{13/2}}{681\,080\,400} + \ldots$$

$$f_2[x] = \frac{1}{\sqrt{x}}\left(1 + x + \frac{x^2}{6} + \frac{x^3}{90} + \frac{x^4}{2520} + \frac{x^5}{113\,400} + \frac{x^6}{7\,484\,400} + \frac{x^7}{681\,080\,400} + \ldots\right)$$



$$s1[x] = 1 + \frac{x}{3} + \frac{x^2}{30} + \frac{x^3}{630} + \frac{x^4}{22\,680} + \frac{x^5}{1\,247\,400} + \frac{x^6}{97\,297\,200} + \frac{x^7}{10\,216\,206\,000} + \frac{x^8}{1\,389\,404\,016\,000} + \frac{x^9}{237\,588\,086\,736\,000}$$



$$s2[x] = \frac{1}{\sqrt{x}} + \sqrt{x} + \frac{x^{3/2}}{6} + \frac{x^{5/2}}{90} + \frac{x^{7/2}}{2520} + \frac{x^{9/2}}{113\,400} + \frac{x^{11/2}}{7\,484\,400} + \frac{x^{13/2}}{681\,080\,400}$$

PRACTICAL-5

5(a) : Monte Carlo Area Simulation

For one Variable:

Find the area beneath the curve y = Cos(x) over the interval $(-\pi/2) \leq x \leq (\pi/2)$.

In[148]:=

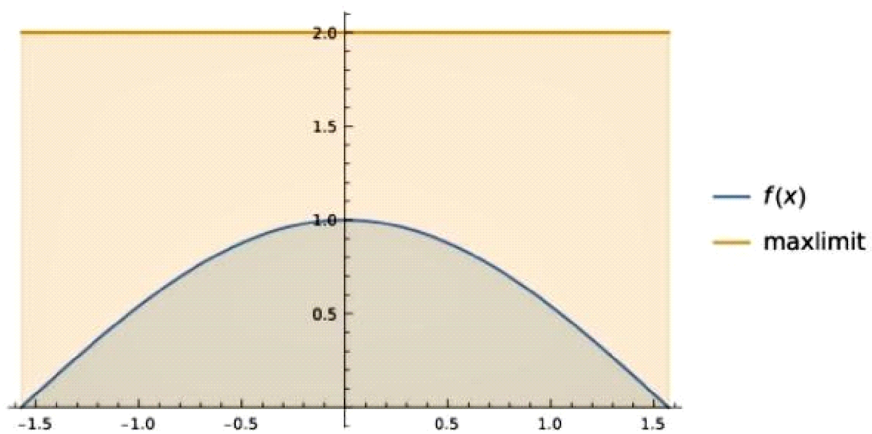
```
Clear[n, counter , a, b, f, c, d, maxlimit , area, x]
```

In[149]:=

```
n = Input["Enter the total number of
      random points to be generated in the simulation : "];
counter = 0;
a = - Pi / 2;
b = Pi / 2;
f[x_] = Cos[x];
c = Minimize [{f[x], x ≥ a && x ≤ b}, x];
d = Maximize [{f[x], x ≥ a && x ≤ b}, x];
maxlimit = d⟦1⟧ + 1
For[i = 1, i ≤ n, i++, x0 = RandomReal [{a, b}];
y0 = RandomReal [{0, maxlimit }];
If[y0 < f[x0], counter ++];];
area = maxlimit * (b - a) * counter / n;
Plot[{f[x], maxlimit}, {x, a, b}, Filling → Bottom, PlotLegends → "Expressions "]
Print["the number of points under the curve is : ", counter ];
Print[
  "The area of the region simulated by Monte Carlo area algoithm is : ", N[area]];
```

Out[156]= 2

Out[159]=




```
the number of points under the curve is : 229
The area of the region simulated by Monte Carlo area algoithm is : 1.88085
```

For more than Variable:

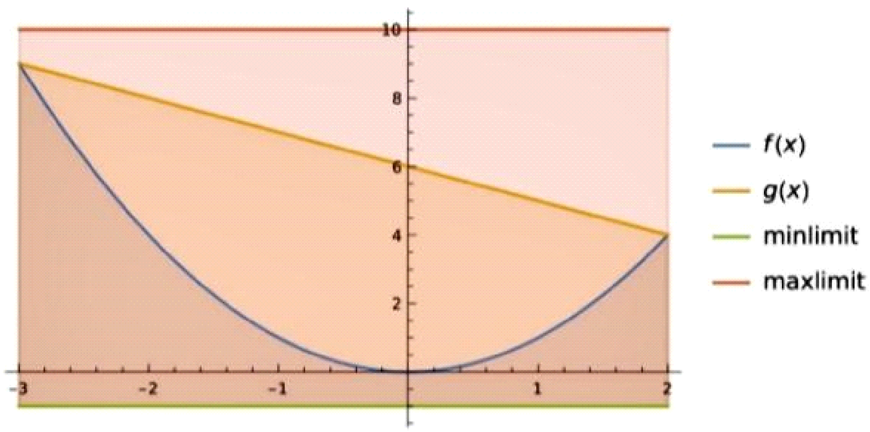
Use Monte Carlo Simulation to calculate the area trapped between the two curves

y = x^2 and y = 6-x .

In[198]:=
```
Clear[n, counter , a, b, f, c, d, maxlimit , area, x];
n = Input["Enter the total number of
      random points to be generated in the simulation : "];
counter = 0;
f[x_] = x ^ 2;
g[x_] = 6 - x;
solution = Solve[f[x] == g[x], x]
a = solution [[1, 1, 2]];
b = solution [[2, 1, 2]];
c1 = Minimize [{f[x], x ≥ a && x ≤ b}, x];
d1 = Maximize [{f[x], x ≥ a && x ≤ b}, x];
c2 = Minimize [{g[x], x ≥ a && x ≤ b}, x];
d2 = Maximize [{g[x], x ≥ a && x ≤ b}, x];
minlimit = Min[c1[[1]], c2[[1]]] - 1;
maxlimit = Max[d1[[1]], d2[[1]]] + 1;
For[i = 1, i ≤ n, i++, x0 = RandomReal [{a, b}];
y0 = RandomReal [{minlimit , maxlimit }];
If[Min[f[x0], g[x0]] < y0 && y0 < Max[f[x0], g[x0]], counter ++ ];];
area = (maxlimit - minlimit ) * (b - a) * counter / n;
Plot[{f[x], g[x], minlimit , maxlimit}, {x, a, b},
 Filling → Bottom , PlotLegends → "Expressions "]
Print["The total numer of points under the curve is : ", counter ];
Print[
  "The area of the region simulated by Monte Carlo area algorithm is : ", N[area]];
```

Out[203]= {{x → -3}, {x → 2}}

Out[214]=




```
The total numer of points under the curve is : 151
The area of the region simulated by Monte Carlo area algorithm is : 20.7625
```

5(b) : Monte Carlo Volume Simulation

For one Variable:

Find the volume of the sphere :

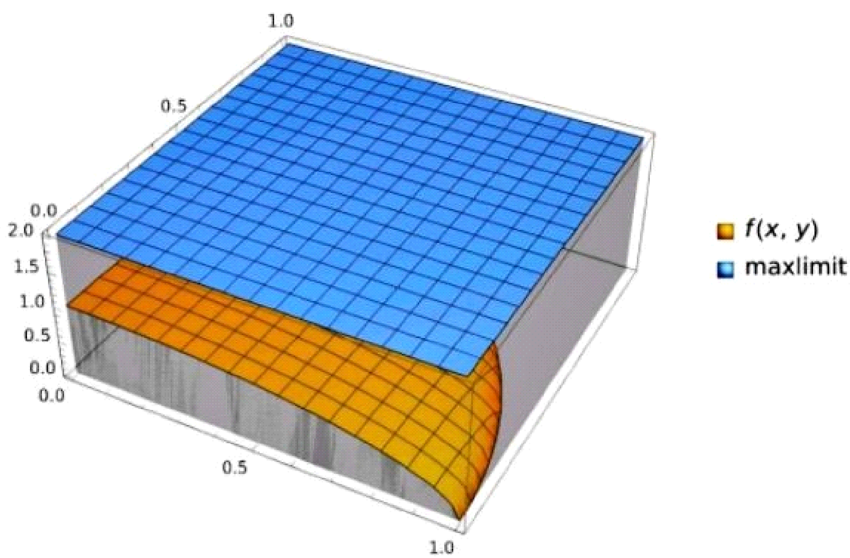
$$x^2 + y^2 + z^2 \le 1$$

in the first octant.

