ASSIGNMENT 2

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Example 9.5

1. Let $\mathbf{x} = (x_1, \dots, x_n)$,where the x_i are non-negative real numbers. Set

$$M_r(\mathbf{x}) = \left(\frac{x_1^r + x_2^r + \cdots + x_n^r}{n} \right)^{\frac{1}{r}}, r \in \mathbf{R} \setminus \{0\},$$

and

$$M_0(\mathbf{x}) = (x_1 x_2 \dots x_n)^{\frac{1}{n}}.$$

We call $M_r(x)$ the *rth power mean* of $\mathbf{x}$.
Claim:

$$\lim_{r \to 0} M_r(\mathbf{x}) = M_0(\mathbf{x}).$$

2. Define

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ x_1 & x_2 & x_3 & \cdots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \cdots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \cdots & x_n^{n-1} \end{bmatrix}$$

We call V_n the Vandermonde matrix of order n.

Claim:

$$det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i)$$

Question 4

- $$3^3 + 4^3 + 5^3 = 6^3$$
- $$\sqrt{100} = 10$$
- $$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
- $$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$
- $$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots$$

- $$\cos\theta = \sin(90^o - \theta)$$
- $$e^{i\theta} = \cos\theta + i\sin\theta$$
- $$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$
- $$\lim_{x\to\infty}\frac{\pi(x)}{x \setminus \log x} = 1$$
- $$\int_{-\infty}^{\infty} e^{-x^2}dx = \sqrt{\pi}$$

Question 5

- Positive numbers a, b and c are the side lengths of a triangle if and only if $a+b>c, b+c>a$ and $c+a>b$.
- The area of a triangle with side lengths a, b and c is given by *Heron's formula*:

$$A=\sqrt{s(s-a)(s-b)(s-c)}$$

  where s is the semi perimeter $\frac{a+b+c}{2}$.
- The volume of a regular tetrahedron of edge length 1 is $\frac{\sqrt{2}}{12}$.
- The quadratic equation $ax^2+bx+c=0$ has roots

$$r_1, r_2=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

- The *derivative* of a function f,denoted f',is defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h-f(x)}{h}$$

- A real-valued function f is convex on an interval I if

$$f(\lambda x + (1-\lambda)y) \leq f(x) + (1 = \lambda)f(y),$$

for all $x, y \in I$ and $0 \leq \lambda \leq 1$.

- The general solution to the differential equation

$$y'' - 3y' + 2y = 0$$

is

$$y = C_1 e^x + C_2 e^{2x}$$

- The *Fermat number* F_n is defined as

$$F_n = 2^{2^n}, n \geq 0$$

.

Question 6

- $$\frac{d}{dx}\left(\frac{x}{x+1}\right) = \frac{1}{(x+1)^2}$$
- $$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$
- $$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$
- $$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

- $$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} + \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

- $$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

- $$f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & 0 \leq x \leq 2 \\ 4, & x > 2 \end{cases}$$

Question 7 Part-1

1.

$$\begin{aligned} 1+2 &= 3 \\ 4+5+6 &= 7+8 \\ 9+10+11+12 &= 13+14+15 \\ 16+17+18+19+20 &= 21+22+23+24 \\ 25+26+27+28+29+30 &= 31+32+33+34+35 \end{aligned}$$

Part-2

2.

$$
\begin{aligned}
(a+b)^2 &= (a+b)(a+b)\\
&= (a+b)a+(a+b)b\\
&= a(a+b)+b(a+b)\\
&= a^2+ab+ba+b^2\\
&= a^2+ab+ab+b^2\\
&= a^2+2ab+b^2
\end{aligned}
$$

Part-3

3.

$$
\begin{aligned}
\tan(\alpha+\beta+\gamma) &= \frac{\tan(\alpha+\beta)+\tan\gamma}{1-\tan(\alpha+\beta)\tan\gamma} \\
&= \frac{\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}+\tan\gamma}{1-\left(\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}\right)\tan\gamma} \\
&= \frac{\tan\alpha+\tan\beta+(1-\tan\alpha\tan\beta)\tan\gamma}{1-\tan\alpha\tan\beta-(\tan\alpha+\tan\beta)\tan\gamma} \\
&= \frac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\beta-\tan\alpha\tan\gamma-\tan\beta\tan\gamma}
\end{aligned}
$$

Part-4

4.

$$\begin{aligned}\prod_p\left(1-\frac{1}{p^2}\right) &= \prod_p \frac{1}{1+\frac{1}{p^2}+\frac{1}{p^4}+\cdots} \\ &= \left(\prod_p\left(1+\frac{1}{p^2}+\frac{1}{p^4}+\cdots\right)\right)^{-1} \\ &= \left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots\right)^{-1} \\ &= \frac{6}{\pi^2}\end{aligned}$$


