



MATA SUNDRI COLLEGE FOR WOMEN
UNIVERSITY OF DELHI



ASSIGNMENT2

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MAT/20/102
20044563030

PG-69 ; PART-1

1. Let $x = (x_1, \dots, x_n)$, where the x_i are nonnegative real numbers. Set

$$M_r(x) = \left(\frac{x_1^r + x_2^r + \dots + x_n^r}{n} \right)^{\frac{1}{r}}, r \in R \setminus \{0\},$$

and

$$M_0(x) = (x_1 x_2 \dots x_n)^{\frac{1}{n}}$$

We call $M_r(x)$ the r th power mean of x .

Claim:

$$\lim_{r \rightarrow 0} M_r(x) = M_0(x)$$

2. Define

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ x_1 & x_2 & x_3 & \cdots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \cdots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \cdots & x_n^{n-1} \end{bmatrix}$$

We call V_n the Vandermonde matrix of order n .

Claim:

$$\det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i)$$

QUESTION-4



$$3^2 + 4^3 + 5^6 = 6^3$$



$$\sqrt{100} = 10$$



$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

QUESTION-4

- $$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

- $$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots$$

- $$\cos \theta = \sin(90^\circ - \theta)$$

- $$e^{i\theta} = \cos \theta + i \sin \theta$$

QUESTION-4



$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$$



$$\lim_{x \rightarrow \infty} \frac{\pi(x)}{\frac{x}{\log x}} = 1$$



$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

QUESTION-5

- Positive numbers a , b , and c are the side lengths of a triangle if and only if $a + b > c$, $b + c > a$, $c + a > b$.
- The area of a triangle with side lengths a , b , c is given by Heron's Formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where s is the semiperimeter $\frac{(a+b+c)}{2}$.

- The volume of a regular tetrahedron of edge length 1 is $\frac{\sqrt{2}}{12}$.
- The Quadratic equation $ax^2 + bx + c = 0$ has roots

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

QUESTION-5

- The derivative of a function f , denoted f' , is defined by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- A real-valued function f is convex on an interval I if

$$f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y),$$

for all $x, y \in I$ and $0 \leq \lambda \leq 1$.

- The general solution of the differential equation

$$y'' - 3y' + 2y = 0$$

is

$$y = C_1 e^x + C_2 e^{2x}.$$

- The Fermat number F_n is defined as

$$F_n = 2^{2^n}, n \geq 0.$$

QUESTION-6

- $$\frac{d}{dx} \left(\frac{x}{x+1} \right) = \frac{1}{(x+1)^2}$$

- $$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^n = e$$

- $$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

- $$R_\theta = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

QUESTION-6

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

$$f(x) = \begin{cases} -x^2 & , x < 0 \\ x^2 & , 0 \leq x \leq 2 \\ 4 & , x > 2 \end{cases}$$

QUESTION-7 (A)

$$1 + 2 = 3$$

$$4 + 5 + 6 = 7 + 8$$

$$9 + 10 + 11 + 12 = 13 + 14 + 15$$

$$16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24$$

$$25 + 26 + 27 + 28 + 29 + 30 = 31 + 32 + 33 + 34 + 35$$

QUESTION-7(B)

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) \\ &= (a + b)a + (a + b)b \\ &= a(a + b) + b(a + b) \\ &= a^2 + ab + ba + b^2 \\ &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2\end{aligned}$$

QUESTION-7(C)

$$\begin{aligned}\tan(\alpha + \beta + \gamma) &= \frac{\tan(\alpha + \beta) + \tan \gamma}{1 - \tan(\alpha + \beta) \tan \gamma} \\ &= \frac{\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} + \tan \gamma}{1 - \left(\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \right) \tan \gamma} \\ &= \frac{\tan \alpha + \tan \beta + (1 - \tan \alpha \tan \beta) \tan \gamma}{1 - \tan \alpha \tan \beta - (\tan \alpha + \tan \beta) \tan \gamma} \\ &= \frac{\tan \alpha + \tan \beta + \tan \gamma - \tan \alpha \tan \beta \tan \gamma}{1 - \tan \alpha \tan \beta - \tan \alpha \tan \gamma - \tan \beta \tan \gamma}\end{aligned}$$

QUESTION-7(D)

$$\begin{aligned}\prod_p \left(1 - \frac{1}{p^2}\right) &= \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots} \\ &= \left(\prod_p \left(1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots\right) \right)^{-1} \\ &= \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots\right)^{-1} \\ &= \frac{6}{\pi^2}\end{aligned}$$

Thank you!



