



Mata Sundri College For women
(University of Delhi)



Assignment 2

Name - Aashi Rajput

Roll no.- MAT/20/118

University roll no.- 20044563040

- 1 Let $\mathbf{x} = (x_1, \dots, x_n)$, where the x_i are nonnegative real numbers. Set

$$M_r(\mathbf{x}) = \left(\frac{x_1^r + x_2^r + \dots + x_n^r}{n} \right)^{1/r}, \quad r \in \mathbf{R} \setminus \{0\},$$

and

$$M_0(\mathbf{x}) = (x_1 x_2 \dots x_n)^{1/n}.$$

We call $M_r(\mathbf{x})$ the *r*th power mean of \mathbf{x}

Claim :

$$\lim_{r \rightarrow 0} M_r(\mathbf{x}) = M_0(\mathbf{x})$$

2 Define

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 & \dots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1} \end{bmatrix}.$$

We call V_n the *Vandermonde matrix* of order n .

Claim:

$$\det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i).$$

Question 4.

Make the following equations;

1 $3^3 + 4^3 + 5^3 = 6^3$

2 $\sqrt{100} = 10$

3 $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

4 $\sum_{k=1}^n k = \frac{n(n+1)}{2}$

5 $\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots$

Remaining parts of question 4

6 $\cos\theta = \sin(90^\circ - \theta)$

7 $e^{i\theta} = \cos\theta + i\sin\theta$

8 $\lim_{\theta \rightarrow 0} \frac{\sin\theta}{\theta} = 1$

9 $\lim_{x \rightarrow \infty} \frac{\frac{\pi(x)}{x}}{\log x} = 1$

10 $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$

Question 5.

Typeset the following sentences :

- 1 Positive number a, b, c are the side lengths of a triangle if and only if $a + b > c, b + c > a$, and $c + a > b$.
- 2 The area of a triangle with side lengths a, b, c is given by *Heron's formula* :

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where s is the semiperimeter $(a + b + c)/2$.

- 3 The Volume of a regular tetrahedron of edge length 1 is $\frac{\sqrt{2}}{12}$.

Remaining parts of question 5.

- 4 The quadratic equation $ax^2 + bx + c = 0$ has roots

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- 5 The *derivative* of a function f , denoted f' , is defined by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- 6 A real valued function f is *convex* on an interval I if

$$f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y),$$

for all x and $y \in I$ and $0 \leq \lambda \leq 1$.

Remaining parts of question 5.

- 7 The general solution to the differential equation

$$y'' - 3y' = 2y = 0$$

is

$$y = C_1 e^x + C_2 e^{2x}.$$

- 8 The *Fermat number* F_n is defined as

$$F_n = 2^{2^n}, n \geq 0.$$

Question 6.

Make the following equations. Notice the large delimiters.

$$1 \quad \frac{d}{dx} \frac{x}{x+1} = \frac{1}{(x+1)^2}$$

$$2 \quad \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$$

$$3 \quad \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$4 \quad R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Remaining Parts of question 6.

$$5 \quad \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

$$6 \quad \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + b_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

$$7 \quad f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & 0 \leq x \leq 2 \\ 4, & x > 2 \end{cases}$$

Question 7.

Make the following multi-line equations;

1

$$1 + 2 = 3$$

$$4 + 5 + 6 = 7 + 8$$

$$9 + 10 + 11 + 12 = 13 + 14 + 15$$

$$16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24$$

$$25 + 26 + 27 = 28 + 30 = 31 + 32 + 33 + 34 + 35$$

2

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) \\ &= (a + b)a + (a + b)b \\ &= a^2 + ab + ba + b^2 \\ &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2\end{aligned}$$

3

$$\begin{aligned}\tan(\alpha + \beta + \gamma) &= \frac{\tan(\alpha + \beta) + \tan\gamma}{1 - \tan(\alpha + \beta)\tan\gamma} \\ &= \frac{\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} + \tan\gamma}{1 - \left(\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}\right)\tan\gamma} \\ &= \frac{\tan\alpha + \tan\beta + (1 - \tan\alpha\tan\beta)\tan\gamma}{1 - \tan\alpha\tan\beta - (\tan\alpha + \tan\beta)\tan\gamma} \\ &= \frac{\tan\alpha + \tan\beta + \tan\gamma - \tan\alpha\tan\beta\tan\gamma}{1 - \tan\alpha\tan\beta - \tan\alpha\tan\gamma - \tan\beta\tan\gamma}\end{aligned}$$

