

Assignment

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College Roll no.- MAT/20/37
University Roll no.- 20044563002

Question no.- 1

- Let $x = (x_1, \dots, x_n)$, where the x_i are non negative real numbers. Set

$$M_r(x) = \left(\frac{x_1^r + x_2^r + \dots + x_n^r}{n} \right)^{1/r}, \quad r \in \mathbb{R} \setminus \{0\},$$

and

$$M_0(x) = (x_1 x_2 \dots x_n)^{1/n}.$$

We call $M_r(x)$ the *rth power mean* of x .

Claim:

$$\lim_{r \rightarrow 0} M_r(x) = M_0(x).$$

Question no.- 2

- Define:

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 & \dots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1} \end{bmatrix}.$$

We call V_n the *Vandermonde matrix* of order n . Claim:

$$\det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i).$$

4. Make the following equations.



$$3^3 + 4^3 + 5^3 = 6^3$$



$$\sqrt{100} = 10$$



$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$



$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$



$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots$$



$$\cos \theta = \sin(90^\circ - \theta)$$



$$e^{i\theta} = \cos\theta + i\sin\theta$$



$$\lim_{\theta \rightarrow 0} \frac{\sin\theta}{\theta} = 1$$



$$\lim_{x \rightarrow \infty} \frac{\pi(x)}{x/\log x} = 1$$



$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

5. Typeset the following sentences.

- Positive numbers a, b , and c are the side lengths of a triangle if and only if $a + b > c$, $b + c > a$, and $c + a > b$.
- The area of a triangle with side lengths a, b, c is given by *Heron's formula*:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

, where s is the semiperimeter $(a + b + c)/2$.

- The volume of a regular tetrahedron of edge length 1 is $\sqrt{2}/12$
- The quadratic equation $ax^2 + bx + c = 0$ has roots

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The derivative of a function f , denoted f' , is defined by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}.$$

- A real-valued function f is convex on an interval I if

$$f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y)$$

, for all $x, y \in I$ and $0 \leq \lambda \leq 1$.

- The general solution to the differential equation

$$y'' - 3y' + 2y = 0$$

$$y = C_1 e^x + C_2 e^{2x}$$

- The Fermat number F_n is defined as

$$F_n = 2^{2^n}, n \geq 0$$

6. Make the following equations, Notice the large elimiters.



$$\frac{d}{dx} \frac{(x)}{(x+1)} = \frac{1}{(x+1)^2}$$



$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$$



$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$



$$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ a_2 & b_3 \end{vmatrix} i - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} j + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} k$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

$$f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & 0 \leq x \leq 2 \\ 4, & x > 2 \end{cases}$$

7. Make the following multi-line equation

$$1 + 2 = 3$$

$$4 + 5 + 6 = 7 + 8$$

$$9 + 10 + 11 + 12 = 21 + 22 + 23 + 24$$

$$16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24$$

$$25 + 26 + 27 + 28 + 29 + 30 = 31 + 32 + 33 + 34 + 35$$

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) \\ &= (a + b)a + (a + b)b \\ &= a(a + b) + b(a + b) \\ &= a^2 + ab + ba + b^2 \\ &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2\end{aligned}$$

$$\begin{aligned}
 \tan(\alpha + \beta + \gamma) &= \frac{\tan(\alpha + \beta) + \tan\gamma}{1 - \tan(\alpha + \beta)\tan\gamma} \\
 &= \frac{\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} + \tan\gamma}{1 - \left(\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}\right)\tan\gamma} \\
 &= \frac{\tan\alpha + \tan\beta + (1 - \tan\alpha\tan\beta)\tan\gamma}{1 - \tan\alpha\tan\beta - (\tan\alpha + \tan\beta)\tan\gamma} \\
 &= \frac{\tan\alpha + \tan\beta + \tan\gamma - \tan\alpha\tan\beta\tan\gamma}{1 - \tan\alpha\tan\beta - \tan\alpha\tan\gamma - \tan\beta\tan\gamma}
 \end{aligned}$$

$$\begin{aligned}
\prod_p \left(1 - \frac{1}{p^2}\right) &= \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots} \\
&= \left(\prod_p \left(1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots\right) \right)^{-1} \\
&= \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \dots\right)^{-1} \\
&= \frac{6}{\pi^2}
\end{aligned}$$

Thank
you!

