MATA SUNDRI COLLEGE FOR WOMEN
UNIVERSITY OF DELHI

ASSIGNMENT2

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MAT/20/102
20044563030

1. Let $x = (x_1, \cdots, x_n)$,where the x_1 are nonnegative real numbers.Set

$$M_r x = \left(\frac{x_1^r + x_2^r + \cdots + x_n^r}{n} \right)^{\frac{1}{r}}, r \in R \setminus \{0\},$$

and

$$M_o(x) = (x_1 x_2 \cdots x_n)^{\frac{1}{n}}$$

We call $M_r(x)$ the rth power mean of x.
Claim:

$$\lim_{r \to 0} M_r(x) = M_o(x)$$

PG-69,PART-2

2. Define

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ x_1 & x_2 & x_3 & \cdots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \cdots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \cdots & x_n^{n-1} \end{bmatrix}$$

We call V_n the Vandermonde matrix of order n.
Claim:

$$det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i)$$

QUESTION-4

- $$3^2 + 4^3 + 5^6 = 6^3$$
- $$\sqrt{100} = 10$$
- $$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

QUESTION-4

- $$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$
- $$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots$$
- $$\cos\theta = \sin(90^o - \theta)$$
- $$e^{\iota\theta} = \cos\theta + \iota\sin\theta$$

QUESTION-4

- $$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$$
- $$\lim_{x \to \infty} \frac{\pi(x)}{\frac{x}{logx}} = 1$$
- $$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

QUESTION-5

- Positive numbers a , b , and c are the side lengths of a triangle if and only if $a + b > c$, $b + c > a$, $c + a > b$.
- The area of a triangle with side lengths a , b , c is given by Heron's Formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where s is the semiperimeter $\frac{(a+b+c)}{2}$.

- The volume of a regular tetrahedron of edge length 1 is $\frac{\sqrt{2}}{12}$.
- The Quadratic equation $ax^2 + bx + c = 0$ has roots

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

QUESTION-5

- The derivative of a function f , denoted f' , is defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- A real-valued function f is convex on an interval I if

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda) f(y),$$

  for all x , $y \in I$ and $0 \leq \lambda \leq 1$.
- The general solution of the diffrential equation

$$y'' - 3y' + 2y = 0$$

  is

$$y = C_1 e^x + C_2 e^{2x}.$$

- The Fermat number F_n is defined as

$$F_n = 2^{2^n}, n \geq 0.$$

QUESTION-6

- $$\frac{d}{dx}\left(\frac{x}{x+1}\right) = \frac{1}{(x+1)^2}$$
- $$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$
- $$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$
- $$R_o = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

QUESTION-6

- $$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

- $$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

- $$f(x) = \begin{cases} -x^2 & , \quad x < 0 \\ x^2 & , \quad 0 \le x \le 2 \\ 4 & , \quad x > 2 \end{cases}$$

QUESTION-7 (A)

$$
\begin{aligned}
1+2 &= 3 \\
4+5+6 &= 7+8 \\
9+10+11+12 &= 13+14+15 \\
16+17+18+19+20 &= 21+22+23+24 \\
25+26+27+28+29+30 &= 31+32+33+34+35
\end{aligned}
$$

QUESTION-7(B)

$$
\begin{aligned}
(a+b)^2 &= (a+b)(a+b) \\
&= (a+b)a+(a+b)b \\
&= a(a+b)+b(a+b) \\
&= a^2+ab+ba+b^2 \\
&= a^2+ab+ab+b^2 \\
&= a^2+2ab+b^2
\end{aligned}
$$

QUESTION-7(C)

$$\begin{aligned}
\tan(\alpha+\beta+\gamma) &= \frac{\tan(\alpha+\beta)+\tan\gamma}{1-\tan(\alpha+\beta)\tan\gamma} \\
&= \frac{\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}+\tan\gamma}{1-\left(\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}\right)\tan\gamma} \\
&= \frac{\tan\alpha+\tan\beta+(1-\tan\alpha\tan\beta)\tan\gamma}{1-\tan\alpha\tan\beta-(\tan\alpha+\tan\beta)\tan\gamma} \\
&= \frac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\beta-\tan\alpha\tan\gamma-\tan\beta\tan\gamma}
\end{aligned}$$

QUESTION-7(D)

$$\begin{aligned}
\prod_p \left(1 - \frac{1}{p^2}\right) &= \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots} \\
&= \left(\prod_p \left(1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots\right)\right)^{-1} \\
&= \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2 + \cdots}\right)^{-1} \\
&= \frac{6}{\pi^2}
\end{aligned}$$


Thank you!

```latex
\documentclass{beamer}
\usepackage[utf8]{inputenc}
\usepackage{graphicx}
\usetheme{CambridgeUS}
\title{ASSIGNMENT2}
\author{ARUSHI SAWHNEY\\MAT/20/102\\20044563030 }
\date{}
\begin{document}
\begin{frame}
\begin{minipage}{0.13\linewidth}
\includegraphics[width=2cm,height=2cm]{DU LOGO.png}
\end{minipage}\hfill
\begin{minipage}{0.69\linewidth}
\centering {MATA SUNDRI COLLEGE FOR WOMEN\\
UNIVERSITY OF DELHI }
\end{minipage}\hfill
\begin{minipage}{0.13\linewidth}
\includegraphics[width=2cm,height=2cm]{COLLEGE LOGO.png}
\end{minipage}
\large\emph\titlepage
\end{frame}
\begin{frame}{PG-69 ; PART-1}
\textbf{1.}
Let $x=(x_1,\cdots,x_n)$,where the $x_1$ are nonnegative real numbers.Set\\
$$M_r{x}=\left(\frac{x_1^r+x_2^r+\cdots+x_n^r}{n}\right)^\frac{1}{r},r\in{R}\setminus\{0\},$$\\
and$$M_o(x)=(x_1 x_2 \cdots x_n)^\frac{1}{n}$$
We call $M_r(x)$ the $r$th power mean of $x.$\\
Claim: $$\lim_{r\rightarrow0}M_r(x)=M_o(x)$$
\end{frame}
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\textbf{2.}\;Define
$$V_n=\left[\begin{array}{ccccc}
1&1&1&\cdots&1  \\
x_1&x_2&x_3&\cdots&x_n\\
x_1^2&x_2^2&x_3^2&\cdots&x_n^2\\
```

