Deepika Sehajpal
MAT/20/120
20044563041

ASSIGNMENT 2

```
\documentclass{beamer}
\usepackage[utf8]{inputenc}
\usepackage{graphicx}
\usetheme{Warsaw}
\date{}
\setbeamertemplate{background}
{
\includegraphics[width=\paperwidth,height=\paperheight]{bg2.jpg}
}
\title{ASSIGNMENT 2}
\author{DEEPIKA SEHAJPAL \\MAT/20/120 \\ 20044563041}
\begin{document}
\begin{frame}
\begin{minipage}{0.11\linewidth}
\includegraphics[width=2cm,height=2cm]{mscw logo.png}
\end{minipage}\hfill
\begin{minipage}{0.6\linewidth}
\centering
\textbf{\textrm{MATA SUNDRI COLLEGE FOR WOMEN}}\\\vspace{0.05in}
\textbf{\textrm{(UNIVERSITY OF DELHI)}}\\
\end{minipage}\hfill
\begin{minipage}{0.11\linewidth}
\includegraphics[width=2cm,height=2cm]{du.png}
\end{minipage}\hfill
\LARGE\textbf\titlepage
\end{frame}

\begin{frame}{Page 69 Part 1}
\large\textbf{1)} Let $\textbf{x}=(x_1,\ldots,x_n),$ where the $x_i$ are nonnegative real numbers. Set
$$M_r(\textbf{x})=\left(\frac{x_1^r+x_2^2+\cdots+x_n^r}{n}\right)^{1/r}, r\in \textbf{R} \setminus \left\{0\right\},
$$
and
$$M_0(\textbf{x})=\left(x_1x_2\ldots x_n \right)^{1/n}.
$$
We call $M_r(\textbf{x})$ the \emph{rth power mean of} \textbf{x}.\\
Claim:
$$\lim_{r\rightarrow 0}M_r(\textbf{x})=M_0(\textbf{x}).
$$
\end{frame}
```

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\begin{frame}{Part 2}
\large\textbf{2)} Define
\[V_n=\left[\begin{array}{ccccc}
1&1&1&\ldots&1\\
x_1&x_2&x_3&\ldots&x_n\\
x_1^2&x_2^2&x_3^2&\ldots&x_n^2\\
\vdots & \vdots&\vdots&\ddots &\vdots\\
x_1^{n-1}&x_2^{n-1}&x_3^{n-1}&\ldots&x_n^{n-1}\\
\end{array}\right]\]
We call $V_n$ the \emph{Vandermonde Matrix} of order $n$.\\
Claim:
\[\det   V_n=\prod_{1\leq i<j\leq n}(x_j-x_i)\]
\end{frame}

\begin{frame}{Ques 4}
\begin{itemize}
    \Large\item \[3^3+4^3+5^3=6^3\]
    \item \[\sqrt{100}=10\]
    \item \[(a+b)^3=a^3+3a^2b+3ab^2+b^3\]
    \item \[\sum_{k=1}^{n}k=\frac{n(n+1)}{2}\]
\end{itemize}
\end{frame}

\begin{frame}
\begin{itemize}
    \Large\item \[\frac{\pi}{4}=\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\cdots\]
    \item \[\cos\theta=\sin(90^o-\theta)\]
    \item \[e^{\iota\theta}=\cos\theta+\iota \sin\theta\]
    \item \[\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}=1\]
\end{itemize}
\end{frame}

\begin{frame}
\begin{itemize}
    \Large\item \[\lim_{x \rightarrow \infty} \frac{\pi (x)}{x/log x}=1\]
    \item \[\int_{\infty}^{-\infty}e^{-x^2} dx=\sqrt{\pi}\]
\end{itemize}
\end{frame}
```

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\begin{frame}{Ques 5}
\begin{itemize}
    \large\item Positive numbers $a$, $b$ and $c$ are the sides of a triangle if and only if $a+b>c$ , $b+c>a$ , and $c+a>b$.
    \item The area of a triangle with sides a, b, c is given by Heron's formula:
    $$A=\sqrt{s(s-a)(s-b)(s-c)} ,$$
    where s is the semiperimeter $(a+b+c)/2$.
    \item The volume of a regular tetrahedron of edge length 1 is $\frac{\sqrt{2}}{12}$.
    \item The quadratic equation $ax^2+bx+c=0$ has roots
    $$r_1, r_2= \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
\end{itemize}
\end{frame}

\begin{frame}
\begin{itemize}
    \large\item The derivative of a function $f$, denoted $f'$, is defined by $$f'(x)=\lim_{h\rightarrow 0} \frac{f(x+h)-f(x)}{h}$$
    \item A real-valued function $f$ is $convex$ on an interval $I$ if
    $$f(\lambda x+(1-\lambda)y)\leq\lambda f(x)+(1-\lambda)f(y),$$
    for all $x,y \in I$ and $0 \leq \lambda\leq 1.$
    \item The general solution to the differential equation
    $$y''-3y'+2y=0$$
    is $$y=C_1e^x+C_2e^{2x}.$$
    \item The \emph{Fermat number} $F_n$ is defined as
    $$F_n=2^{2^{n}}, n\geq 0.$$
\end{itemize}
\end{frame}

\begin{frame}{Ques 6}
\begin{itemize}
    \item \[\frac{d}{dx} \left(\frac{x}{x+1}\right)=\frac{1}{(x+1)^2}\]
    \item \[\lim_{n \rightarrow \infty}\left(1+\frac{1}{n}\right)^n=e\]
    \item \[\lim_{n \rightarrow \infty}\left(1+\frac{1}{n}\right)^n=e\]
    \item \[\left|\begin{array}{rcl}
    a&b\\
    c&d\\
    \end{array}\right|=ad-bc\]
    \item \[R_\theta= \left[\begin{array}{rcl}
    \cos \theta & -\sin \theta\\
    \sin \theta & \cos \theta\\
    \end{array} \right]\]
\end{itemize}
```