In[231]:=

```
Clear[n, counter , a, b, c, d, maxlimit , volume , x, y, z, x0, y0, z0];
n = Input[
   "Enter the total number of random points to be generated in the simulation "];
counter = 0;
a = 1;
b = 1;
f[x_, y_] = Sqrt[1 - x ^ 2 - y ^ 2];
c = Minimize [{f[x, y], x ≥ 0 && x ≤ a && y ≥ 0 && y ≤ b}, {x, y}];
d = Maximize [{f[x, y], x ≥ 0 && x ≤ a && y ≥ 0 && y ≤ b}, {x, y}];
maxlimit = d[[1]] + 1;
For[i = 1, i ≤ n, i++, x0 = RandomReal [{0, a}];
y0 = RandomReal [{0, b}];
z0 = RandomReal [{0, maxlimit }];
If[z0 ^ 2 < f[x0, y0]^ 2, counter ++];];
volume = maxlimit * b * a * counter / n;
Plot3D[{f[x, y], maxlimit}, {x, 0, a}, {y, 0, b},
 Filling → Bottom, PlotLegends → "Expressions "]
 Print["The number of points in the sphere is : ", counter ];
Print["The volume of the sphere in first
    octant simulated by Monte Carlo algorithm is :", N[volume]];
```

Out[242]=



```
The number of points in the sphere is : 117
The volume of the sphere in first octant simulated by Monte Carlo algorithm is :0.531818
```

For more than Variable:

Calculate the volume trapped between the two paraboloids z=8-x^2-y^2 and z=x^2+3y^2.

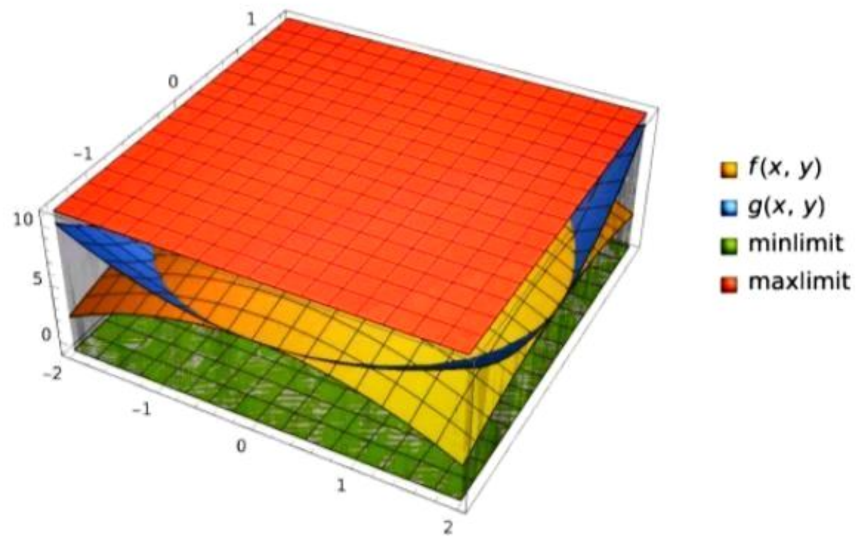
In[377]:=

```
Clear[n, counter , a, b, c, d, volume , maxlimit , minlimit , solution , x, y, z]
n = Input["Enter the total number of
     random points to be generated in the simulation :"];
counter = 0;
f[x_, y_] = 8 - x ^ 2 - y ^ 2;
g[x_, y_] = x ^ 2 + 3 y ^ 2;
solution = Solve[f[x, y] == g[x, y], {x, y}]
f1[x] = solution [[1, 1, 2]];
f2[x] = solution [[2, 1, 2]];
solution1 = Solve[f1[x] == f2[x], x]
a = solution1 [[1, 1, 2]];
b = solution1 [[2, 1, 2]];
c1y = Minimize [{f1[x], x ≥ a && x ≤ b}, x];
d1y = Maximize [{f1[x], x ≥ a && x ≤ b}, x];
c2y = Minimize [{f2[x], x ≥ a && x ≤ b}, x];
d2y = Maximize [{f2[x], x ≥ a && x ≤ b}, x];
minlimity = Min[c1y[[1]], c2y[[1]]];
maxlimity = Max[d1y[[1]], d2y[[1]]];
c1 = Minimize [{f[x, y], x ≥ a && x ≤ b && y ≤ maxlimity && y ≥ minlimity }, {x, y}];
d1 = Maximize [{f[x, y], x ≥ a && x ≤ b && y ≤ maxlimity && y ≥ minlimity }, {x, y}];
c2 = Minimize [{g[x, y], x ≥ a && x ≤ b && y ≤ maxlimity && y ≥ minlimity }, {x, y}];
d2 = Maximize [{g[x, y], x ≥ a && x ≤ b && y ≤ maxlimity && y ≥ minlimity }, {x, y}];
minlimit = Min[c1[[1]], c2[[1]]] - 1;
maxlimit = Max[d1[[1]], d2[[1]]] + 1;
For[i = 1, i ≤ n, i++, x0 = RandomReal [{a, b}];
y0 = RandomReal [{minlimity , maxlimity }];
z0 = RandomReal [{minlimit , maxlimit }];
If[Min[f[x0, y0], g[x0, y0]] < z0 < Max[f[x0, y0], g[x0, y0]], counter ++];];
volume = (maxlimit - minlimit ) * (maxlimity - minlimity ) * (b - a) * counter / n;
Plot3D[{f[x, y], g[x, y], minlimit , maxlimit}, {x, a, b},
 {y, minlimity , maxlimity }, Filling → Bottom , PlotLegends → "Expressions "]
Print["The total number of points under the curve is : ", counter ];
Print["The volume of the region simulated by Monte Carlo area algorithm is : ",
  N[volume ]];
```