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x_1^2&x_2^2&x_3^2&\cdots&x_n^2\\
\vdots&\vdots&\vdots&\ddots&\vdots\\
x_1^{n-1}&x_2^{n-1}&x_3^{n-1}&\cdots&x_n^{n-1}\\
\end{array}\right]$$
 We call $V_n$ the Vandermonde matrix of order $n$.\\
Claim:$$detV_n=\prod_{1\leq i < j \leq n}\left(x_j-x_i\right)$$
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\begin{frame}{QUESTION-4}
\begin{itemize}
\item  $$3^2+4^3+5^6=6^3$$
\item $$\sqrt{100}=10$$
\item $$(a+b)^3=a^3+3a^2b+3ab^2+b^3$$
\end{itemize}
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\item $$\frac{\pi}{4}=\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\cdots$$
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\item Positive numbers $a$ , $b$ , and $c$ are the side lengths of a triangle if and only if $a+b>c$ , $b+c>a$ , $c+a>b$.
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$$A=\sqrt{s(s-a)(s-b)(s-c)},$$\\
where $s$ is the semiperimeter $\frac{(a+b+c)}{2}$.
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\item The volume of a regular tetrahedron of edge length $1$ is  $
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\item A real-valued function $f$ is convex on an interval $I$ if $$f(\lambda x +(1-\lambda)y)\leq\lambda f(x)+(1-\lambda)f(y),$$
for all $x$ , $y$ $\in$   $I$ and   $0 \leq \lambda \leq 1.$
\item The general solution of the diffrential equation $$y''-3y'+2y=0$$\\
is $$y=C_1e^x+C_2e^{2x}.$$
\item The Fermat number $F_n$ is defined as $$F_n=2^{2^{n}},n\geq 0.$$
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\item $$\frac{d}{dx}\left(\frac{x}{x+1}\right)=\frac{1}{(x+1)^2}$$
\item $$\lim_{n\rightarrow{\infty}}\left(1+\frac{1}{n}\right)^n=e$$
\item $$ \left|\begin{array}{cc}
a&b\\
c&d
\end{array}\right|=ad-bc$$
\item $$  R_o=\left[ \begin{array}{cc}
\cos\theta   & -\sin\theta \\
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\item $$ \left|\begin{array}{ccc}
\textbf i& \textbf j& \textbf k\\
a_1&a_2&a_3\\
b_1&b_2&b_3
```

```
b_1&b_2&b_3
\end{array}\right| = \left|\begin{array}{ccc}
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b_2&b_3
\end{array}\right|\textbf i -\left|\begin{array}{cc}
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b_1&b_3
\end{array}\right|\textbf j +\left|\begin{array}{cc}
a_1&a_2  \\
b_1&b_2
\end{array}\right|\textbf k$$
\item $$\left[\begin{array}{cc}
a_{11}&a_{12} \\
a_{21}&a_{22}
\end{array}\right]\left[\begin{array}{cc}
b_{11}&b_{12} \\
b_{21}&b_{22}
\end{array}\right]=\left[\begin{array}{cc}
a_{11}b_{11}+a_{12}b_{21}&a_{11}b_{12}+a_{12}b_{22} \\
a_{21}b_{11}+a_{22}b_{21}&a_{21}b_{12}+a_{22}b_{22}
\end{array}\right]$$
\item $$f(x)=\left\{\begin{array}{ccl}
-x^2 & ,&x<0 \\
x^2 & ,&0\leq x \leq 2\\
4 &,&x>2
\end{array}\right.$$
\end{itemize}
\end{frame}
\begin{frame}{QUESTION-7 (A)}
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\tan(\alpha+\beta+\gamma)&=&\frac{\tan(\alpha+\beta)+\tan\gamma}{1-\tan(\alpha+\beta)\tan\gamma}  \\
&=&\frac{\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} +\tan\gamma}{1-\left(\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}\right)\tan\gamma}\\
&=& \frac{\tan\alpha+\tan\beta+(1-\tan\alpha\tan\beta)\tan\gamma}{1-\tan\alpha\tan\beta-(\tan\alpha+\tan\beta)\tan\gamma}\\
&=&\frac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\beta-\tan\alpha\tan\gamma-\tan\beta\tan\gamma}
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\begin{frame}{QUESTION-7(D)}
\begin{eqnarray*}
\prod_{p} \left(1-\frac{1}{p^2}\right)
&=&\prod_{p} \frac{1}{1+\frac{1}{p^2}+\frac{1}{p^4}+\cdots}\\
&=&\left(\prod_{p}\left(1+\frac{1}{p^2}+\frac{1}{p^4}+\cdots\right)\right)^{-1}\\
&=&\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2+\cdots}\right)^{-1}\\
&=&\frac{6}{\pi^2}
\end{eqnarray*}
\end{frame}

\begin{frame}
    \includegraphics[width=10cm,height=10cm]{Thank Youpeg.jpeg}
\end{frame}
```