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1 \documentclass{beamer}
2 \usepackage[utf8]{inputenc}
3 \usepackage{graphicx}
4 \usetheme{CambridgeUS}
5 \title{ASSIGNMENT2}
6 \author{ARUSHI SAWHNEY\MAT/20/102\20044563030 }
7 \date{}
8 \begin{document}
9 \begin{frame}
10 \begin{minipage}{0.13\linewidth}
11 \includegraphics[width=2cm,height=2cm]{DU LOGO.png}
12 \end{minipage}\hfill
13 \begin{minipage}{0.69\linewidth}
14 \centering {MATA SUNDRI COLLEGE FOR WOMEN\
15 UNIVERSITY OF DELHI }
16 \end{minipage}\hfill
17 \begin{minipage}{0.13\linewidth}
18 \includegraphics[width=2cm,height=2cm]{COLLEGE LOGO.png}
19 \end{minipage}
20 \large\emph\titlepage
21 \end{frame}
22 \begin{frame}{PG-69 ; PART-1}
23 \textbf{1.}
24 Let  $x=(x_1, \dots, x_n)$ , where the  $x_i$  are nonnegative real numbers. Set\
 $M_r(x)=\left(\frac{x_1^r+x_2^r+\dots+x_n^r}{n}\right)^{\frac{1}{r}}, r \in \mathbb{R} \setminus \{0\}, r > 0$ \
25 and  $M_0(x)=(x_1 x_2 \dots x_n)^{\frac{1}{n}}$ \
26 We call  $M_r(x)$  the  $r$ th power mean of  $x$ .\
27 Claim:  $\lim_{r \rightarrow 0} M_r(x) = M_0(x)$ \
28 \end{frame}
29 \begin{frame}{PG-69, PART-2}
30 \textbf{2.} \; Define
31  $V_n = \left[ \begin{array}{c} \dots \\ 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 & \dots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \end{array} \right]$ \
34

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34 x_1^2&x_2^2&x_3^2&\cdots&x_n^2\\
35 \vdots&\vdots&\vdots&\ddots&\vdots\\
36 x_1^{n-1}&x_2^{n-1}&x_3^{n-1}&\cdots&x_n^{n-1}\\
37 \end{array}\right]$$
38 We call  $V_n$  the Vandermonde matrix of order  $n$ .\\
39 Claim:  $\det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i)$ 
40 \end{frame}
41 \begin{frame}{QUESTION-4}
42 \begin{itemize}
43 \item  $3^2+4^3+5^6=6^3$ 
44 \item  $\sqrt{100}=10$ 
45 \item  $(a+b)^3=a^3+3a^2b+3ab^2+b^3$ 
46 \end{itemize}
47 \end{frame}
48 \begin{frame}{QUESTION-4}
49 \begin{itemize}
50 \item  $\sum_{k=1}^n k = \frac{n(n+1)}{2}$ 
51 \item  $\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots$ 
52 \item  $\cos \theta = \sin(90^\circ - \theta)$ 
53 \item  $e^{i\theta} = \cos \theta + i \sin \theta$ 
54 \end{itemize}
55 \end{frame}
56 \begin{frame}{QUESTION-4}
57 \begin{itemize}
58 \item  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ 
59 \item  $\lim_{x \rightarrow \infty} \frac{\pi(x)}{\frac{x}{\log x}} = 1$ 
60 \item  $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$ 
61 \end{itemize}
62 \end{frame}
63 \begin{frame}{QUESTION-5}
64 \begin{itemize}
65 \item Positive numbers  $a$ ,  $b$ , and  $c$  are the side lengths of a triangle if and only if  $a+b>c$ ,  $b+c>a$ ,  $c+a>b$ .
66 \item The area of a triangle with side lengths  $a$ ,  $b$ ,  $c$  is given by Heron's Formula:
67  $A = \sqrt{s(s-a)(s-b)(s-c)}$ 
68 where  $s$  is the semiperimeter  $\frac{(a+b+c)}{2}$ .

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69 \item The volume of a regular tetrahedron of edge length  $l$  is  $\frac{\sqrt{2}}{12}l^3$ .
70 \frac{\sqrt{2}}{12}l^3 .
71 \item The Quadratic equation  $ax^2+bx+c=0$  has roots  $r_1, r_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ 
72 \end{itemize}
73 \end{frame}
74 \begin{frame}{QUESTION-5}
75 \begin{itemize}
76 \item The derivative of a function  $f$ , denoted  $f'$ , is defined by  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ 
77 \item A real-valued function  $f$  is convex on an interval  $I$  if  $f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda)f(y)$ , for all  $x, y \in I$  and  $0 \leq \lambda \leq 1$ .
78 for all  $x, y \in I$  and  $0 \leq \lambda \leq 1$ .
79 \item The general solution of the differential equation  $y'' - 3y' + 2y = 0$  is  $y = C_1e^x + C_2e^{2x}$ .
80 is  $y = C_1e^x + C_2e^{2x}$ .
81 \item The Fermat number  $F_n$  is defined as  $F_n = 2^{2^n} + 1, n \geq 0$ .
82 \end{itemize}
83 \end{frame}
84 \begin{frame}{QUESTION-6}
85 \begin{itemize}
86 \item  $\frac{d}{dx} \left( \frac{x}{x+1} \right) = \frac{1}{(x+1)^2}$ 
87 \item  $\lim_{n \rightarrow \infty} \left( 1 + \frac{1}{n} \right)^n = e$ 
88 \item  $\begin{array}{cc} a & b \\ c & d \end{array}$ 
89  $a \quad b$ 
90  $c \quad d$ 
91  $\begin{array}{r} a & b \\ c & d \end{array}$ 
92 \item  $R_o = \begin{array}{cc} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{array}$ 
93  $\cos \theta \quad -\sin \theta$ 
94  $\sin \theta \quad \cos \theta$ 
95  $\begin{array}{r} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{array}$ 
96 \end{itemize}
97 \end{frame}
98 \begin{frame}{QUESTION-6}
99 \begin{itemize}
100 \item  $\begin{array}{ccc} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{array}$ 
101  $\text{itbf } i \ \text{itbf } j \ \text{itbf } k$ 
102  $a_1 \ a_2 \ a_3$ 
103  $b_1 \ b_2 \ b_3$ 

