

Assignment-2

Muskan Tyagi

College Roll no.- MAT/20/28

University Roll no.- 20044563004



Mata Sundri College for Women,
University of Delhi

- Let $x = (x_1, \dots, x_n)$, where the x_i are non-negative real numbers. Set

$$M_r(x) = \left(\frac{x_1^r + x_2^r + \dots + x_n^r}{n} \right)^{1/r}, \quad r \in \mathbb{R} \setminus \{0\},$$

and

$$M_0(x) = (x_1 x_2 \dots x_n)^{1/n}.$$

We call $M_r(x)$ the *r*th power mean of x .

Claim:

$$\lim_{r \rightarrow 0} M_r(x) = M_0(x).$$

- Define

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 & \dots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1} \end{bmatrix}.$$

We call V_n the *Vandermonde matrix* of order n .

Claim:

$$\det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i).$$

Question 4

- $$3^3 + 4^3 + 5^3 = 6^3$$

- $$\sqrt{100} = 10$$

- $$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

- $$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

- $$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots$$

Remaining parts of Question 4

- $\cos\theta = \sin(90^\circ - \theta)$

- $e^{i\theta} = \cos\theta + i\sin\theta$

- $\lim_{\theta \rightarrow 0} \frac{\sin\theta}{\theta} = 1$

- $\lim_{x \rightarrow \infty} \frac{\pi(x)}{x/\log x} = 1$

- $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$

Question 5

- Positive numbers a , b , and c are the side lengths of a triangle if and only if $a + b > c$, $b + c > a$, and $c + a > b$.
- The area of a triangle with side lengths a , b , c is given by *Heron's formula* :

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where s is the semiperimeter $(a + b + c)/2$.

- The volume of a regular tetrahedron of edge length 1 is $\sqrt{2}/12$.
- The quadratic equation $ax^2 + bx + c = 0$ has roots

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Remaining Parts of Question 5

- The derivative of a function f , denoted f' , is defined by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- A real-valued function f is *convex* on a interval I if

$$f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y),$$

for all $a, y \in I$ and $0 \leq \lambda \leq 1$.

Remaining Parts of Question 5

- The general solution to the differential equation

$$y'' - 3y' + 2y = 0$$

is

$$y = C_1 e^x + C_2 e^{2x}.$$

- The *Fermat number* F_n is defined as

$$f_n = 2^{2^n}, \quad n \geq 0.$$

Question 6

$$\frac{d}{dx} \left(\frac{x}{x+1} \right) = \frac{1}{(x+1)^2}$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^n = e$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Remaining Parts of Question 6

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

$$f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & 0 \leq x \leq 2 \\ 4, & x > 2 \end{cases}$$

Question 7



$$1 + 2 = 3$$

$$4 + 5 + 6 = 7 + 8$$

$$9 + 10 + 11 + 12 = 13 + 14 + 15$$

$$16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24$$

$$25 + 26 + 27 + 28 + 29 + 30 = 31 + 32 + 33 + 34 + 35$$

Remaining Parts of Question 7

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) \\ &= (a + b)a + (a + b)b \\ &= a(a + b) + b(a + b) \\ &= a^2 + ab + ba + b^2 \\ &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2\end{aligned}$$

Remaining Parts of Question 7

$$\begin{aligned} \tan(\alpha + \beta + \gamma) &= \frac{\tan(\alpha + \beta) + \tan\gamma}{1 - \tan(\alpha + \beta)\tan\gamma} \\ &= \frac{\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} + \tan\gamma}{1 - \left(\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}\right)\tan\gamma} \\ &= \frac{\tan\alpha + \tan\beta + (1 - \tan\alpha\tan\beta)\tan\gamma}{1 - \tan\alpha\tan\beta - (\tan\alpha + \tan\beta)\tan\gamma} \\ &= \frac{\tan\alpha + \tan\beta + \tan\gamma - \tan\alpha\tan\beta\tan\gamma}{1 - \tan\alpha\tan\beta - \tan\alpha\tan\gamma - \tan\beta\tan\gamma} \end{aligned}$$

Remaining Parts of Question 7

- $$\begin{aligned}\prod_p \left(1 - \frac{1}{p^2}\right) &= \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots} \\ &= \left(\prod_p \left(\frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots} \right) \right)^{-1} \\ &= \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots \right)^{-1} \\ &= \frac{6}{\pi^2}\end{aligned}$$

Thank you