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\end{itemize}
\end{frame}

\begin{frame}
\begin{itemize}
    \item \[\left |\begin{array}{ccc}
    \textbf{i}&\textbf{j}&\textbf{k}\\
    a_1&a_2&a_3\\
    b_1&b_2&b_3\\
    \end{array} \right|=\left|\begin{array}{cc}
    a_2&a_3\\
    b_2&b_3\\
    \end{array} \right| \textbf{i} - \left|\begin{array}{cc}
    a_1&a_3\\
    b_1&b_3\\
    \end{array}\right| \textbf{j} + \left|\begin{array}{cc}
    a_1&a_2\\
    b_1&b_2\\
    \end{array} \right| \textbf{k} \]
    \item \[\left [\begin{array}{cc}
    a_{11}&a_{12}\\
    a_{21}&a_{22}\\
    \end{array} \right] \left [\begin{array}{cc}
    b_{11}&b_{12}\\
    b_{21}&b_{22}\\
    \end{array} \right] = \left[\begin{array}{ccc}
    a_{11} b_{11}+a_{12} b_{21} & a_{11} b_{12}+a_{12} b_{22}\\
    a_{21} b_{11}+a_{22} b_{21} & a_{21} b_{12}+a_{22} b_{22}\\
    \end{array}\right]\]
    \item \[f(x)=\left\{\begin{array}{cc}
    -x^2, &x<0\\
    x^2, &0\leq x \leq 2\\
    4, &x>2
    \end{array}\right.\]
\end{itemize}
\end{frame}

\begin{frame}{Ques 7 Part 1}
\begin{itemize}
    \item $$\begin{array}{rcl}
1+2&=&3\\
4+5+6&=&7+8\\
9+10+11+12&=&13+14+15\\
```

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9+10+11+12&=&13+14+15\\
16+17+18+19+20&=&21+22+23+24\\
25+26+26+27+28+29+30&=&31+32+33+34+35\\
\end{array}$$
\end{itemize}
\end{frame}

\begin{frame}{Part 2}
\begin{itemize}
    \Large\item \begin{eqnarray*}
(a+b)^2&=&(a+b)(a+b)\\
&=&(a+b)a+(a+b)b\\
&=&a(a+b)+b(a+b)\\
&=&a^2+ab+ba+b^2\\
&=&a^2+2ab+b^2
\end{eqnarray*}
\end{itemize}
\end{frame}

\begin{frame}{Part 3}
\begin{itemize}
    \large\item \begin{eqnarray*}
\tan(\alpha+ \beta +\gamma)&=&\frac{\tan(\alpha + \beta)+\tan \gamma}{1-\tan(\alpha + \beta) \tan \gamma}\\
&=&\frac{\frac{\tan \alpha +\tan \beta}{1-\tan \alpha \tan \beta}+\tan \gamma}{1-(\frac{\tan \alpha +\tan \beta}{1-\tan \alpha \tan \beta})\tan \gamma}\\
&=&\frac{\tan \alpha +\tan \beta +(1-\tan \alpha \tan \beta)\tan \gamma}{1-\tan \alpha \tan \beta -(\tan \alpha +\tan \beta)\tan \gamma}\\
&=&\frac{\tan \alpha +\tan \beta +\tan \gamma -\tan \alpha \tan \beta \tan \gamma}{1-\tan \alpha \tan \beta -\tan \alpha \tan \gamma +\tan \beta \tan \gamma}\\
\end{eqnarray*}
\end{itemize}
\end{frame}

\begin{frame}{Part 4}
\begin{itemize}
    \large\item \begin{eqnarray*}
\prod_{p}\left(1-\frac{1}{p^2}\right)&=&\prod_{p}\frac{1}{1+\frac{1}{p^2}+\frac{1}{p^4}+\cdots}\\
&=&\left(\prod_{p}\left(1+\frac{1}{p^2}+\frac{1}{p^4}+\cdots\right)\right)^{-1}\\
&=&\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}\cdots\right)^{-1}\\
&=&\frac{6}{\pi^2}\\
```

