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- Let $\mathrm{x} = (x_1, \ldots, x_n)$, where the x_i are non-negative real numbers. Set

$$M_r(\mathrm{x}) = \left(\frac{x_1^r + x_2^r + \cdots + x_n^r}{n} \right)^{1/r}, \quad r \in \mathbb{R} \setminus \{0\},$$

and

$$M_0(\mathrm{x}) = (x_1 x_2 \ldots x_n)^{1/n}.$$

We call $M_r(\mathrm{x})$ the *rth power mean* of x.
Claim:

$$\lim_{r \to 0} M_r(\mathrm{x}) = M_0(\mathrm{x}).$$

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- Define

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 & \dots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1} \end{bmatrix}.$$

  We call V_n the *Vandermonde matrix* of order n.
  Claim:

$$\det V_n = \prod_{1 \le i < j \le n} (x_j - x_i).$$

Question 4

- $$3^3 + 4^3 + 5^3 = 6^3$$
- $$\sqrt{100} = 10$$
- $$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
- $$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$
- $$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots$$

Remaining parts of Question 4

- $$cos\theta = sin(90^\circ - \theta)$$
- $$e^{\iota\theta} = cos\theta + \iota sin\theta$$
- $$\lim_{\theta \to 0} \frac{sin\theta}{\theta} = 1$$
- $$\lim_{x \to \infty} \frac{\pi(x)}{x/\log x} = 1$$
- $$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

Question 5

- Positive numbers a, b, and c are the side lengths of a triangle if and only if $a+b>c$, $b+c>a$, and $c+a>b$.
- The area of a triangle with side lengths a, b, c is given by $Heron's \;\; formula$:

$$A=\sqrt{s(s-a)(s-b)(s-c)},$$

  where s is the semiperimeter $(a+b+c)/2$.
- The volume of a regular tetrahedron of edge length 1 is $\sqrt{2}/12$.
- The quadratic equation $ax^2+bx+c=0$ has roots

$$r_1,r_2=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

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Remaining Parts of Question 5

- The derivative of a function f, denoted f', is defined by

$$f'(x) = \lim_{h \to o} \frac{f(x+h) - f(x)}{h}$$

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- A real-valued function f is *convex* on a interval I if

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda)f(y),$$

for all $a, y \in I$ and $0 \leq \lambda \leq 1$.

Remaining Parts of Question 5

- The general solution to the differential equation

$$y" - 3y' + 2y = 0$$

  is

$$y = C_1 e^x + C_2 e^{2x}.$$

- The *Fermat number* F_n is defined as

$$f_n = 2^{2^n}, \quad n \geq 0.$$

Question 6

- $$\frac{d}{dx}\left(\frac{x}{x+1}\right) = \frac{1}{(x+1)^2}$$
- $$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$
- $$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$
- $$R_\theta = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$$

Remaining Parts of Question 6

- $$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$
- $$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$
- $$f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & 0 \leq x \geq 2 \\ 4, & x > 2 \end{cases}$$

Question 7

- $$\begin{aligned} 1+2 &= 3 \\ 4+5+6 &= 7+8 \\ 9+10+11+12 &= 13+14+15 \\ 16+17+18+19+20 &= 21+22+23+24 \\ 25+26+27+28+29+30 &= 31+32+33+34+35 \end{aligned}$$

Remaining Parts of Question 7

- $$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= (a+b)a+(a+b)b\\ &= a(a+b)+b(a+b)\\ &= a^2+ab+ba+b^2\\ &= a^2+ab+ab+b^2\\ &= a^2+2ab+b^2\end{aligned}$$

Remaining Parts of Question 7

- $$\begin{aligned} tan(\alpha+\beta+\gamma) &= \frac{tan(\alpha+\beta)+tan\gamma}{1-tan(\alpha+\beta)tan\gamma} \\ &= \frac{\frac{tan\alpha+tan\beta}{1-tan\alpha\, tan\beta}+tan\gamma}{1-\left(\frac{tan\alpha+tan\beta}{1-tan\alpha\, tan\beta}\right)tan\gamma} \\ &= \frac{tan\alpha+tan\beta+(1-tan\alpha tan\beta)tan\gamma}{1-tan\alpha tan\beta-(tan\alpha+tan\beta)tan\gamma} \\ &= \frac{tan\alpha+tan\beta+tan\gamma-tan\alpha tan\beta tan\gamma}{1-tan\alpha tan\beta-tan\alpha tan\gamma-tan\beta tan\gamma} \end{aligned}$$

Remaining Parts of Question 7

- $$\begin{aligned}\prod_p \left(1 - \frac{1}{p^2}\right) &= \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots} \\ &= \left(\prod_p \left(\frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots}\right)\right)^{-1} \\ &= \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\right)^{-1} \\ &= \frac{6}{\pi^2}\end{aligned}$$

Thank you