Thank You

Source Rich Text

```
\documentclass{beamer}
\usepackage[utf8]{inputenc}
\usepackage{graphicx}
\setbeamertemplate{background}{\includegraphics[height=12cm,width=14cm]{bg.jpg}}
\title{ASSIGNMENT 2}
\author{Muskan Manocha\\
MAT/20/106\\
20044563032\\
\vspace{0.2in}
MATA SUNDRI COLLEGE FOR WOMEN\\
UNIVERSITY OF DELHI}
\date{}
\usetheme{Berlin}
\usecolortheme{beaver}
\begin{document}
\begin{frame}
    \LARGE \titlepage

\end{frame}
\begin{frame}{Example 9.5}
     \textbf{1.}
     Let $\mathbf{x}=(x_1,\ldots,x_n)$,where the $x_i$ are non-negative real numbers. Set\\
$$M_r(\mathbf{x})=\left(\frac{x_1^r+x_2^r+\cdots+x_n^r}{n}\right)^\frac{1}{r},r\in
\mathbf{R}\setminus \{0\},$$\\
and $$M_0(\mathbf{x})=(x_1 x_2 \ldots x_n)^\frac{1}{n}.$$\\
We call $M_r(x)$ the \emph{rth power mean} of $\mathbf{x}$.\\
Claim:$$\lim_{r\rightarrow0}M_r(\mathbf{x})=M_0(\mathbf{x}).$$
\end{frame}
\begin{frame}
 \textbf{2.}\;Define
$$V_n=\left[\begin{array}{ccccc}
1&1&1&\cdots&1  \\
x_1&x_2&x_3&\cdots&x_n\\
x_1^2&x_2^2&x_3^2&\cdots&x_n^2\\
\vdots&\vdots&\vdots&\ddots&\vdots\\
x_1^{n-1}&x_2^{n-1}&x_3^{n-1}&\cdots&x_n^{n-1}\\
\end{array}\right]$$
 We call $V_n$ the Vandermonde matrix of order $n$.\\
Claim:$$detV_n=\prod_{1\leq i < j \leq n}\left(x_j-x_i\right)$$
\end{frame}
\begin{frame}{Question 4}
\begin{itemize}
\item $${3^3+4^3+5^3=6^3}$$\\
\item$$\sqrt{100}=10$$\\
\item$$(a+b)^3=a^3+3a^2b+3ab^2+b^3$$\\
\item$$\sum^n_{k=1} k=\frac{n(n+1)}{2}$$\\
\item$$\frac{\pi}{4}=\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\cd
ts$$\\
\end{itemize}
\end{frame}
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\item $$\cos\theta=\sin(90^o-\theta)$$\\
\item$$e^{i\theta}=\cos\theta+i\sin\theta$$\\
\item$$\lim_{\theta\rightarrow0}\frac{\sin\theta}{\theta}=1$$\\
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\item$$\int_{-\infty}^{\infty} e^{-x^2} dx=\sqrt{\pi}$$\\
\end{itemize}
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\begin{frame}{Question 5}
\begin{itemize}
\item Positive numbers $a,b$ and $c$ are the side lengths of a triangle if and only if
$a+b>c,b+c>a$ and $c+a>b$.
\item The area of a triangle with side lengths $a,b$ and $c$ is given by \emph {Heron's
formula}:$$A=\sqrt{s(s-a)(s-b)(s-c)}$$\\where $s$ is the semi perimeter $\frac{a+b+c}{2}$.
\item The volume of a regular tetrahedron of edge length 1 is $\frac{\sqrt{2}}{12}$.
\item The quadratic equation $ax^2+bx+c=0$ has roots$$r_1,r_2=\frac{-b\pm \sqrt{b^2-4ac}}{2a}$$
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\begin{frame}
\begin{itemize}
\item The \emph{derivative} of a function $f$,denoted $f'$,is defined
by$$f'(x)=\lim_{h\rightarrow0}\frac{f(x+h-f(x)}{h}$$\\
\item A real-valued function $f$ is convex on an interval $I$ if $$f(\lambda x+(1-\lambda)y)\leq
f(x)+(1=\lambda)f(y),$$\\
for all $x,y\in I$ and $0\leq\lambda\leq1$.
\item The general solution to the differential equation$$y''-3y'+2y=0$$ is$$y=C_1e^x+C_2e^{2x}$$\