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&=&\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}\cdots\right)^{-1}\\
&=&\frac{6}{\pi^2}\\
\end{eqnarray*}
\end{itemize}
\end{frame}

\begin{frame}
\includegraphics[width=11cm,height=8cm]{thankyou.jpg}
\end{frame}
\end{document}

```

MATA SUNDRI COLLEGE
FOR WOMEN
(UNIVERSITY OF DELHI)

ASSIGNMENT 2

DEEPIKA SEHAJPAL
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$$M_r(\mathbf{x}) = \left(\frac{x_1^r + x_2^2 + \cdots + x_n^r}{n} \right)^{1/r}, r \in \mathbf{R} \setminus \{0\},$$

and

$$M_0(\mathbf{x}) = (x_1 x_2 \ldots x_n)^{1/n}.$$

We call $M_r(\mathbf{x})$ the *rth power mean of* $\mathbf{x}$.
Claim:

$$\lim_{r \to 0} M_r(\mathbf{x}) = M_0(\mathbf{x}).$$

Part 2

2) Define

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 & \dots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1} \end{bmatrix}$$

We call V_n the *Vandermonde Matrix* of order n.
Claim:

$$\det V_n = \prod_{1 \le i < j \le n} (x_j - x_i)$$

Ques 4

- $$3^3 + 4^3 + 5^3 = 6^3$$
- $$\sqrt{100} = 10$$
- $$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
- $$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

- $$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots$$

- $$\cos\theta = \sin(90^o - \theta)$$

- $$e^{\iota\theta} = \cos\theta + \iota\sin\theta$$

- $$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$$

- $$\lim_{x\to\infty} \frac{\pi(x)}{x/logx} = 1$$
- $$\int_{\infty}^{-\infty} e^{-x^2} dx = \sqrt{\pi}$$

Ques 5

- Positive numbers a, b and c are the sides of a triangle if and only if $a+b>c$, $b+c>a$, and $c+a>b$.
- The area of a triangle with sides a, b, c is given by Heron's formula:

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$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda) f(y),$$

for all $x, y \in I$ and $0 \leq \lambda \leq 1$.

- The general solution to the differential equation

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is

$$y = C_1 e^x + C_2 e^{2x}.$$

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$$F_n = 2^{2^n}, n \geq 0.$$

Ques 6

- $$\frac{d}{dx}\left(\frac{x}{x+1}\right) = \frac{1}{(x+1)^2}$$
- $$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$
- $$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$
- $$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$
- $$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

- $$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

- $$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

- $$f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & 0 \leq x \leq 2 \\ 4, & x > 2 \end{cases}$$

Ques 7 Part 1

- $$\begin{aligned} 1+2 &= 3 \\ 4+5+6 &= 7+8 \\ 9+10+11+12 &= 13+14+15 \\ 16+17+18+19+20 &= 21+22+23+24 \\ 25+26+26+27+28+29+30 &= 31+32+33+34+35 \end{aligned}$$

Part 2

- $$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= (a+b)a+(a+b)b\\ &= a(a+b)+b(a+b)\\ &= a^2+ab+ba+b^2\\ &= a^2+2ab+b^2\end{aligned}$$

Part 3

- $$\begin{aligned}\tan(\alpha+\beta+\gamma) &= \frac{\tan(\alpha+\beta)+\tan\gamma}{1-\tan(\alpha+\beta)\tan\gamma}\\ &= \frac{\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}+\tan\gamma}{1-(\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta})\tan\gamma}\\ &= \frac{\tan\alpha+\tan\beta+(1-\tan\alpha\tan\beta)\tan\gamma}{1-\tan\alpha\tan\beta-(\tan\alpha+\tan\beta)\tan\gamma}\\ &= \frac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\beta-\tan\alpha\tan\gamma+\tan\beta\tan\gamma}\end{aligned}$$

- $$\begin{aligned}\prod_p \left(1 - \frac{1}{p^2}\right) &= \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots} \\ &= \left(\prod_p \left(1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots\right)\right)^{-1} \\ &= \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \cdots\right)^{-1} \\ &= \frac{6}{\pi^2}\end{aligned}$$


Thank
You