4

$$\begin{aligned}\prod_p \left(1 - \frac{1}{p^2}\right) &= \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots} \\ &= \left(\prod_p \left(1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots\right) \right)^{-1} \\ &= \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots\right)^{-1} \\ &= \frac{6}{\pi^2}\end{aligned}$$

Thank You



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1 \documentclass{beamer}
2 \usepackage[utf8]{inputenc}
3 \usepackage{xcolor}
4 \usepackage{shadowtext}
5 \usepackage{graphicx}
6 \geometry{papersize={13cm,12cm}}
7 \usetheme{Berlin}
8 \setbeamerfont{title}{size=\huge}
9 \setbeamerfont{author}{size=\LARGE}
10 \setbeamertemplate{background}
11 {
12 \includegraphics[width=\paperwidth,height=\paperheight]{th (5).jpg}
13 }
14 \title{\textcolor{black}{Assignment 2}}
15 \author{\textcolor{purple}{Name - Aashi Rajput}\vspace{0.5cm}{\textcolor{purple}{Roll no.-
MAT/20/118}}\vspace{0.5cm}{\textcolor{purple}{University roll no.- 20044563040}}}
16 \date{}
17 \begin{document}
18 \begin{frame}
19 \begin{minipage}[0.13\linewidth]
20 \includegraphics[width=1.5cm,height=1.5cm]{th (2).jpg}
21 \end{minipage}\hfill
22 \begin{minipage}[0.7\linewidth]
23 \centering\Large{\textcolor{purple}{\shadowtext{Mata Sundri College For women}}}
\\{\Large{\textcolor{purple}{\shadowtext{(University of Delhi)}}}}
24 \end{minipage}\hfill
25 \begin{minipage}[0.13\linewidth]
26 \includegraphics[width=1.5cm,height=1.5cm]{th (3).jpg}
27 \end{minipage}\hfill\vspace{0.5cm}
28 \Large\titlepage
29 \end{frame}
30 \begin{enumerate}
31 \begin{frame}{My Document}
32 \begin{block}
33
34 \item \Large{Let  $\textbf{x}=(x_1,\dots,x_n)$ , where the  $x_i$  are nonnegative real numbers. Set
35  $M_r(\textbf{x})=\left(\frac{x_1^r+x_2^r+\dots+x_n^r}{n}\right)^{1/r}$ ,  $\forall r \in \textbf{R}$  \setminus \{0\}, \} \\
and
36  $M_0(\textbf{x})=\left(x_1x_2 \dots x_n\right)^{1/n}$ . \\
37 We call  $M_r(\textbf{x})$  the  $r$ th power mean of  $\textbf{x}$  \\
38 Claim :
39  $\lim_{r \rightarrow 0} M_r(\textbf{x})=M_0(\textbf{x})$  \\
40
41 \end{block}
42 \end{frame}
43 \begin{frame}
44 \begin{block}
45
46 \item \Large{Define
47  $V_n=$ 
48 \left[
49 \begin{array}{ccccc}
50 1 & 1 & 1 & \dots & 1 \\
51 x_1 & x_2 & x_3 & \dots & x_n \\
52 x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\
53 \vdots & \vdots & \vdots & \ddots & \vdots \\
54 x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1}
55 \end{array} \right]. \\
56 We call  $V_n$  the Vandermonde matrix of order  $n$ . \\
57 Claim:
58  $\det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i)$ . \\
59 \end{block}
60 \end{frame}
61 \end{enumerate}
62 \begin{enumerate}
63 \begin{frame}[fragile]{Question 4.}
64 \begin{block}{\LARGE{Make the following equations;}}
65 \item \LARGE{\framebox[\textwidth]{ $3^3+4^3+5^3=6^3$ }}
66 \item \framebox[\textwidth]{ $\sqrt{100}=10$ }
67 \item \framebox[\textwidth]{ $(a+b)^3=a^3+3a^2b+3ab^2+b^3$ }
68 \item \framebox[\textwidth]{ $\sum_{k=1}^n k = \frac{n(n+1)}{2}$ }
69 \item \framebox[\textwidth]{ $\frac{\pi}{4}=\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\dots$ }
70 \end{block}
71 \end{frame}
72 \begin{frame}{Remaining parts of question 4}
73 \begin{block}
74
75 \item \LARGE{\framebox[\textwidth]{ $\cos\theta=\sin(90^\circ-\theta)$ }}
76 \item \framebox[\textwidth]{ $e^{i\theta}=\cos\theta+i\sin\theta$ }
77 \item \framebox[\textwidth]{ $\lim_{\theta \rightarrow 0} \frac{\sin\theta}{\theta}=1$ }
78 \item \framebox[\textwidth]{ $\lim_{x \rightarrow \infty} \frac{\log x}{x}=0$ }
79 \item \framebox[\textwidth]{ $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$ }
80 \end{block}
81 \end{frame}
82 \end{enumerate}
83 \begin{enumerate}
84 \begin{frame}{Question 5.}
85 \begin{block}{\LARGE{Typeset the following sentences :}}
86 \item \Large{Positive number  $a,b,c$  are the side lengths of a triangle if and only if  $a+b>c$ ,  $b+c>a$ , and  $c+a>b$ .}

