

ASSIGNMENT 2

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1 \documentclass{beamer}
2 \usepackage[utf8]{inputenc}
3 \usepackage{graphicx}
4 \usetheme{Warsaw}
5 \date{}
6 \setbeamertemplate{background}
7 {
8 \includegraphics[width=\paperwidth,height=\paperheight]{bg2.jpg}
9 }
10 \title{ASSIGNMENT 2}
11 \author{DEEPIKA SEHAJPAL \MAT/20/120 \ 20044563041}
12 \begin{document}
13 \begin{frame}
14 \begin{minipage}{0.11\linewidth}
15 \includegraphics[width=2cm,height=2cm]{mscw_logo.png}
16 \end{minipage}\hfill
17 \begin{minipage}{0.6\linewidth}
18 \centering
19 \textbf{\textrm{MATA SUNDRI COLLEGE FOR WOMEN}}\vspace{0.05in}
20 \textbf{\textrm{(UNIVERSITY OF DELHI)}}\vspace{0.05in}
21 \end{minipage}\hfill
22 \begin{minipage}{0.11\linewidth}
23 \includegraphics[width=2cm,height=2cm]{du.png}
24 \end{minipage}\hfill
25 \LARGE\textbf\titlepage
26 \end{frame}
27
28 \begin{frame}{Page 69 Part 1}
29 \large\textbf{1} Let  $\textbf{x}=(x_1, \dots, x_n)$ , where the  $x_i$  are nonnegative real
numbers. Set
30  $M_r(\textbf{x})=\left(\frac{x_1^r+x_2^r+\dots+x_n^r}{n}\right)^{1/r}$ ,  $r \in \textbf{R}$ 
\setminus \left\{0\right\},
31  $M_0$ 
32 and
33  $M_0(\textbf{x})=\left(x_1x_2\dots x_n\right)^{1/n}$ .
34  $M_0$ 
35 We call  $M_r(\textbf{x})$  the rth power mean of  $\textbf{x}$ .
36 Claim:
37  $\lim_{r \rightarrow 0} M_r(\textbf{x})=M_0(\textbf{x})$ .
38  $M_0$ 
39 \end{frame}
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40
41 ▾ \begin{frame}{Part 2}
42 \large\textbf{2} Define
43 \[V_n=\left[\begin{array}{ccccc}
44 1&1&1&\dots&1\\
45 x_1&x_2&x_3&\dots&x_n\\
46 x_1^2&x_2^2&x_3^2&\dots&x_n^2\\
47 \vdots & \vdots&\vdots&\ddots &\vdots\\
48 x_1^{n-1}&x_2^{n-1}&x_3^{n-1}&\dots&x_n^{n-1}\end{array}\right]\]
49 \end{array}\right]\]
50 We call  $V_n$  the Vandermonde Matrix of order  $n$ .
51 Claim:
52 \[\det V_n=\prod_{1\leq i<j\leq n}(x_j-x_i)\]
53 \end{frame}
54
55 ▾ \begin{frame}{Ques 4}
56 ▾ \begin{itemize}
57 \Large\item \[3^3+4^3+5^3=6^3\]
58 \item \[\sqrt{100}=10\]
59 \item \[(a+b)^3=a^3+3a^2b+3ab^2+b^3\]
60 \item \[\sum_{k=1}^n k=\frac{n(n+1)}{2}\]
61 \end{itemize}
62 \end{frame}
63
64 ▾ \begin{frame}
65 ▾ \begin{itemize}
66 \Large\item \[\frac{\pi}{4}=\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}
67 ]-\frac{1}{11}+\dots\]
68 \item \[\cos\theta=\sin(90^\circ-\theta)\]
69 \item \[e^{i\theta}=\cos\theta+i\sin\theta\]
70 \item \[\lim_{\theta\rightarrow 0} \frac{\sin\theta}{\theta}=1\]
71 \end{itemize}
72 \end{frame}
73 ▾ \begin{frame}
74 ▾ \begin{itemize}
75 \Large\item \[\lim_{x\rightarrow \infty} \frac{\pi(x)}{x/\log x}=1\]
76 \item \[\int_{-\infty}^{\infty} e^{-x^2} dx=\sqrt{\pi}\]
77 \end{itemize}
78 \end{frame}