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1 \documentclass{beamer}
2 \usepackage[utf8]{inputenc}
3 \usepackage{graphicx}
4 \title{Assignment }
5 \author{Khushi}
6 \institute{Mata Sundri College for women\ University of Delhi}
7 \date{}
8 \usetheme{Berlin}
9 \begin{document}
10 \begin{frame}
11 \titlepage
12 \textbf{College Roll no.- MAT/20/37}
13
14
15 \textbf{University Roll no.- 20044563002}
16 \end{frame}
17
18 \begin{frame}{Question no.- 1}
19 \begin{itemize}
20 \item Let  $\mathbf{x}=(x_1,\dots,x_n)$ ,
21 where the  $x_i$  are non negative real numbers.
22 Set  $M_r(\mathbf{x})= \left(\frac{x_1^r+x_2^r+\dots+x_n^r}{n}\right)^{1/r},\;\; r\in \mathbf{R}$ 
23 and  $M_0(\mathbf{x})=\left(x_1 x_2 \dots x_n\right)^{1/n}.$ 
24 We call  $M_r(\mathbf{x})$  the  $r$ th power mean of  $\mathbf{x}$ .
25
26 Claim:
27 
$$\lim_{r \rightarrow 0} M_r(x) = M_0(\mathbf{x}).$$

28 \end{itemize}
29 \end{frame}
30

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31 = \begin{frame}{Question no.- 2}
32 =   \begin{itemize}
33     \item Define:
34     \[V_n=\left[\begin{array}{ccccc}
35       1 & 1 & 1 & \dots & 1 \\
36       x_1 & x_2 & x_3 & \dots & x_n \\
37       x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\
38       \vdots & \vdots & \vdots & \ddots & \vdots \\
39       x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1}
40     \end{array}\right]
41     \right].\]
42 We call  $V_n$  the Vandermonde matrix of order  $n$ .
43 Claim:
44 \[ \det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i) .\]
45   \end{itemize}
46 \end{frame}
47
48 = \begin{frame}{4. Make the following equations.}
49 = \begin{itemize}
50 \item  $3^3+4^3+5^3=6^3$ 
51 \item  $\sqrt{100}=10$ 
52 \item  $(a+b)^3=a^3+3a^2b+3ab^2+b^3$ 
53 \item  $\sum_{k=1}^n k = \frac{n(n+1)}{2}$ 
54 \item  $\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots$ 
55 \end{itemize}
56 \end{frame}
57 = \begin{frame}
58 = \begin{itemize}
59 \item  $\cos \theta = \sin(90^\circ - \theta)$ 
60 \item  $e^{i\theta} = \cos \theta + i \sin \theta$ 
61 \item  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ 
62 \item  $\lim_{x \rightarrow \infty} \frac{\pi(x)}{x \log x} = 1$ 
63 \item  $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$ 
64 \end{itemize}
65 \end{frame}
66
67 = \begin{frame}{5. Typeset the following sentences.}
68 = \begin{itemize}
69 \item Positive numbers  $a, b,$  and  $c$  are the side lengths of a triangle if and only if  $a+b > c,$ 
 $b+c > a,$  and  $c+a > b.$ 
70 \item The area of a triangle with side lengths  $a, b, c$  is given by Heron's formula:

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66
67 * \begin{frame}{5. Typeset the following sentences.}
68 * \begin{itemize}
69     \item Positive numbers  $a, b,$  and  $c$  are the side lengths of a triangle if and only if  $a+b>c,$ 
70      $b+c>a,$  and  $c+a>b.$ 
71     \item The area of a triangle with side lengths  $a, b, c$  is given by \emph{Heron's formula}:
72      $[A=\sqrt{s(s-a)(s-b)(s-c)}],$ 
73     where  $s$  is the semiperimeter  $(a+b+c)/2.$ 
74     \item The volume of a regular tetrahedron of edge length  $1$  is  $\sqrt{2}/12.$ 
75     \item The quadratic equation  $ax^2+bx+c=0$  has roots
76      $[r_1, r_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}].$ 
77 \end{itemize}
78 \end{frame}
79 * \begin{frame}
80 \begin{itemize}
81     \item The derivative of a function  $f,$  denoted  $f',$  is defined by
82      $[f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}].$ 
83     \item A real-valued function  $f$  is convex on an interval  $I$  if
84      $[f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda)f(y)],$ 
85     for all  $x, y \in I$  and  $0 \leq \lambda \leq 1.$ 
86     \item The general solution to the differential equation
87      $[y'' - 3y' + 2y = 0]$ 
88     is  $[y = C_1 e^x + C_2 e^{2x}].$ 
89     \item The Fermat number  $F_n$  is defined as
90      $[F_n = 2^{2^n}, \quad n \geq 0]$ 
91 \end{itemize}
92 \end{frame}

