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Example 9.6

1. Let $\mathrm{x} = (x_1, \ldots, x_n)$, where the x_i are nonnegative real numbers. Set

$$M_r(\mathrm{x}) = \left(\frac{x_1^r + x_2^r + \cdots + x_n^r}{n} \right)^{1/r}, \quad r \in \mathrm{R} \setminus \{0\},$$

and

$$M_0(\mathrm{x}) = (x_1 x_2 \ldots x_n)^{1/n}.$$

We call $M_r(\mathrm{x})$ the *rth power mean* of x. Claim:

$$\lim_{r \to 0} M_r(\mathrm{x}) = M_0(\mathrm{x}).$$

1. Define

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 & \dots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1} \end{bmatrix}.$$

We call V_n the *Vandermonde matrix* of order n. Claim:

$$\det V_n = \prod_{1 \le i < j \le n} (x_j - x_i).$$

Question 4

$$
\begin{aligned}
3^3 + 4^3 + 5^3 &= 6^3 \\
\sqrt{100} &= 10 \\
(a+b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\
\sum_{k=1}^{n} k &= \frac{n(n+1)}{2} \\
\frac{\pi}{4} &= \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots
\end{aligned}
$$

$$\lim_{\theta \to 0} \frac{Sin\theta}{\theta} = 1$$

$$\lim_{x \to \infty} \frac{pi(x)}{x/log(x)} = 1$$

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{pi}$$

Question 5

1. Positive numbers a,b,c are the side lengths of a triangle if and only if $a + b > c, b + c > a, c + a > b$
2. The area of a triangle with side lenghts a,b,c is given by *Heron's formula*:

   $$\mathrm{A} = \sqrt{s(s-a)(s-b)(s-c)},$$

   where$s$ is the semiperimeter (a+b+c)/2.
3. The quadratic equation $ax^2 + bx + c = 0$ has roots

   $$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

.

1. The derivative of a function f,denotedf',is defined by,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

2. A real-valued function f is *convex* on an interval I if

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda) f(y)$$

,

$$forall x, y \in land 0 \leq \lambda \leq 1$$

.

3. The general solution to the differential equation

$$y" - 3y' + 2y = 0$$

is

$$y = C_1 e^x + C_2 e^{2x}.$$

4. The Fermat numberF_n is defined as

$$F_n = 2^{2^n}$$

, n¿0.

Question 6

- $$\frac{d}{dx}\left(\frac{x}{x+1}\right) = \frac{1}{(x+1)^2}$$

- $$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$

- $$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

- $$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

- $$\begin{vmatrix} i & j & k \\ a_1 & b_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_1 & b_3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} a_1 & b_3 \\ b_1 & b_3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\mathbf{k}$$

- $$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

- $$f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & 0 \leq x \leq 2 \\ 4, & x > 2 \end{cases}$$

Question 7(MULTI LINE EQUATIONS)

1 st part

$1+2 = 3$

$4+5+6 = 7+8$

$9+10+11+12 = 13+14+15$

$16+17+18+19+20 = 21+22+23+24$

$25+26+27+28+29+30 = 31+32+33+34+35$

2nd part

$$
\begin{aligned}
(a+b)^2 &= (a+b)(a+b) \\
&= (a+b)a + (a+b)b \\
&= a(a+b) + b(a+b) \\
&= a^2 + ab + ba + b^2 \\
&= a^2 + ab + ab + b^2 \\
&= a^2 + 2ab + b^2
\end{aligned}
$$

3rd part

$$tan(\alpha+\beta+\gamma) = \frac{tan(\alpha+\beta)+tan\gamma}{1-tan(\alpha+\beta)tan\gamma}$$

$$= \frac{\frac{tan\alpha+tan\beta}{1-tan\alpha tan\beta}+tan\gamma}{1-(\frac{tan\alpha+tan\beta}{1-tan\alpha tan\beta})tan\gamma}$$

$$= \frac{tan\alpha+tan\beta+(1-tan\alpha tan\beta)tan\gamma}{1-tan\alpha tan\beta-(tan\alpha+tan\beta)tan\gamma}$$

$$= \frac{tan\alpha+tan\beta+tan\gamma-tan\alpha tan\beta tan\gamma}{1-tan\alpha tan\beta-tan\alpha tan\gamma-tan\beta tan\gamma}$$

4th part

$$\prod_p \left(1 - \frac{1}{p^2}\right) = \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots}$$
$$= \left(\left(\prod_p (1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots\right)\right)^{-1}$$
$$= \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\right)^{-1}$$
$$= \frac{6}{\pi^2}$$