

## ASSIGNMENT 2

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MAT/20/106

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## Example 9.5

1. Let  $\mathbf{x} = (x_1, \dots, x_n)$ , where the  $x_i$  are non-negative real numbers. Set

$$M_r(\mathbf{x}) = \left( \frac{x_1^r + x_2^r + \dots + x_n^r}{n} \right)^{\frac{1}{r}}, \quad r \in \mathbf{R} \setminus \{0\},$$

and

$$M_0(\mathbf{x}) = (x_1 x_2 \dots x_n)^{\frac{1}{n}}.$$

We call  $M_r(x)$  the *r*th power mean of  $\mathbf{x}$ .

Claim:

$$\lim_{r \rightarrow 0} M_r(\mathbf{x}) = M_0(\mathbf{x}).$$

## 2. Define

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ x_1 & x_2 & x_3 & \cdots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \cdots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \cdots & x_n^{n-1} \end{bmatrix}$$

We call  $V_n$  the Vandermonde matrix of order  $n$ .

Claim:

$$\det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i)$$

## Question 4



$$3^3 + 4^3 + 5^3 = 6^3$$



$$\sqrt{100} = 10$$



$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$



$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$



$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots$$

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$$\cos \theta = \sin(90^\circ - \theta)$$

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$$

$$\lim_{x \rightarrow \infty} \frac{\pi(x)}{x \log x} = 1$$

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

## Question 5

- Positive numbers  $a$ ,  $b$  and  $c$  are the side lengths of a triangle if and only if  $a + b > c$ ,  $b + c > a$  and  $c + a > b$ .
- The area of a triangle with side lengths  $a$ ,  $b$  and  $c$  is given by *Heron's formula*:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where  $s$  is the semi perimeter  $\frac{a+b+c}{2}$ .

- The volume of a regular tetrahedron of edge length 1 is  $\frac{\sqrt{2}}{12}$ .
- The quadratic equation  $ax^2 + bx + c = 0$  has roots

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The *derivative* of a function  $f$ , denoted  $f'$ , is defined by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- A real-valued function  $f$  is convex on an interval  $I$  if

$$f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y),$$

for all  $x, y \in I$  and  $0 \leq \lambda \leq 1$ .

- The general solution to the differential equation

$$y'' - 3y' + 2y = 0$$

is

$$y = C_1 e^x + C_2 e^{2x}$$

- The *Fermat number*  $F_n$  is defined as

$$F_n = 2^{2^n}, n \geq 0$$

## Question 6



$$\frac{d}{dx} \left( \frac{x}{x+1} \right) = \frac{1}{(x+1)^2}$$



$$\lim_{n \rightarrow \infty} \left( 1 + \frac{1}{n} \right)^n = e$$



$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$



$$R_\theta = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$



$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} + \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

$$f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & 0 \leq x \leq 2 \\ 4, & x > 2 \end{cases}$$

## Question 7 Part-1

1.

$$1 + 2 = 3$$

$$4 + 5 + 6 = 7 + 8$$

$$9 + 10 + 11 + 12 = 13 + 14 + 15$$

$$16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24$$

$$25 + 26 + 27 + 28 + 29 + 30 = 31 + 32 + 33 + 34 + 35$$

## Part-2

2.

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) \\ &= (a + b)a + (a + b)b \\ &= a(a + b) + b(a + b) \\ &= a^2 + ab + ba + b^2 \\ &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2\end{aligned}$$

## Part-3

3.

$$\begin{aligned}\tan(\alpha + \beta + \gamma) &= \frac{\tan(\alpha + \beta) + \tan \gamma}{1 - \tan(\alpha + \beta) \tan \gamma} \\ &= \frac{\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} + \tan \gamma}{1 - \left( \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \right) \tan \gamma} \\ &= \frac{\tan \alpha + \tan \beta + (1 - \tan \alpha \tan \beta) \tan \gamma}{1 - \tan \alpha \tan \beta - (\tan \alpha + \tan \beta) \tan \gamma} \\ &= \frac{\tan \alpha + \tan \beta + \tan \gamma - \tan \alpha \tan \beta \tan \gamma}{1 - \tan \alpha \tan \beta - \tan \alpha \tan \gamma - \tan \beta \tan \gamma}\end{aligned}$$

## Part-4

4.

$$\begin{aligned}\prod_p \left(1 - \frac{1}{p^2}\right) &= \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots} \\ &= \left( \prod_p \left(1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots\right) \right)^{-1} \\ &= \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots\right)^{-1} \\ &= \frac{6}{\pi^2}\end{aligned}$$



