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1 \documentclass{beamer}
2 \usepackage[utf8]{inputenc}
3 \usepackage{framed}
4 \usepackage{xcolor}
5 \usepackage{graphicx}
6
7 \title{{\Huge{\textcolor{green}{ASSIGNMENT 2}}}}
8
9 \institute{{\Large{\textbf{Mata Sundri College for
Women}}}
10
11
12 \textbf{{University of Delhi}}}
13
14 \author{{\LARGE{\textcolor{red}{Sneha Paliwal}} \ \
15 \normalfont{College Roll no.} \ $=$
16 \textbf{{\large{MAT \$/\$ 20 \$/\$ 127}}} \ \
17 \normalfont{University Roll no.} \ $=$
18 \textbf{{\large{20044563049}}}}
19
20 \date{}
21
22 \usetheme{Frankfurt}
23 \usecolortheme{beetle}
24
25 \begin{document}
26
27 \begin{frame}
28 \titlepage{}
29 \end{frame}
30
31 \begin{frame}{EXAMPLE 9.5 : Part 1}
32 \begin{enumerate}
33 \item [\cdot]
34 Let  $\mathbf{x}=(x_1, \dots, x_n)$ ,
35 where the  $x_i$  are nonnegative real numbers.
36 Set
37 \[
38 M_r(\mathbf{x})=\left(\frac{x_1^r+x_2^r+\dots+x_n^r}{n}\right)^{1/r},
39 \];  $r \in \mathbf{R}$  \setminus  $\{0\}$ ,
40 \]
41 and
42 \[
43 M_0(\mathbf{x})=\left(x_1 x_2 \dots x_n\right)^{1/n}.
44 \]
45 We call  $M_r(\mathbf{x})$  the  $r$ th power mean of  $\mathbf{x}$ .

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44
45 Claim :  $\lim_{r \rightarrow 0} M_r(\mathbf{x}) = M_0(\mathbf{x})$ .
46
47 \end{enumerate}
48 \end{frame}
49
50 \begin{frame}{Part 2}
51 \begin{enumerate}
52 \item [\cdot]
53 Define
54 \[
55 V_n=
56 \left[
57 \begin{array}{cccc}
58 1 & 1 & 1 & \dots & 1 \\
59 x_1 & x_2 & x_3 & \dots & x_n \\
60 x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\
61 \vdots & \vdots & \vdots & \ddots & \vdots \\
62 x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1}
63 \end{array}
64 \right].
65 \]
66
67 We call  $V_n$  the Vandermonde
matrix of order  $n$ .
68
69 Claim:
70 \[
71 \det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i).
72 \]
73 \end{enumerate}
74 \end{frame}
75
76 \begin{frame}{QUESTION 4}
77
78 \begin{framed}
79  $3^3+4^3+5^3=6^3$ 
80 \end{framed}
81
82 \begin{framed}
83  $\sqrt{100}=10$ 
84 \end{framed}
85
86 \begin{framed}
87  $(a+b)^3=a^3+3a^2b+3ab^2+b^3$ 
88 \end{framed}
89
90 \begin{framed}

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88 \end{framed}
89
90 \begin{framed}
91 $\sum_{k=1}^n k$ = $\frac{n(n+1)}{2}$
92 \end{framed}
93
94 \begin{framed}
95 $\frac{\pi}{4}$ =
   $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$
96 \end{framed}
97 \end{frame}
98
99 \begin{frame}{Remaining parts of QUESTION 4}
100
101 \begin{framed}
102 $\cos\theta$ = $\sin(90^\circ - \theta)$
103 \end{framed}
104
105 \begin{framed}
106 $e^{i\theta}$ = $\cos\theta + i \sin\theta$
107 \end{framed}
108
109 \begin{framed}
110 $\lim_{\theta \rightarrow 0} \frac{\sin\theta}{\theta} = 1$
111 \end{framed}
112
113 \end{frame}
114
115 \begin{frame}{Remaining parts of QUESTION 4}
116
117 \begin{framed}
118 $\lim_{x \rightarrow \infty} \frac{\pi x}{x \log x}$
   & = & 1.
119 \end{framed}
120
121 \begin{framed}
122 $\int_{-\infty}^{\infty} e^{-x^2} dx$ & = & $\sqrt{\pi}$
123 \end{framed}
124
125 \end{frame}
126
127 \begin{frame}{QUESTION 5 : Typeset the following
   sentences.}
128 $\ast$ Positive numbers a,b and c are the side
   lengths of a triangle if and only if $a+b>c$,
   $b+c>a$ & and & $c+a>b$ \\ [0.20cm]
129 $\ast$ The area of a triangle with side lengths
   a,b,c is given by \emph{Heron's}
   \emph{formula}: \\

