Assignment

Khushi

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University of Delhi

College Roll no.- MAT/20/37
University Roll no.- 20044563002

Question no.- 1

- Let $x = (x_1, \ldots, x_n)$, where the x_i are non negative real numbers. Set

$$M_r(x) = \left(\frac{x_1^r + x_2^r + \cdots + x_n^r}{n} \right)^{1/r}, \quad r \in \mathrm{R} \setminus \{0\},$$

and

$$M_0(x) = (x_1 x_2 \ldots x_n)^{1/n}.$$

We call $M_r(x)$ the *rth power mean* of x.
Claim:

$$\lim_{r \to 0} M_r(x) = M_0(x).$$

Question no.- 2

- Define:

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 & \dots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1} \end{bmatrix}.$$

We call V_n the *Vandermonde matrix* of order n. Claim:

$$\det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i).$$

4. Make the following equations.

- $$3^3 + 4^3 + 5^3 = 6^3$$
- $$\sqrt{100} = 10$$
- $$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
- $$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$
- $$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots$$

- $$\cos\theta = sin(90^\circ - \theta)$$

- $$e^{i\theta} = cos\theta + isin\theta$$

- $$\lim_{\theta \to 0} \frac{sin\theta}{\theta} = 1$$

- $$\lim_{x \to \infty} \frac{\pi(x)}{x/logx} = 1$$

- $$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

5. Typeset the following sentences.

- Positive numbers a,b, and c are the side lengths of a triangle if and only if $a + b > c$, $b + c > a$, and $c + a > b$.
- The area of a triangle with side lengths a,b,c is given by *Heron's formula*:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

, where s is the semiperimeter $(a + b + c)/2$.

- The volume of a regular tetrahedron of edge length 1 is $\sqrt{2}/12$
- The quadratic equation $ax^2 + bx + c = 0$ has roots

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

.

- The derivative of a function f , denoted f', is defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

- A real-valued function f is convex on an interval I if

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda) f(y)$$

, for all$x, y \in I$ and $0 \leq \lambda \leq 1$.

- The general solution to the differential equation

$$y'' - 3y' + 2y = 0$$

$$y = C_1 e^x + C_2 e^2 x$$

.

- The Fermat number F_n is defined as

$$F_n = 2^{2n}, n \geq 0$$

6. Make the following equations, Notice the large elimiters.

- $$\frac{d}{dx}\frac{(x)}{(x+1)} = \frac{1}{(x+1)^2}$$
- $$\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$$
- $$\left| \begin{array}{cc} a & b \\ c & d \end{array} \right| = ad - bc$$
- $$R_\theta = \left[\begin{array}{cc} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{array} \right]$$

- $$\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ a_2 & b_3 \end{vmatrix} \boldsymbol{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \boldsymbol{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \boldsymbol{k}$$

- $$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

- $$f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & 0 \leq x \leq 2 \\ 4, & x > 2 \end{cases}$$

7. Make the following multi-line equation

$$\begin{aligned}
1+2 &= 3 \\
4+5+6 &= 7+8 \\
9+10+11+12 &= 21+22+23+24 \\
16+17+18+19+20 &= 21+22+23+24 \\
25+26+27+28+29+30 &= 31+32+33+34+35
\end{aligned}$$

$$
\begin{aligned}
(a+b)^2 &= (a+b)(a+b) \\
&= (a+b)a+(a+b)b \\
&= a(a+b)+b(a+b) \\
&= a^2+ab+ba+b^2 \\
&= a^2 2+ab+ab+b^2 \\
&= a^2+2ab+b^2
\end{aligned}
$$

$$\begin{aligned}
tan(\alpha+\beta+\gamma) &= \frac{tan(\alpha+\beta)+tan\gamma}{1-tan(\alpha+\beta)tan\gamma} \\
&= \frac{\frac{tan\alpha+tan\beta}{1-tan\alpha tan\beta}+tan\gamma}{1-(\frac{tan\alpha+tan\beta}{1-tan\alpha tan\beta})tan\gamma} \\
&= \frac{tan\alpha+tan\beta+(1-tan\alpha tan\beta)tan\gamma}{1-tan\alpha tan\beta-(tan\alpha+tan\beta)tan\gamma} \\
&= \frac{tan\alpha+tan\beta+tan\gamma-tan\alpha tan\beta tan\gamma}{1-tan\alpha tan\beta-tan\alpha tan\gamma-tan\beta tan\gamma}
\end{aligned}$$

$$
\begin{aligned}
\prod_p \left(1 - \frac{1}{p^2}\right) &= \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots} \\
&= \left(\prod_p \left(1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots \right) \right)^{-1} \\
&= \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \cdots \right)^{-1} \\
&= \frac{6}{\pi^2}
\end{aligned}
$$