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91 \end{frame}
92 * \begin{frame}{6. Make the following equations, Notice the large elimiters.}
93 * \begin{itemize}
94   \item \[\frac{d}{dx}\frac{(x)}{(x+1)}=\frac{1}{(x+1)^2}\]
95   \item \[\lim_{n \rightarrow \infty} \left(1+\frac{1}{n}\right)^n=e\]
96   \item \left[ \begin{array}{cc}
97     a & & b \\
98     c & & d \\
99   \end{array} \right] = ad-bc
100   \item \left[ R_{\theta} = \begin{array}{cc}
101     \cos\theta & & -\sin\theta \\
102     \sin\theta & & \cos\theta \\
103   \end{array} \right]
104 \end{itemize}
105 \end{frame}
106 * \begin{frame}
107 * \begin{itemize}
108   \item \left[ \begin{array}{ccc}
109     \boldsymbol{i} & & \boldsymbol{j} & & \boldsymbol{k} \\
110     a_1 & & a_2 & & a_3 \\
111     b_1 & & b_2 & & b_3 \\
112   \end{array} \right] = \left[ \begin{array}{cc}
113     a_2 & & a_3 \\
114     a_2 & & b_3 \\
115   \end{array} \right] \boldsymbol{i} - \left[ \begin{array}{cc}
116     a_1 & & a_3 \\
117     b_1 & & b_3 \\
118   \end{array} \right] \boldsymbol{j} + \left[ \begin{array}{cc}
119     a_1 & & a_2 \\
120     b_1 & & b_2 \\
121   \end{array} \right] \boldsymbol{k}
122   \item \left[ \begin{array}{cc}
123     a_{11} & & a_{12} \\
124     a_{21} & & a_{22} \\
125   \end{array} \right] \left[ \begin{array}{cc}
126     b_{11} & & b_{12} \\
127     b_{21} & & b_{22} \\
128   \end{array} \right] = \left[ \begin{array}{cc}
129     a_{11}b_{11}+a_{12}b_{21} & & a_{11}b_{12}+a_{12}b_{22} \\
130     a_{21}b_{11}+a_{22}b_{21} & & a_{21}b_{12}+a_{22}b_{22} \\
131   \end{array} \right]

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119     a_1 & a_2 \\
120     b_1 & b_2
121 \end{array}\right] \boldsymbol{k} \ ]
122 \item \left[ \begin{array}{cc}
123     a_{11} & a_{12} \\
124     a_{21} & a_{22}
125 \end{array}\right] \left[ \begin{array}{cc}
126     b_{11} & b_{12} \\
127     b_{21} & b_{22}
128 \end{array}\right] = \left[ \begin{array}{cc}
129     a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\
130     a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22}
131 \end{array}\right] \ ]
132 \item \left[ f(x) = \begin{array}{l}
133     -x^2, \text{ \& } x < 0 \\
134     x^2, \text{ \& } 0 \leq x \leq 2 \\
135     4, \text{ \& } x > 2
136 \end{array}\right]. \ ]
137 \end{itemize}
138 \end{frame}
139 \begin{frame}{7. Make the following multi-line equation}
140 \begin{eqnarray}
141 1+2&=&3 \ \text{\nonumber} \\
142 4+5+6&=&7+8 \ \text{\nonumber} \\
143 9+10+11+12&=&21+22+23+24 \ \text{\nonumber} \\
144 16+17+18+19+20&=&21+22+23+24 \ \text{\nonumber} \\
145 25+26+27+28+29+30&=&31+32+33+34+35 \\
146 \ \text{\nonumber} \end{eqnarray}
147 \end{frame}
148

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148
149 * \begin{frame}
150 * \begin{eqnarray}
151 (a+b)^2&=&(a+b)(a+b) \nonumber \\
152 &=&(a+b)a+(a+b)b \nonumber \\
153 &=&a(a+b)+b(a+b) \nonumber \\
154 &=&a^2+ab+ba+b^2 \nonumber \\
155 &=&a^2+ab+ab+b^2 \nonumber \\
156 &=&a^2+2ab+b^2 \nonumber \\
157 \end{eqnarray}
158 \end{frame}
159
160 * \begin{frame}
161 * \begin{eqnarray}
162 \tan(\alpha + \beta + \gamma)&=&\frac{\tan(\alpha + \beta) + \tan \gamma}{1 - \tan(\alpha + \beta) \tan \gamma} \nonumber \\
163 &=&\frac{\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} + \tan \gamma}{1 - (\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}) \tan \gamma} \nonumber \\
164 &=&\frac{\tan \alpha + \tan \beta + (1 - \tan \alpha \tan \beta) \tan \gamma}{1 - \tan \alpha \tan \beta - (\tan \alpha + \tan \beta) \tan \gamma} \nonumber \\
165 &=&\frac{\tan \alpha + \tan \beta + \tan \gamma - \tan \alpha \tan \beta \tan \gamma}{1 - \tan \alpha \tan \beta - \tan \alpha \tan \beta \tan \gamma} \nonumber \\
166 \end{eqnarray}
167 \end{frame}
168
169 * \begin{frame}
170 * \begin{eqnarray}
171 \prod_p (1 - \frac{1}{p^2})&=&\prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots} \nonumber \\
172 &=&\left( \prod_p \left( 1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots \right) \right)^{-1} \nonumber \\
173 &=&\left( 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \dots \right)^{-1} \nonumber \\
174 &=&\frac{6}{\pi^2} \nonumber \\
175 \end{eqnarray}
176 \end{frame}
177 * \begin{frame}
178 * \begin{center}
179 \includegraphics{images.jpg}
180 \end{center}
181 \end{frame}
182 \end{document}
183

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