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129  $\ast$  The area of a triangle with side lengths
a,b,c is given by  $\emph{Heron's}$ 
 $\emph{formula}$ : $\backslash\backslash$ 
130  $\begin{center}$ 
131  $A=\sqrt{s(s-a)(s-b)(s-c)}$ , $\backslash\backslash$ 
132  $\end{center}$ 
133
134 where s is the semiperimeter
 $(a+b+c)/2$ . $\backslash\backslash[0.20cm]$ 
135  $\ast$  The volume of a regular tetrahedron of
edge length 1 is  $\sqrt{2}/12$ . $\backslash\backslash[0.20cm]$ 
136  $\ast$  The quadratic equation  $ax^2+bx+c=0$  has
roots
137  $\begin{center}\{r_1,r_2=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\}$ 
138  $\end{center}$ 
139  $\end{frame}$ 
140
141  $\begin{frame}$ {Remaining parts of QUESTION 5}
142  $\ast$  The  $\emph{derivative}$  of a function
f,denoted by  $f^\prime$  , is  $\quad$  defined  $\quad$ 
by $\backslash\backslash$ 
143  $\begin{center}$ 
144  $f^\prime(x)=\lim_{h\rightarrow 0}\frac{f(x+h)-f(x)}{h}$ 
145  $\end{center}$ 
146  $\ast$  A real-valued function f is convex on an
interval I if  $\backslash\backslash$ 
147  $\begin{center}\{f(\lambda x + (1-\lambda)y) \leq$ 
 $\lambda f(x) + (1-\lambda)f(y)\}$ , $\backslash\backslash$ 
148  $\end{center}$ 
149 for all  $x,y \in I$  and  $0 \leq \lambda \leq$ 
 $1$ . $\backslash\backslash$ 
150
151
152  $\ast$  The general solution to the differential
equation  $\backslash\backslash$ 
153  $\begin{center}$ 
154  $y''-3y'+2y=0$ 
155  $\end{center}$ 
156 is $\backslash\backslash$ 
157  $\begin{center}$ 
158  $y=C_1e^x+C_2e^{2x}$ . $\backslash\backslash$ 
159  $\end{center}$ 
160
161  $\ast$  The  $\emph{Fermat}$   $\emph{number}$   $F_n$  is defined
as $\backslash\backslash$ 
162  $\begin{center}$ 
163  $\{F_n=2^{2^n} - 1, n \geq 0\}$ 
164  $\end{center}$ 
165  $\end{frame}$ 

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EXAMP

Part 2

QUEST

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166
167 * \begin{frame}{QUESTION 6}
168 * \begin{framed}
169 $\frac{d}{dx} \left( \frac{x}{x+1} \right) =
\frac{1}{(x+1)^2}$ \\
170 \end{framed}
171 * \begin{framed}
172 $\lim_{n \rightarrow \infty} \left( 1 +
\frac{1}{n} \right)^n = e$ \\
173 \end{framed}
174 * \begin{framed}
175
176 $\left|
177 * \begin{array}{cc}
178 a & b \\
179 c & d
180 \end{array}
181 \right| = ad - bc$ \\
182
183 \end{framed}
184 \end{frame}
185
186 * \begin{frame}{Remaining parts QUESTION 6}
187 * \begin{framed}
188 $R_{\theta} =
189 \left[
190 * \begin{array}{cc}
191 \cos\theta & -\sin\theta \\
192 \sin\theta & \cos\theta
193 \end{array}
194 \right]$ \\
195
196 \end{framed}
197 * \begin{framed}
198 $\left|
199 * \begin{array}{ccc}
200 \mathbf{i} & \mathbf{j} & \mathbf{k} \\
201 a_1 & a_2 & a_3 \\
202 b_1 & b_2 & b_3
203 \end{array}
204 \right| = $
205 $\left|
206 * \begin{array}{cc}
207 a_2 & a_3 \\
208 b_2 & b_3
209 \end{array}
210 \right| \mathbf{i} - $
211 $\left|
212 * \begin{array}{cc}
213 a_1 & a_3 \\
214 b_1 & b_3
215 \end{array}