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79
80 ▾ \begin{frame}{Ques 5}
81 ▾ \begin{itemize}
82     \large\item Positive numbers  $a$ ,  $b$  and  $c$  are the sides of a triangle if and only
      if  $a+b>c$  ,  $b+c>a$  , and  $c+a>b$ .
83     \item The area of a triangle with sides  $a$ ,  $b$ ,  $c$  is given by Heron's formula:
84      $A=\sqrt{s(s-a)(s-b)(s-c)}$  ,
85     where  $s$  is the semiperimeter  $(a+b+c)/2$ .
86     \item The volume of a regular tetrahedron of edge length 1 is  $\frac{\sqrt{2}}{12}$ .
87     \item The quadratic equation  $ax^2+bx+c=0$  has roots
88      $r_1, r_2= \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ 
89 \end{itemize}
90 \end{frame}
91
92 ▾ \begin{frame}
93 ▾ \begin{itemize}
94     \large\item The derivative of a function  $f$ , denoted  $f'$ , is defined by
       $f'(x)=\lim_{h\rightarrow 0} \frac{f(x+h)-f(x)}{h}$ 
95     \item A real-valued function  $f$  is convex on an interval  $I$  if
96      $f(\lambda x+(1-\lambda)y)\leq\lambda f(x)+(1-\lambda)f(y)$ ,
97     for all  $x,y \in I$  and  $0 \leq \lambda \leq 1$ .
98     \item The general solution to the differential equation
99      $y''-3y'+2y=0$ 
100     is  $y=C_1e^x+C_2e^{2x}$ .
101     \item The Fermat number  $F_n$  is defined as
102      $F_n=2^{2^n}+1$ ,  $n \geq 0$ .
103 \end{itemize}
104 \end{frame}
105
106 ▾ \begin{frame}{Ques 6}
107 ▾ \begin{itemize}
108     \item  $\frac{d}{dx} \left(\frac{x}{x+1}\right)=\frac{1}{(x+1)^2}$ 
109     \item  $\lim_{n \rightarrow \infty} \left(1+\frac{1}{n}\right)^n=e$ 
110     \item  $\lim_{n \rightarrow \infty} \left(1+\frac{1}{n}\right)^n=e$ 
111     \item  $\left[\begin{array}{rcl}
112         a&b&\\
113         c&d&\\
114         \end{array}\right]_{\text{right}}=ad-bc$ 
115     \item  $[R_\theta = \left[\begin{array}{rcl}
116         \cos \theta & -\sin \theta & \\
117         \sin \theta & \cos \theta & \\
118         \end{array}\right]_{\text{right}}]$ 
119 \end{itemize}