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86 \item \Large{Positive number  $a, b, c$  are the side lengths of a triangle if and only if  $a+b>c, b+c>a$ , and  $c+a>b$ }.
87 \item The area of a triangle with side lengths  $a, b, c$  is given by \emph{Heron's formula} :
88  $[A = \sqrt{s(s-a)(s-b)(s-c)}]$  where \emph{s} is the semiperimeter  $(a+b+c)/2$ .
89 \item The Volume of a regular tetrahedron of edge length 1 is  $\frac{\sqrt{2}}{12}$ .
90 \end{block}
91 \end{frame}
92 \begin{frame}{Remaining parts of question 5.}
93 \begin{block}
94
95 \item \Large{The quadratic equation  $ax^2+bx+c=0$  has roots
96  $[r_1, r_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}]$ 
97 \item The \emph{derivative} of a function  $f$ , denoted  $f'$ , is defined by
98  $[f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}]$ 
99 \item A real valued function  $f$  is \emph{convex} on an interval  $I$  if
100  $[f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda)f(y)]$ ,
101  $[f$  for all  $x, y \in I$  and  $0 \leq \lambda \leq 1$ .]
102 \end{block}
103 \end{frame}
104 \begin{frame}{Remaining parts of question 5.}
105 \begin{block}
106
107 \item \Large{The general solution to the differential equation
108  $[y''-3y'+2y=0]$ 
109 is  $[y=C_1 e^x + C_2 e^{2x}]$ .]
110 \item The \emph{Fermat number}  $F_n$  is defined as
111  $[F_n=2^{2^n}, n \geq 0.]$ 
112 \end{block}
113 \end{frame}
114 \end{enumerate}
115 \begin{enumerate}
116 \begin{frame}{Question 6.}
117 \begin{block}{\Large{Make the following equations. Notice the large delimiters.}}
118 \item \Large{ $\frac{d}{dx} \frac{x}{x+1} = \frac{1}{(x+1)^2}$ }
119 \item  $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n = e$ 
120 \item  $\left[ \begin{array}{l} a & b \\ c & d \end{array} \right]$ 
121 \begin{array}{cc}
122 a & b \\
123 c & d
124 \end{array} \right]
125 =ad-bc
126 \item  $R_\theta =$ 
127  $\left[ \begin{array}{cc} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{array} \right]$ 
128 \begin{array}{cc}
129 \cos \theta & -\sin \theta \\
130 \sin \theta & \cos \theta
131 \end{array} \right]
132 \end{block}
133 \end{frame}
134 \begin{frame}{Remaining Parts of question 6.}
135 \begin{block}
136
137 \item \Large{ $\left[ \begin{array}{ccc} \textbf{i} & \textbf{j} & \textbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{array} \right]$ 
138 \begin{array}{ccc}
139 \textbf{i} & \textbf{j} & \textbf{k} \\
140 a_1 & a_2 & a_3 \\
141 b_1 & b_2 & b_3
142 \end{array} \right] = \left[ \begin{array}{cc}
143 a_2 & a_3 \\
144 b_2 & b_3
145 \end{array} \right] \textbf{i} - \left[ \begin{array}{cc}
146 a_1 & a_3 \\
147 b_1 & b_3
148 \end{array} \right] \textbf{j} + \left[ \begin{array}{cc}
149 a_1 & a_2 \\
150 b_1 & b_2
151 \end{array} \right] \textbf{k}
152 \begin{array}{cc}
153 a_1 & a_2 \\
154 b_1 & b_2
155 \end{array} \right] \textbf{k}
156 \item \small{ $\left[ \begin{array}{cc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right]$ 
157 \begin{array}{cc}
158 a_{11} & a_{12} \\
159 a_{21} & a_{22}
160 \end{array} \right]
161 \left[ \begin{array}{cc}
162 b_{11} & b_{12} \\
163 b_{21} & b_{22}
164 \end{array} \right] =
165 \left[ \begin{array}{cc}
166 a_{11} b_{11} + b_{12} b_{21} & a_{11} b_{12} + a_{12} b_{22} \\
167 a_{21} b_{11} + a_{22} b_{21} & a_{21} b_{12} + a_{22} b_{22}
168 \end{array} \right]
169 \begin{array}{cc}
170 a_{11} b_{11} + b_{12} b_{21} & a_{11} b_{12} + a_{12} b_{22} \\
171 a_{21} b_{11} + a_{22} b_{21} & a_{21} b_{12} + a_{22} b_{22}
172 \end{array} \right]
173  $[f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & 0 \leq x \leq 2 \\ 4, & x > 2 \end{cases}]$ 
174 \end{block}