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215 \end{array}
216 \right| \textbf{j} + \$
217 $\left|
218 \begin{array}{cc}
219 a_1 & a_2 \\
220 b_1 & b_2
221 \end{array}
222 \right| \textbf{k} \$
223 \end{framed}
224
225 \end{frame}
226
227 \begin{frame}{Remaining parts of QUESTION 6}
228 \begin{framed}
229 $\left[
230 \begin{array}{cc}
231 a_{1,1} & a_{1,2} \\
232 a_{2,1} & a_{2,2}
233 \end{array}
234 \right]$
235 $\left[
236 \begin{array}{cc}
237 b_{1,1} & b_{1,2} \\
238 b_{2,1} & b_{2,2}
239 \end{array}
240 \right] = \$
241 $\left[
242 \begin{array}{cc}
243 a_{1,1} b_{1,1} + a_{1,2} b_{2,1} & a_{1,1} b_{1,2} + a_{1,2} b_{2,2} \\
244 a_{2,1} b_{1,1} + a_{2,2} b_{2,1} & a_{2,1} b_{1,2} + a_{2,2} b_{2,2}
245 \end{array}
246 \right]$
247 \end{framed}
248 \begin{framed}
249 $f(x) = \$
250 $\left\{
251 \begin{array}{cc}
252 -x^2, & x < 0 \\
253 x^2, & 0 \leq x \leq 2 \\
254 4, & x > 2
255 \end{array}
256 \right\} \cdot \$
257 \end{framed}
258 \end{frame}
259
260 \begin{frame}{QUESTION 7 : Part 1}
261 \begin{framed}
262 \begin{center}
263 1+2 = 3 \\
264 4+5+6 = 7+8 \\
265 9+10+11+12 = 13+14+15 \\
266 16+17+18+19+20 = 21+22+23+24