Out[382]= $\left\{\left\{y \to -\frac{\sqrt{4-x^2}}{\sqrt{2}}\right\}, \left\{y \to \frac{\sqrt{4-x^2}}{\sqrt{2}}\right\}\right\}$

Out[385]= $\{\{x \to -2\}, \{x \to 2\}\}$

Out[402]=




```
The total number of points under the curve is : 111
The volume of the region simulated by Monte Carlo area algorithm is : 37.6746
```

PRACTICAL-6

Programming of single server queue (e.g. Harbor system algorithm)

Terms used in the algorithm :

betweeni = Time between successive arrival of ships i and i -1.(A random integer varying between 15 and 145 mm)
arrivei = Time from start of clock at t = 0 when ship i arrives at the harbor for unloading .
unloadi = time required to unload ship i at the dock (a random integer varying between 45 and 90 mm.)
starti = Time from start of clock at which ship i commences its unloading .
idlei = time for which dock facilities are idle immediately before commencement of unloading ship.
waiti = time ship i waits in the harbor after arrival before unloading commences .
finishi = time from start of clock at which service for ship i is completed at the unloading facilities .
harbori = total time ship i spends in the harbor .
HARTIME : Average time per ship in the harbor .
MAXHAR : maximum time of a ship in the harbor .
WAITTIME : average waiting time per ship before unloading .
MAX WAIT : maximum waiting time of a ship.
IDLETIME : Percentage of total simulation time unloading facilities are idle.