Thank you!

```latex
\documentclass{beamer}
\usepackage[utf8]{inputenc}
\usepackage{graphicx}
\title{Assignment }
\author{Khushi}
\institute{Mata Sundri College for women\\ University of Delhi}
\date{}
\usetheme{Berlin}
\begin{document}
\begin{frame}
 \titlepage
 \textbf{College Roll no.- MAT/20/37}


\textbf{University Roll no.- 20044563002}
\end{frame}

\begin{frame}{Question no.- 1}
  \begin{itemize}
      \item Let $\mathbf{x}=(x_1,\ldots,x_n)$,
      where the $x_i$ are non negative real numbers.
Set\[M_r(\mathbf{x})= \left(\frac{x_1^r+x_2^r+\cdots+x_n^r}{n}\right)^{1/r},\;\; r\in \mathbf{R}
\setminus \{0\},\]
and \[M_0(\mathbf{x})=\left(x_1 x_2 \ldots x_n \right)^{1/n}.\]
We call $M_r(\mathbf{x})$ the \emph{$r$th  power mean} of $\mathbf{x}$.

Claim:
\[\lim_{r \rightarrow 0} M_r(x)= M_0(\mathbf{x}).\]
  \end{itemize}
\end{frame}

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\begin{frame}{Question no.- 2}
    \begin{itemize}
        \item Define:
\[V_n=\left[\begin{array}{ccccc}
   1  & 1 & 1 & \ldots & 1  \\
    x_1 & x_2 & x_3 & \ldots & x_n\\
    x_1^2 & x_2^2 & x_3^2 & \ldots & x_n^2\\
    \vdots & \vdots & \vdots & \ddots & \vdots\\
    x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \ldots & x_n^{n-1}
\end{array}
\right].\]
We call $V_n$ the \emph{Vandermonde matrix} of order $n$.
Claim:
\[\det V_n=\prod_{1 \leq i < j \leq n}(x_j-x_i).\]
    \end{itemize}
\end{frame}

\begin{frame}{4. Make the following equations.}
\begin{itemize}
\item  \[3^3+4^3+5^3=6^3\]\\
 \item $$  \sqrt{100}=10 $$\\
 \item \[(a+b)^3=a^3+3a^2b+3ab^2+b^3\]\\
\item $$ \sum_{k=1}^n k=\frac{n(n+1)}{2}$$\\
 \item \[\frac{\pi}{4}=\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\cdots\]\\
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 \item   $$\cos\theta=sin(90^{\circ}-\theta)$$\\
  \item  \[e^{i\theta}=cos\theta+isin\theta\]\\
\item $$ \lim_{\theta \rightarrow 0} \frac{sin\theta }{\theta}=1$$\\
\item $$\lim_{x \rightarrow \infty}\frac{\pi(x)}{x/logx}=1$$\\
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\end{frame}

\begin{frame}{5. Typeset the following sentences.}
  \begin{itemize}
      \item Positive numbers a,b, and c are the side lengths of a triangle if and only if $a+b>c, b+c>a,$ and    $c+a>b.$
      \item The area of a triangle with side lengths a,b,c is given by \emph{Heron's formula}:
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\end{frame}

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      \[A=\sqrt{s(s-a)(s-b)(s-c)}\],
      where s is the semiperimeter $(a+b+c)/2$.
      \item The volume of a regular tetrahedron of edge length 1 is $\sqrt{2}/12$
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  \begin{itemize}
  \item The derivative of a function f , denoted f', is defined by
  \[f'(x)=\lim_{h\rightarrow 0}\frac{f(x+h)-f(x)}{h}.\]
  \item A real-valued function f is convex on an interval I if
  \[f(\lambda x+(1-\lambda)y)\leq\lambda f(x)+(1-\lambda)f(y)\],
  for all$ x,y\in I$  and    $0 \leq \lambda \leq 1$.
  \item The general solution to the differential equation
  \[y''-3y'+2y=0\]
  \[y=C_1e^x+C_2e^2x\].
  \item The Fermat number $F_n$ is defined as
  \[F_n=2^{2n},     n \geq 0\]
\end{itemize}
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\begin{frame}{6. Make the following equations. Notice the large delimiters.}
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\end{frame}
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\begin{itemize}
    \item \[\frac{d}{dx}\frac{(x)}{(x+1)}=\frac{1}{(x+1)^2}\]