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265 9+10+11+12 & = & 13+14+15 \\
266 16+17+18+19+20 & = & 21+22+23+24 \\
267 25+26+27+28+29+30 & = & 31+32+33+34+35
268 \end{center}
269 \end{framed}

```



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270 \end{frame}
271
272 * \begin{frame}{Part 2}
273 * \begin{framed}
274 * \begin{align*}
275 (a+b)^2 \quad &= \quad (a+b)(a+b) \\
276 &= \quad (a+b)a + (a+b)b \\
277 &= \quad a(a+b) + b(a+b) \\
278 &= \quad a^2 + ab + \underline{ba} + b^2 \\
279 &= \quad a^2 + ab + ab + b^2 \\
280 &= \quad a^2 + 2ab + b^2
281 \end{align*}
282 \end{framed}
283 \end{frame}

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284
285
286 * \begin{frame}{ Part 3}
287 * \begin{framed}
288 * \begin{align*}
289 \tan(\alpha + \beta + \gamma) \quad &= \quad \frac{\tan(\alpha + \beta) + \tan\gamma}{1 - \tan(\alpha + \beta) \tan\gamma} \\
290 &= \quad \frac{\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta} + \tan\gamma}{1 - \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta} \tan\gamma} \\
291 &= \quad \frac{\tan\alpha + \tan\beta + (1 - \tan\alpha \tan\beta) \tan\gamma}{1 - \tan\alpha \tan\beta - (\tan\alpha + \tan\beta) \tan\gamma} \\
292 &= \quad \frac{\tan\alpha + \tan\beta + \tan\gamma - \tan\alpha \tan\beta \tan\gamma}{1 - \tan\alpha \tan\beta - \tan\alpha \tan\gamma - \tan\beta \tan\gamma} \\
293 \end{align*}
294 \end{framed}
295 \end{frame}

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296
297 * \begin{frame}{ Part 4}
298 * \begin{framed}
299 * \begin{align*}
300 \prod_{p} ( 1 - \frac{1}{p^2} ) \quad &= \quad \prod_{p} \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots} \\
301 &= \quad ( \prod_{p} ( 1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots ) )^{-1} \\
302 &= \quad ( 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots )^{-1}

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293     \end{align*}
294 \end{framed}
295 \end{frame}
296
297 * \begin{frame}{ Part 4}
298 * \begin{framed}
299 * \begin{align*}
300     \prod_{p} ( 1- \frac{1}{p^2} ) \quad &= \\
&\prod_{p} \frac{1}{1+ \frac{1}{p^2} + \\
&\frac{1}{p^4} + \cdots } \\
301     &= \quad ( \prod_{p} ( 1 + \frac{1}{p^2} + \\
&\frac{1}{p^4} + \cdots ) )^{-1} \\
302     &= \quad ( 1 + \frac{1}{2^2} + \frac{1}{3^2} \\
&+ \frac{1}{4^2} + \cdots )^{-1} \\
303     &= \quad \frac{6}{\pi^2} \\
304 \end{align*}
305 \end{framed}
306 \end{frame}
307
308
309
310 * \begin{frame}
311 \includegraphics[angle=15,scale=0.50]{Pic1.jpg}
312 \end{frame}
313 \end{document}

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# ASSIGNMENT 2

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University of Delhi**

## EXAMPLE 9.5 : Part 1

- Let  $x = (x_1, \dots, x_n)$ , where the  $x_i$  are nonnegative real numbers. Set

$$M_r(x) = \left( \frac{x_1^r + x_2^r + \dots + x_n^r}{n} \right)^{1/r}, \quad r \in \mathbb{R} \setminus \{0\},$$

and

$$M_0(x) = (x_1 x_2 \dots x_n)^{1/n}.$$

We call  $M_r(x)$  the *r*th power mean of  $x$ .

Claim :  $\lim_{r \rightarrow 0} M_r(x) = M_0(x)$ .

## Part 2

- Define

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 & \dots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1} \end{bmatrix}.$$

We call  $V_n$  the *Vandermonde matrix* of order  $n$ .

Claim:

$$\det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i).$$

## QUESTION 4

$$3^3 + 4^3 + 5^3 = 6^3$$

$$\sqrt{100} = 10$$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots$$

## Remaining parts of QUESTION 4

$$\cos\theta = \sin(90^\circ - \theta)$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\lim_{\theta \rightarrow 0} \frac{\sin\theta}{\theta} = 1$$



## Remaining parts of QUESTION 4

$$\lim_{x \rightarrow \infty} \frac{\pi x}{x / \log x} = 1.$$

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

## QUESTION 5 : Typeset the following sentences.

- \* Positive numbers  $a, b$  and  $c$  are the side lengths of a triangle if and only if  $a + b > c$ ,  $b + c > a$  and  $c + a > b$
- \* The area of a triangle with side lengths  $a, b, c$  is given by *Heron's formula*:

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where  $s$  is the semiperimeter  $(a + b + c)/2$ .

- \* The volume of a regular tetrahedron of edge length 1 is  $\sqrt{2}/12$ .
- \* The quadratic equation  $ax^2 + bx + c = 0$  has roots

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

## Remaining parts of QUESTION 5

- \* The *derivative* of a function  $f$ , denoted by  $f'$ , is defined by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- \* A real-valued function  $f$  is convex on an interval  $I$  if

$$f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y),$$

for all  $x, y \in I$  and  $0 \leq \lambda \leq 1$ .

- \* The general solution to the differential equation

$$y'' - 3y' + 2y = 0$$

is

$$y = C_1 e^x + C_2 e^{2x}.$$

- \* The *Fermat number*  $F_n$  is defined as

$$F_n = 2^{2^n}, n \geq 0.$$

## QUESTION 6

$$\frac{d}{dx} \left( \frac{x}{x+1} \right) = \frac{1}{(x+1)^2}$$

$$\lim_{n \rightarrow \infty} \left( 1 + \frac{1}{n} \right)^n = e$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

## Remaining parts QUESTION 6

$$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$



## Remaining parts of QUESTION 6

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

$$f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & 0 \leq x \leq 2 \\ 4, & x > 2 \end{cases} .$$

## QUESTION 7 : Part 1

$$1+2 = 3$$

$$4+5+6 = 7+8$$

$$9+10+11+12 = 13+14+15$$

$$16+17+18+19+20 = 21+22+23+24$$

$$25+26+27+28+29+30 = 31+32+33+34+35$$

## Part 2

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) \\ &= (a + b)a + (a + b)b \\ &= a(a + b) + b(a + b) \\ &= a^2 + ab + ba + b^2 \\ &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2\end{aligned}$$

## Part 3

$$\begin{aligned} \tan(\alpha + \beta + \gamma) &= \frac{\tan(\alpha + \beta) + \tan\gamma}{1 - \tan(\alpha + \beta)\tan\gamma} \\ &= \frac{\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} + \tan\gamma}{1 - \left(\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}\right)\tan\gamma} \\ &= \frac{\tan\alpha + \tan\beta + (1 - \tan\alpha\tan\beta)\tan\gamma}{1 - \tan\alpha\tan\beta - (\tan\alpha + \tan\beta)\tan\gamma} \\ &= \frac{\tan\alpha + \tan\beta + \tan\gamma - \tan\alpha\tan\beta\tan\gamma}{1 - \tan\alpha\tan\beta - \tan\alpha\tan\gamma - \tan\beta\tan\gamma} \end{aligned}$$

## Part 4

$$\begin{aligned}\prod_p \left(1 - \frac{1}{p^2}\right) &= \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots} \\ &= \left(\prod_p \left(1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots\right)\right)^{-1} \\ &= \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots\right)^{-1} \\ &= \frac{6}{\pi^2}\end{aligned}$$



**Thank  
You**