In[342]:=

```
n = Input["Enter the total number of ships for the the simulation . "];
Array[table , {5, 7}];
table[1, 1] = "Average time of ship in the harbor .";
table[2, 1] = "Maximum time of ship in the harbor .";
table[3, 1] = "Average waiting time of a ship.";
table[4, 1] = "Maximum waiting time of a ship.";
table[5, 1] = "Percentage of time dock facilities are idle.";
For[j = 1, j ≤ 6, j++, Subscript [between , 1] = RandomInteger [{15, 145}];
Subscript [unload , 1] = RandomInteger [{45, 90}];
Subscript [arrive , 1] = Subscript [between , 1];
HARTIME = Subscript [unload , 1];
MAXHAR = Subscript [unload , 1];
WAITTIME = 0;
MAXWAIT = 0;
IDLETIME = Subscript [arrive , 1];
Subscript [finish , 1] = Subscript [arrive , 1] + Subscript [unload , 1];
For[i = 2, i ≤ n, i++,
Subscript [between , i] = RandomInteger [{15, 145}];
Subscript [unload , i] = RandomInteger [{45, 90}];
Subscript [arrive , i] = Subscript [arrive , i - 1] + Subscript [between , i];
timediff = Subscript [arrive , i] - Subscript [finish , i - 1];
If[timediff ≥ 0, Subscript [idle, i] = timediff ;
Subscript [wait, i] = 0,
Subscript [wait, i] = - timediff ;
Subscript [idle, i] = 0];
Subscript [start , i] = Subscript [arrive , i] + Subscript [wait, i];
Subscript [finish , i] = Subscript [start , i] + Subscript [unload , i];
Subscript [harbor , i] = Subscript [wait, i] + Subscript [unload , i];
HARTIME += Subscript [harbor , i];
 If[Subscript [harbor , i] > MAXHAR , MAXHAR = Subscript [harbor , i]];
 WAITTIME += Subscript [wait, i];
 IDLETIME += Subscript [idle, i];
 If[Subscript [wait, i] > MAXWAIT , MAXWAIT = Subscript [wait, i]];];
 HARTIME /= n;
 WAITTIME /= n;
 IDLETIME /= Subscript [finish , n];
 table[1, j + 1] = Round[HARTIME ];
 table[2, j + 1] = MAXHAR ;
 table[3, j + 1] = Round[WAITTIME ];
 table[4, j + 1] = MAXWAIT ;
 table[5, j + 1] = N[IDLETIME ];];
 TableForm [Array[table , {5, 7}]]
```

Out[350]//TableForm=

Average time of ship in the harbor .	68	54	84	8
Maximum time of ship in the harbor .	76	69	124	1
Average waiting time of a ship.	0	2	9	1
Maximum waiting time of a ship.	0	7	35	5
Percentage of time dock facilities are idle.	0.457256	0.520642	0.392276	0

PRACTICAL-7

Solve the following linear programming problem
Minimize x + 2y
Subject to $-5x + y = 7$
$x + y \geq 26$
$x \geq 3$, $y \geq 4$

In[54]:=
```
Minimize[{x + 2 * y, -5 * x + y == 7 && x + y ≥ 26 && x ≥ 3 && y ≥ 4}, {x, y}]
```
Out[54]=
$$\left\{\frac{293}{6}, \left\{x \to \frac{19}{6}, y \to \frac{137}{6}\right\}\right\}$$

In[11]:=
```
f[{x_, y_}] = x + 2 y;
constraints[x_, y_] = {-5 x + y == 7, x + y ≥ 26, x ≥ 3, y ≥ 4};
solution = Minimize[f[{x, y}], constraints[x, y], {x, y}];
Print["Find the minimum of"]
Print["f[{x,y}]=", f[{x, y}]];
Print["Subject to the constraints :"];
Print[TableForm [constraints [x, y]]];
Print["The solution found by Mathematica is "];
Print[solution ];
```

```
Find the minimum of
f[{x,y}]=x + 2 y
Subject to the constraints :
-5 x + y == 7
x + y ≥ 26
x ≥ 3
y ≥ 4
The solution found by Mathematica is
```
$$\left\{\frac{293}{6}, \left\{x \to \frac{19}{6}, y \to \frac{137}{6}\right\}\right\}$$

Solve the following linear programming problem
Maximize 2x + y
Subject to $x + y \leq 6$
$x + y \geq 1$
$x \geq 0$, $y \geq 0$

```
In[205]:= Maximize[{2 * x + y, x + y ≤ 6 && x + y ≥ 1 && x ≥ 0 && y ≥ 0}, {x, y}]

Out[1]= {12, {x → 6, y → 0}}

In[20]:= f[{x_, y_}] = 2 x + y;
constraints[x_, y_] = {x + y ≤ 6, x + y ≥ 1, x ≥ 0, y ≥ 0};
solution = Maximize[f[{x, y}], constraints[x, y], {x, y}];
Print["Find the maximum of"]
Print["f[{x,y}]=", f[{x, y}]];
Print["Subject to the constraints :"];
Print[TableForm [constraints [x, y]]];
Print["The solution found by Mathematica is "];
Print[solution ];

Find the maximum of

f[{x,y}]=2 x + y

Subject to the constraints :

x + y ≤ 6
x + y ≥ 1
x ≥ 0
y ≥ 0

The solution found by Mathematica is

{12, {x → 6, y → 0}}
```