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103 b_1&b_2&b_3
104 \end{array}\right| = \left|\begin{array}{ccc}
105 a_2&a_3 \\
106 b_2&b_3 \\
107 \end{array}\right|\textbf{i} -\left|\begin{array}{cc}
108 a_1&a_3 \\
109 b_1&b_3 \\
110 \end{array}\right|\textbf{j} +\left|\begin{array}{cc}
111 a_1&a_2 \\
112 b_1&b_2 \\
113 \end{array}\right|\textbf{k}$$
114 \item $$\left[\begin{array}{cc}
115 a_{11}&a_{12} \\
116 a_{21}&a_{22} \\
117 \end{array}\right]\left[\begin{array}{cc}
118 b_{11}&b_{12} \\
119 b_{21}&b_{22} \\
120 \end{array}\right]=\left[\begin{array}{cc}
121 a_{11}b_{11}+a_{12}b_{21}&a_{11}b_{12}+a_{12}b_{22} \\
122 a_{21}b_{11}+a_{22}b_{21}&a_{21}b_{12}+a_{22}b_{22} \\
123 \end{array}\right]$$
124 \item $$f(x)=\left\{\begin{array}{c}
125 -x^2 \quad ,x<0 \\
126 x^2 \quad ,0\leq x \leq 2 \\
127 4 \quad ,x>2 \\
128 \end{array}\right. $$
129 \end{itemize}
130 \end{frame}
131 \begin{frame}{QUESTION-7 (A)}
132 \begin{eqnarray*}
133 1+2=&3 \\
134 4+5+6=&7+8 \\
135 9+10+11+12=&13+14+15 \\
136 16+17+18+19+20=&21+22+23+24 \\
137 25+26+27+28+29+30=&31+32+33+34+35

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136 16+17+18+19+20&=&21+22+23+24\
137 25+26+27+28+29+30&=&31+32+33+34+35
138 \end{eqnarray*}
139 \end{frame}
140 \begin{frame}{QUESTION-7(B)}
141 \begin{eqnarray*}
142 (a+b)^2&=&(a+b)(a+b) \quad \\\
143 &=&(a+b)a+(a+b)b\ \\\
144 &=&a(a+b)+b(a+b)\ \\\
145 &=&a^2+ab+ba+b^2\ \\\
146 &=&a^2+ab+ab+b^2\ \\\
147 &=&a^2+2ab+b^2
148 \end{eqnarray*}
149 \end{frame}
150 \begin{frame}{QUESTION-7(C)}
151 \begin{eqnarray*}
152 \tan(\alpha+\beta+\gamma)&=&\frac{\tan(\alpha+\beta)+\tan\gamma}{1-\tan(\alpha+\beta)\tan\gamma} \quad \\\
153 &=&\frac{\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}+\tan\gamma}{1-\left(\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}\right)\tan\gamma}\ \\\
154 &=&\frac{\tan\alpha+\tan\beta+(1-\tan\alpha\tan\beta)\tan\gamma}{1-\tan\alpha\tan\beta-(\tan\alpha+\tan\beta)\tan\gamma}\ \\\
155 &=&\frac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\beta-\tan\alpha\tan\gamma-\tan\beta\tan\gamma}
156 \end{eqnarray*}
157 \end{frame}
158 \begin{frame}{QUESTION-7(D)}
159 \begin{eqnarray*}
160 \prod_p \left(1-\frac{1}{p^2}\right)
161 &=&\prod_p \left(1+\frac{1}{p^2}+\frac{1}{p^4}+\cdots\right)\ \\\
162 &=&\left(\prod_p \left(1+\frac{1}{p^2}+\frac{1}{p^4}+\cdots\right)\right)^{-1}\ \\\
163 &=&\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots\right)^{-1}\ \\\
164 &=&\frac{6}{\pi^2}
165 \end{eqnarray*}
166 \end{frame}
167
168 \begin{frame}
169 \includegraphics[width=10cm,height=10cm]{Thank Youpeg.jpeg}
170 \end{frame}

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