    \item \[\lim_{n \rightarrow \infty}\left(1+\frac{1}{n}\right)^n=e\]
    \item \[ \left|\begin{array}{cc}
    a     & b \\
      c   & d
    \end{array} \right| = ad-bc \]
    \item \[R_\theta = \left[ \begin{array}{cc}
     cos\theta    &  -sin\theta\\
       sin\theta  & cos\theta
    \end{array} \right] \]
\end{itemize}
\end{frame}
\begin{frame}
\begin{itemize}
\item \[ \left|\begin{array}{ccc}
  \boldsymbol{i}  &  \boldsymbol{j} &  \boldsymbol{k} \\
   a_1     &  a_2 &  a_3 \\
   b_1 & b_2 & b_3
  \end{array} \right|= \left|\begin{array}{cc}
     a_2  & a_3 \\
    a_2   &  b_3
  \end{array}\right| \boldsymbol{i}-\left|\begin{array}{cc}
    a_1   &  a_3\\
    b_1   &  b_3
  \end{array}\right| \boldsymbol{j}+\left|\begin{array}{cc}
    a_1   &  a_2\\
    b_1   & b_2
  \end{array}\right| \boldsymbol{k} \]
\item  \[ \left[ \begin{array}{cc}
     a_{11}  & a_{12} \\
     a_{21}  & a_{22}
  \end{array}\right] \left[ \begin{array}{cc}
     b_{11}  & b_{12} \\
    b_{21}   &  b_{22}
  \end{array}\right]= \left[ \begin{array}{cc}
    a_{11}b_{11}+a_{12}b_{21}   &  a_{11}b_{12}+a_{12}b_{22}\\
    a_{21}b_{11}+a_{22}b_{21}   &  a_{21}b_{12}+a_{22}b_{22}
  \end{array}\right] \]
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    a_1   &  a_2\\
    b_1   & b_2
  \end{array}\right| \boldsymbol{k} \]
\item  \[ \left[ \begin{array}{cc}
     a_{11}  & a_{12} \\
     a_{21}  & a_{22}
  \end{array}\right] \left[ \begin{array}{cc}
     b_{11}  & b_{12} \\
    b_{21}   &  b_{22}
  \end{array}\right]= \left[ \begin{array}{cc}
    a_{11}b_{11}+a_{12}b_{21}   &  a_{11}b_{12}+a_{12}b_{22}\\
    a_{21}b_{11}+a_{22}b_{21}   &  a_{21}b_{12}+a_{22}b_{22}
  \end{array}\right] \]
\item \[ f(x)=\left\{ \begin{array}{lr}
    -x^2, & x<0 \\
    x^2, &  0 \leq x \leq 2 \\
    4 , &  x>2
 \end{array}\right. \]
 \end{itemize}
\end{frame}
\begin{frame}{7. Make the following multi-line equation}
 \begin{eqnarray}
1+2&=&3 \nonumber\\
 4+5+6&=&7+8 \nonumber\\
 9+10+11+12&=&21+22+23+24 \nonumber\\
 16+17+18+19+20&=&21+22+23+24 \nonumber\\
 25+26+27+28+29+30&=&31+32+33+34+35
 \nonumber \end{eqnarray}
\end{frame}
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```latex

\begin{frame}
  \begin{eqnarray}
  (a+b)^2&=&(a+b)(a+b) \nonumber \\
  &=&(a+b)a+(a+b)b  \nonumber \\
  &=&a(a+b)+b(a+b)  \nonumber \\
  &=&a^2+ab+ba+b^2 \nonumber \\
  &=&a^22+ab+ab+b^2  \nonumber \\
  &=&a^2+2ab+b^2  \nonumber
  \end{eqnarray}
\end{frame}

\begin{frame}
  \begin{eqnarray}
  tan(\alpha + \beta+ \gamma)&=&\frac{tan(\alpha + \beta)+tan \gamma}{1-tan(\alpha + \beta)tan\gamma}\nonumber \\
  &=&\frac{\frac{tan\alpha+tan\beta}{1-tan\alpha tan\beta}+tan\gamma}{1-(\frac{tan\alpha+tan\beta}{1-tan\alpha tan\beta})tan\gamma}  \nonumber \\
  &=&\frac{tan\alpha+tan\beta+(1-tan\alpha tan\beta)tan\gamma}{1-tan\alpha tan\beta-(tan\alpha+tan\beta)tan\gamma} \nonumber \\
  &=&\frac{tan\alpha+tan\beta+tan\gamma-tan\alpha tan\beta tan\gamma}{1-tan\alpha tan\beta-tan\alpha tan\gamma-tan\beta tan\gamma} \nonumber
  \end{eqnarray}
\end{frame}

\begin{frame}
  \begin{eqnarray}
\prod_p (1-\frac{1}{p^2})&=&\prod_p \frac{1}{1+\frac{1}{p^2}+\frac{1}{p^4}+\cdots} \nonumber\\
&=&\left(\prod_p\left(1+\frac{1}{p^2}+\frac{1}{p^4}+\cdots\right)\right)^{-1} \nonumber\\
&=&\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}\cdots\right)^{-1} \nonumber\\
&=&\frac{6}{\pi^2} \nonumber
 \end{eqnarray}
\end{frame}
\begin{frame}
\begin{center}
  \includegraphics{images.jpg}
\end{center}
\end{frame}
\end{document}

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