\item The \emph{Fermat number} $F_n$ is defined as$$F_n=2^{2^n},n\geq0$$.
\end{itemize}
\end{frame}
\begin{frame}{Question 6}
\begin{itemize}
\item $$\frac{d}{dx} \left(\frac{x}{x+1}\right)=\frac{1}{(x+1)^2}$$\\
\item$$\lim_{n\rightarrow\infty}\left(1+\frac{1}{n}\right)^n=e$$\\
\item$$\left|\begin{array}{cc}
  a&b \\
    c&d
\end{array}\right|=ad-bc$$\\
\item $$R_{\theta}=\left[\begin{array}{cc}
\cos\theta&-\sin\theta\\
\sin\theta&\cos\theta
\end{array}\right]$$\\
\end{itemize}
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```latex
\end{array}\right|=ad-bc$$\\
\item $$R_{\theta}=\left[\begin{array}{cc}
\cos\theta&-\sin\theta\\
\sin\theta&\cos\theta
\end{array}\right]$$\\
\end{itemize}
\end{frame}
\begin{frame}
\begin{itemize}
\item$$\left|\begin{array}{ccc}
\textbf{i}&\textbf{j}&\textbf{k}\\
a_1&a_2&a_3\\
b_1&b_2&b_3
\end{array}\right|=\left|\begin{array}{cc}
a_2&a_3\\
b_2&b_3
\end{array}\right|\textbf{i}+\left|\begin{array}{cc}
a_1&a_3\\
b_1&b_3
\end{array}\right|\textbf{j}+\left|\begin{array}{cc}
a_1&a_2\\
b_1&b_2
\end{array}\right|\textbf{k}$$\\
\item$$\left[\begin{array}{cc}
a_{11}&a_{12}\\
a_{21}&a_{22}
\end{array}\right]\left[\begin{array}{cc}
b_{11}&b_{12}\\
b_{21}&b_{22}
\end{array}\right]=\left[\begin{array}{cc}
a_{11}b_{11}+a_{12}b_{21}&a_{11}b_{12}+a_{12}b_{22}\\
a_{21}b_{11}+a_{22}b_{21}&a_{21}b_{12}+a_{22}b_{22}
\end{array}\right]$$\\
\item$$f(x)=\left\{\begin{array}{cc}
-x^2,&x<0\\
x^2,&0\leq x\leq2\\
4,&x>2
\end{array}\right.$$\\
\end{itemize}
\end{frame}
\begin{frame}{Question 7 Part-1}
\textbf{1.}
\begin{eqnarray*}
1+2&=&3\\
4+5+6&=&7+8\\
9+10+11+12&=&13+14+15\\
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25+26+27+28+29+30&=&31+32+33+34+35\\
\end{eqnarray*}
\end{frame}
\begin{frame}{Part-2}

\textbf{2.}
\begin{eqnarray*}
(a+b)^2&=&(a+b)(a+b)\\
&=&(a+b)a+(a+b)b\\
&=&a(a+b)+b(a+b)\\
&=&a^2+ab+ba+b^2\\
&=&a^2+ab+ab+b^2\\
&=&a^2+2ab+b^2
\end{eqnarray*}
\end{frame}
\begin{frame}{Part-3}
\textbf{3.}
\begin{eqnarray*}
\tan(\alpha+\beta+\gamma)&=&\frac{\tan(\alpha+\beta)+\tan\gamma}{1-\tan(\alpha+\beta) \tan\gamma}\\
&=&\frac{\frac{\tan\alpha+\tan\beta}{1-\tan\alpha \tan\beta}+\tan\gamma}{1-\left(\frac{\tan\alpha+\tan\beta}{1-\tan\alpha \tan\beta}\right) \tan\gamma}\\
&=&\frac{\tan\alpha+\tan\beta+(1-\tan\alpha \tan\beta)\tan\gamma}{1-\tan\alpha \tan\beta-(\tan\alpha+\tan\beta)\tan\gamma}\\
&=&\frac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha \tan\beta \tan\gamma}{1-\tan\alpha \tan\beta-\tan\alpha \tan\gamma -\tan\beta \tan\gamma}
\end{eqnarray*}
\end{frame}
\begin{frame}{Part-4}
\textbf{4.}
\begin{eqnarray*}
\prod_{p} \left(1-\frac{1}{p^2}\right)&=&\prod_{p} \frac{1}{1+\frac{1}{p^2}+\frac{1}{p^4}+\cdots}\\
&=&\left(\prod_{p} \left(1+\frac{1}{p^2}+\frac{1}{p^4}+\cdots\right)\right)^{-1}\\
&=&\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots\right)^{-1}\\
&=&\frac{6}{\pi^2}
\end{eqnarray*}
\end{frame}
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