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172 x^2 , \hspace{0.2cm} 0 \leq x \leq 2 \\
173 4, \hspace{0.2cm} x>2\end{cases}$}
174 \end{block}
175 \end{frame}
176 \end{enumerate}
177 \begin{enumerate}
178 \begin{frame}{\LARGE{Question 7.}}
179 \begin{block}{\LARGE{Make the following multi-line equations;}}
180 \item \Large{\{1+2=3\}}
181 \{4+5+6=7+8\}}
182 \{9+10+11+12=13+14+15\}}
183 \{16+17+18+19+20=21+22+23+24\}}
184 \{25+26+27+28+30=31+32+33+34+35\}}
185 \end{block}
186 \end{frame}
187 \begin{frame}
188 \begin{block}
189
190 \item \LARGE{
191 \begin{eqnarray*}
192 (a+b)^2&=&(a+b)(a+b)\\
193 &=&(a+b)a+(a+b)b\\
194 &=&a^2+ab+ba+b^2\\
195 &=&a^2+ab+ab+b^2\\
196 &=&a^2+2ab+b^2
197 \end{eqnarray*}}
198 \end{block}
199 \end{frame}
200 \begin{frame}
201 \begin{block}
202
203 \item \large{
204 \begin{eqnarray*}
205 \tan(\alpha+\beta+\gamma)&=&\frac{\tan(\alpha+\beta)+\tan\gamma}{1-\tan(\alpha+\beta)\tan\gamma}\\
206 &=&\frac{\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}+\tan\gamma}{1-(\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta})\tan\gamma}\\
207 &=&\frac{\tan\alpha+\tan\beta+(1-\tan\alpha\tan\beta)\tan\gamma}{1-\tan\alpha\tan\beta-(\tan\alpha+\tan\beta)\tan\gamma}\\
208 &=&\frac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\beta-\tan\alpha\tan\gamma-\tan\beta\tan\gamma}
209 \end{eqnarray*}}
210 \end{block}
211 \end{frame}
212 \begin{frame}
213 \begin{block}
214
215 \item \Large{
216 \begin{eqnarray*}
217 \prod_p\left(1-\frac{1}{p^2}\right)&=&\prod_p\frac{1}{1+\frac{1}{p^2}+\frac{1}{p^4}+\dots}\\
218 &=&\left(\prod_p\left(1+\frac{1}{p^2}+\frac{1}{p^4}+\dots\right)\right)^{-1}\\
219 &=&\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\dots\right)^{-1}\\
220 &=&\frac{6}{\pi^2}
221 \end{eqnarray*}}
222 \end{block}
223 \end{frame}
224 \end{enumerate}
225 \begin{frame}{Thank You}
226 \includegraphics[angle=360,scale=1.2]{th (6).jpg}
227 \end{frame}
228 \end{document}

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