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119 \end{itemize}
120 \end{frame}
121
122 \begin{frame}
123 \begin{itemize}
124 \item \left | \begin{array}{ccc}
125 \textbf{i}&\textbf{j}&\textbf{k} \\
126 a_1&a_2&a_3 \\
127 b_1&b_2&b_3 \\
128 \end{array} \right| = \left| \begin{array}{cc}
129 a_2&a_3 \\
130 b_2&b_3 \\
131 \end{array} \right| \textbf{i} - \left| \begin{array}{cc}
132 a_1&a_3 \\
133 b_1&b_3 \\
134 \end{array} \right| \textbf{j} + \left| \begin{array}{cc}
135 a_1&a_2 \\
136 b_1&b_2 \\
137 \end{array} \right| \textbf{k} \\
138 \item \left [ \begin{array}{cc}
139 a_{11}&a_{12} \\
140 a_{21}&a_{22} \\
141 \end{array} \right] \left [ \begin{array}{cc}
142 b_{11}&b_{12} \\
143 b_{21}&b_{22} \\
144 \end{array} \right] = \left[ \begin{array}{ccc}
145 a_{11} b_{11}+a_{12} b_{21} & a_{11} b_{12}+a_{12} b_{22} \\
146 a_{21} b_{11}+a_{22} b_{21} & a_{21} b_{12}+a_{22} b_{22} \\
147 \end{array} \right] \\
148 \item [f(x) = \left\{ \begin{array}{cc}
149 -x^2, & x < 0 \\
150 x^2, & 0 \leq x \leq 2 \\
151 4, & x > 2 \\
152 \end{array} \right. \\
153 \end{itemize}
154 \end{frame}
155
156 \begin{frame}{Ques 7 Part 1}
157 \begin{itemize}
158 \item $$ \begin{array}{rcl}
159 1+2 & = & 3 \\
160 4+5+6 & = & 7+8 \\
161 9+10+11+12 & = & 13+14+15

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161 9+10+11+12&=&13+14+15\\
162 16+17+18+19+20&=&21+22+23+24\\
163 25+26+26+27+28+29+30&=&31+32+33+34+35\\
164 \end{array}$$
165 \end{itemize}
166 \end{frame}
167
168 \begin{frame}{Part 2}
169 \begin{itemize}
170 \Large\item \begin{eqnarray*}
171 (a+b)^2&=&(a+b)(a+b)\\
172 &=&(a+b)a+(a+b)b\\
173 &=&a(a+b)+b(a+b)\\
174 &=&a^2+ab+ba+b^2\\
175 &=&a^2+2ab+b^2
176 \end{eqnarray*}
177 \end{itemize}
178 \end{frame}
179
180 \begin{frame}{Part 3}
181 \begin{itemize}
182 \large\item \begin{eqnarray*}
183 \tan(\alpha + \beta) &=& \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha) \tan(\beta)} \\
184 &=& \frac{\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} + \tan \gamma}{1 - (\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}) \tan \gamma} \\
185 &=& \frac{\tan \alpha + \tan \beta + (1 - \tan \alpha \tan \beta) \tan \gamma}{\tan \beta - (\tan \alpha + \tan \beta) \tan \gamma} \\
186 &=& \frac{\tan \alpha + \tan \beta + \tan \gamma - \tan \alpha \tan \beta \tan \gamma}{\alpha \tan \beta - \tan \alpha \tan \gamma + \tan \beta \tan \gamma} \\
187 \end{eqnarray*}
188 \end{itemize}
189 \end{frame}
190
191 \begin{frame}{Part 4}
192 \begin{itemize}
193 \large\item \begin{eqnarray*}
194 \prod_{p} \left(1 - \frac{1}{p^2}\right) &=& \prod_{p} \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots} \\
195 &=& \left(\prod_{p} \left(1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots\right)\right)^{-1} \\
196 &=& \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots\right)^{-1} \\
197 &=& \frac{6}{\pi^2}

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196 &=&\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}\cdots\right)^{-1}\\
197 &=&\frac{6}{\pi^2}\\
198 \end{eqnarray*}
199 \end{itemize}
200 \end{frame}
201
202 \begin{frame}
203 \includegraphics[width=11cm,height=8cm]{thankyou.jpg}
204 \end{frame}
205 \end{document}
206
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**MATA SUNDRI COLLEGE
FOR WOMEN
(UNIVERSITY OF DELHI)**



ASSIGNMENT 2

**DEEPIKA SEHAJPAL
MAT/20/120
20044563041**

1) Let $\mathbf{x} = (x_1, \dots, x_n)$, where the x_i are nonnegative real numbers. Set

$$M_r(\mathbf{x}) = \left(\frac{x_1^r + x_2^r + \dots + x_n^r}{n} \right)^{1/r}, r \in \mathbf{R} \setminus \{0\},$$

and

$$M_0(\mathbf{x}) = (x_1 x_2 \dots x_n)^{1/n}.$$

We call $M_r(\mathbf{x})$ the *r*th power mean of \mathbf{x} .

