Presentation

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Question 1

- Let $x = (x_1, \ldots, x_n)$ where the x_i are non negative real numbers. Set
$$M_r(x) = \left(\frac{x_1^r + x_2^r + \cdots + x_n^r}{n}\right)^{1/r}, \quad r \in \mathbb{R} \setminus \{0\},$$
and
$$M_0(x) = (x_1 x_2 \ldots x_n)^{1/n}.$$
We call $M_r(x)$ the *rth power mean* of x.
Claim:
$$\lim_{r \to 0} M_r(x) = M_0(x)$$

- Define

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 & \dots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1} \end{bmatrix}.$$

We will call V_n the *Vandermonde matrix* of order n

Claim:

$$\det V_n = \prod_{1 \le i < j \le n} (x_j - x_i)$$

Question4

- $$3^3 + 4^3 + 5^3 = 6^3$$
- $$\sqrt{100} = 10$$
- $$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
- $$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$
- $$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots$$

- $$cos\theta = sin(90^\circ - \theta)$$
- $$e^{i\theta} = cos\theta + isin\theta$$
- $$\lim_{\theta \to \infty} \frac{sin\theta}{\theta} = 1$$
- $$\lim_{x \to \infty} \frac{\pi(x)}{x/logx} = 1$$
- $$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

Question 5

- Positive numbers a, b and c are the side lengths of a triangle if and only if $a + b > c, b + c > a and c + a > b$.
- The area of a triangle with side lengths a,b,c is given by *Heron's formula*: $A = \sqrt{s(s-a)(s-b)(s-c)}$,
  where $s$is the semiperimeter (a+b+c)/2.

- The volume of a regular tetrahedron of edge length 1 is $\sqrt{2}/12$.

- The quadratic equation $ax^2 + bx + c = 0$ has roots

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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- The *derivative* of a function f,denoted f', is defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- A real-valued function f is *convex* on an interval I if

$$f(\lambda + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda) f(y)$$

, for all x,y $\in$ $Iand0 \leq \lambda \leq 1$.

- The general solution to the differential equataion is

$$y'' - 3y' + 2y = 0$$

$$y = C_1e^x + C_2e^{2x}$$

- The *Fermat number* Fn is defined as

$$F_n = 2^{2n}, n \geq 0.$$

Question 6

- $$\frac{d}{dx}\left(\frac{x}{x+1}\right) = \frac{1}{(x+1)^2}$$
- $$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$
- $$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$
- $$R_\theta = \begin{bmatrix} Cos\theta & -Sin\theta \\ Sin\theta & Cos\theta \end{bmatrix}$$

- $\begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} i - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} j + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} k$

- $$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

- $$f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & 0 \le x \le 2 \\ 4, & x > 2 \end{cases}$$

Question 7

Part 1

$$1 + 2 = 3$$
$$4 + 5 + 6 = 7 + 8$$
$$9 + 10 + 11 + 12 = 13 + 14 + 15$$
$$16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24$$
$$25 + 26 + 27 + 28 + 29 + 30 = 31 + 32 + 33 + 34 + 35$$

Part 2

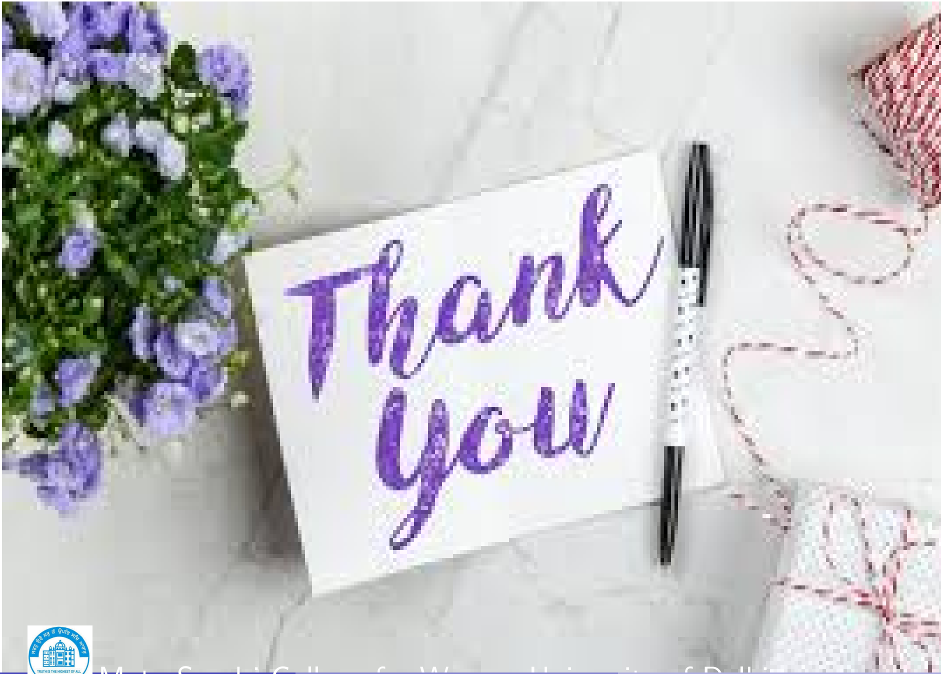
$$
\begin{aligned}
(a+b)^2 &= (a+b)(a-b) \\
&= (a+b)a + (a-b)b \\
&= a(a+b) + b(a+b) \\
&= a^2 + ab + ba + b^2 \\
&= a^2 + ab + ab + b^2 \\
&= a^2 + 2ab + b^2
\end{aligned}
$$

Part 3

$$
\begin{aligned}
tan(\alpha+\beta+\gamma) &= \frac{tan(\alpha+\beta)+tan\gamma}{1-tan(\alpha+\beta)tan\gamma} \\
&= \frac{\frac{tan\alpha+tan\beta}{1-tan\alpha tan\beta}+tan\gamma}{1-(\frac{tan\alpha+tan\beta}{1-tan\alpha tan\beta})tan\gamma} \\
&= \frac{tan\alpha+tan\beta+(1-tan\alpha tan\beta)tan\gamma}{1-tan\alpha tab\beta-(tan\alpha+tan\beta)tan\gamma} \\
&= \frac{tan\alpha+tan\beta+tan\gamma+tan\alpha tan\beta tan\gamma}{1-tan\alpha tan\beta-tan\alpha tan\gamma-tan\beta tan\gamma}
\end{aligned}
$$

Part 4

$$\prod_p \left(1-\frac{1}{p^2}\right) = \prod_p \frac{1}{1+\frac{1}{p^2}+\frac{1}{p^4}+\cdots}$$
$$= \left(\prod_p \left(1+\frac{1}{p^2}+\frac{1}{p^4}+\cdots\right)\right)^{-1}$$
$$= \left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots\right)^{-1}$$
$$= \frac{6}{\pi^2}$$


Thank
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