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6 \author{Muskan Manocha}
7 MAT/20/106
8 20044563032
9 \vspace{0.2in}
10 MATA SUNDRI COLLEGE FOR WOMEN
11 UNIVERSITY OF DELHI
12 \date{}
13 \usetheme{Berlin}
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20 \begin{frame}{Example 9.5}
21 \textbf{1.}
22 Let  $\mathbf{x}=(x_1, \dots, x_n)$ , where the  $x_i$ 's are non-negative real numbers. Set
23  $M_r(\mathbf{x})=\left(\frac{x_1^r+x_2^r+\dots+x_n^r}{n}\right)^{\frac{1}{r}}$ ,  $r \in \mathbb{R}$ ,  $r \neq 0$ ,
24 and  $M_0(\mathbf{x})=x_1 x_2 \dots x_n$ .
25 We call  $M_r(x)$  the  $r$ th power mean of  $\mathbf{x}$ .
26 Claim:  $\lim_{r \rightarrow 0} M_r(\mathbf{x})=M_0(\mathbf{x})$ .
27 \end{frame}
28 \begin{frame}
29 \textbf{2.} Define
30  $V_n=\begin{bmatrix} 1 & & & \\ & \ddots & & \\ & & \ddots & \\ & & & \ddots & \\ & & & & 1 \end{bmatrix}$ 
31  $x_1 x_2 x_3 \dots x_n$ 
32  $x_1^2 x_2^2 x_3^2 \dots x_n^2$ 
33  $\vdots \vdots \vdots \ddots \vdots$ 
34  $x_1^{n-1} x_2^{n-1} x_3^{n-1} \dots x_n^{n-1}$ 
35 \end{array} \right)
36 We call  $V_n$  the Vandermonde matrix of order  $n$ .
37 Claim:  $\det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i)$ 
38 \end{frame}
39 \begin{frame}{Question 4}
40 \begin{itemize}
41 \item  $3^3+4^3+5^3=6^3$ 
42 \item  $\sqrt{100}=10$ 
43 \item  $(a+b)^3=a^3+3a^2b+3ab^2+b^3$ 
44 \item  $\sum_{k=1}^n k = \frac{n(n+1)}{2}$ 
45 \item  $\frac{d}{dx} \ln x = \frac{1}{x}$ ,  $\frac{d}{dx} \ln 3 = \frac{1}{3}$ ,  $\frac{d}{dx} \ln 5 = \frac{1}{5}$ ,  $\frac{d}{dx} \ln 7 = \frac{1}{7}$ ,  $\frac{d}{dx} \ln 9 = \frac{1}{9}$ ,  $\frac{d}{dx} \ln 11 = \frac{1}{11}$ ,  $\frac{d}{dx} \ln 13 = \frac{1}{13}$ 
46 \end{itemize}
47 \end{frame}
48 \begin{frame}
49 \begin{itemize}
50 \item  $\cos \theta = \sin(90^\circ - \theta)$ 
51 \item  $e^{i\theta} = \cos \theta + i \sin \theta$ 
52 \item  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ 
53 \item  $\lim_{x \rightarrow \infty} \frac{\ln x}{x} = 0$ 
54 \item  $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$ 
55 \end{itemize}
56 \end{frame}
57 \begin{frame}{Question 5}
58 \begin{itemize}
59 \item Positive numbers  $a, b, c$  are the side lengths of a triangle if and only if  $a+b > c, b+c > a$  and  $c+a > b$ .
60 \item The area of a triangle with side lengths  $a, b, c$  is given by Heron's formula:  $S = \sqrt{s(s-a)(s-b)(s-c)}$  where  $s$  is the semi perimeter  $\frac{a+b+c}{2}$ .
61 \item The volume of a regular tetrahedron of edge length 1 is  $\frac{\sqrt{2}}{12}$ .
62 \item The quadratic equation  $ax^2+bx+c=0$  has roots  $r_1, r_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ 
63 \end{itemize}
64 \end{frame}
65 \begin{frame}
66 \begin{itemize}
67 \item The derivative of a function  $f$ , denoted  $f'$ , is defined by  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$ 
68 \item A real-valued function  $f$  is convex on an interval  $I$  if  $f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda)f(y)$  for all  $x, y \in I$  and  $0 \leq \lambda \leq 1$ .
69 \item The general solution to the differential equation  $yy'' - 3y' + 2y = 0$  is  $y = C_1 e^x + C_2 e^{2x}$ 
70 \item The Fermat number  $F_n$  is defined as  $F_n = 2^{2^n} + 1, n \geq 0$ .
71 \end{itemize}
72 \end{frame}
73 \begin{frame}{Question 6}
74 \begin{itemize}
75 \item  $\frac{d}{dx} \left( \frac{x}{x+1} \right) = \frac{1}{(x+1)^2}$ 
76 \item  $\lim_{n \rightarrow \infty} \left( 1 + \frac{1}{n} \right)^n = e$ 
77 \item  $\frac{d}{dx} (a^x) = a^x \ln a$ 
78 \item  $\frac{d}{dx} (x^a) = a x^{a-1}$ 
79 \end{itemize}
80 \end{frame}
81 \begin{frame}
82 \begin{itemize}
83 \item  $\cos(\theta - \sin \theta)$ 
84  $\sin \theta \cos \theta$ 
85 \end{itemize}
86 \end{frame}
87 \end{frame}