Claim:

$$\lim_{r \rightarrow 0} M_r(\mathbf{x}) = M_0(\mathbf{x}).$$

2) Define

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 & \dots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1} \end{bmatrix}$$

We call V_n the *Vandermonde Matrix* of order n .

Claim:

$$\det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i)$$

Ques 4



$$3^3 + 4^3 + 5^3 = 6^3$$



$$\sqrt{100} = 10$$



$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$



$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

- $$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots$$

- $$\cos \theta = \sin(90^\circ - \theta)$$

- $$e^{i\theta} = \cos \theta + i \sin \theta$$

- $$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$$

-
-

$$\lim_{x \rightarrow \infty} \frac{\pi(x)}{x/\log x} = 1$$

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

Ques 5

- Positive numbers a , b and c are the sides of a triangle if and only if $a + b > c$, $b + c > a$, and $c + a > b$.
- The area of a triangle with sides a , b , c is given by Heron's formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where s is the semiperimeter $(a + b + c)/2$.

- The volume of a regular tetrahedron of edge length 1 is $\frac{\sqrt{2}}{12}$.
- The quadratic equation $ax^2 + bx + c = 0$ has roots

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The derivative of a function f , denoted f' , is defined by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- A real-valued function f is *convex* on an interval I if

$$f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y),$$

for all $x, y \in I$ and $0 \leq \lambda \leq 1$.

- The general solution to the differential equation

$$y'' - 3y' + 2y = 0$$

is

$$y = C_1 e^x + C_2 e^{2x}.$$

- The *Fermat number* F_n is defined as

$$F_n = 2^{2^n}, n \geq 0.$$

Ques 6

- $$\frac{d}{dx} \left(\frac{x}{x+1} \right) = \frac{1}{(x+1)^2}$$

- $$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^n = e$$

- $$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^n = e$$

- $$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

- $$R_\theta = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

$$f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & 0 \leq x \leq 2 \\ 4, & x > 2 \end{cases}$$

Ques 7 Part 1



$$1 + 2 = 3$$

$$4 + 5 + 6 = 7 + 8$$

$$9 + 10 + 11 + 12 = 13 + 14 + 15$$

$$16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24$$

$$25 + 26 + 26 + 27 + 28 + 29 + 30 = 31 + 32 + 33 + 34 + 35$$



$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) \\ &= (a + b)a + (a + b)b \\ &= a(a + b) + b(a + b) \\ &= a^2 + ab + ba + b^2 \\ &= a^2 + 2ab + b^2\end{aligned}$$



$$\begin{aligned}
 \tan(\alpha + \beta + \gamma) &= \frac{\tan(\alpha + \beta) + \tan \gamma}{1 - \tan(\alpha + \beta) \tan \gamma} \\
 &= \frac{\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} + \tan \gamma}{1 - \left(\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}\right) \tan \gamma} \\
 &= \frac{\tan \alpha + \tan \beta + (1 - \tan \alpha \tan \beta) \tan \gamma}{1 - \tan \alpha \tan \beta - (\tan \alpha + \tan \beta) \tan \gamma} \\
 &= \frac{\tan \alpha + \tan \beta + \tan \gamma - \tan \alpha \tan \beta \tan \gamma}{1 - \tan \alpha \tan \beta - \tan \alpha \tan \gamma + \tan \beta \tan \gamma}
 \end{aligned}$$



$$\begin{aligned}\prod_p \left(1 - \frac{1}{p^2}\right) &= \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots} \\ &= \left(\prod_p \left(1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots\right) \right)^{-1} \\ &= \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \dots\right)^{-1} \\ &= \frac{6}{\pi^2}\end{aligned}$$

Thank You