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Source	Rich Text
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83	<code>\item \$R_{\theta}=\left[\begin{array}{cc}</code>
84	<code>\cos\theta&amp;-\sin\theta\\</code>
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94	<code>b_1&amp;b_2&amp;b_3</code>
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100	<code>b_1&amp;b_3</code>
101	<code>\end{array}\right \textbf{j}+\left[\begin{array}{cc}</code>
102	<code>a_1&amp;a_2\\</code>
103	<code>b_1&amp;b_2</code>
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105	<code>\item\$\$\left[\begin{array}{cc}</code>
106	<code>a_{11}&amp;a_{12}\\</code>
107	<code>a_{21}&amp;a_{22}</code>
108	<code>\end{array}\right]\left[\begin{array}{cc}</code>
109	<code>b_{11}&amp;b_{12}\\</code>
110	<code>b_{21}&amp;b_{22}</code>
111	<code>\end{array}\right]=\left[\begin{array}{cc}</code>
112	<code>a_{11}b_{11}+a_{12}b_{21}&amp;a_{11}b_{12}+a_{12}b_{22}\\</code>
113	<code>a_{21}b_{11}+a_{22}b_{21}&amp;a_{21}b_{12}+a_{22}b_{22}</code>
114	<code>\end{array}\right]\$\$\\</code>
115	<code>\item\$\$f(x)=\left[\begin{array}{cc}</code>
116	<code>-x^2,&amp;x&lt;0\\</code>
117	<code>x^2,&amp;0\leq x\leq 2\\</code>
118	<code>4,&amp;x&gt;2</code>
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124	<code>\begin{eqnarray*}</code>
125	<code>1+2=&amp;3\\</code>
126	<code>4+5+6=&amp;7+8\\</code>
127	<code>9+10+11+12=&amp;13+14+15\\</code>
128	<code>16+17+18+19+20=&amp;21+22+23+24\\</code>
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132	<code>\begin{frame}{Part-2}</code>
133	
134	<code>\textbf{2.}</code>
135	<code>\begin{eqnarray*}</code>
136	<code>(a+b)^2=&amp;(a+b)(a+b)\\</code>
137	<code>=&amp;(a+b)a+(a+b)b\\</code>
138	<code>=&amp;a(a+b)+b(a+b)\\</code>
139	<code>=&amp;a^2+ab+ba+b^2\\</code>
140	<code>=&amp;a^2+ab+ab+b^2\\</code>
141	<code>=&amp;a^2+2ab+b^2</code>
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144	<code>\begin{frame}{Part-3}</code>
145	<code>\textbf{3.}</code>
146	<code>\begin{eqnarray*}</code>
147	<code>\tan(\alpha+\beta+\gamma)=\frac{\tan(\alpha+\beta)+\tan\gamma}{1-\tan(\alpha+\beta)\tan\gamma}</code>
148	<code>=\frac{\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}+\tan\gamma}{1-\left(\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}\right)\tan\gamma}</code>
149	<code>=\frac{\tan\alpha+\tan\beta+(1-\tan\alpha\tan\beta)\tan\gamma}{\tan\beta-(\tan\alpha+\tan\beta)\tan\gamma}</code>
150	<code>=\frac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{\tan\beta-\tan\alpha\tan\gamma-\tan\beta\tan\gamma}</code>
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157	<code>=\left(\prod_{p\leftarrow\frac{1}{p^2}}\left(1+\frac{1}{p^2}+\frac{1}{p^4}+\dots\right)\right)^{-1}</code>
158	<code>=\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\dots\right)^{-1}</code>
159	<code>=\frac{6}{\pi^2}</code>
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