ASSIGNMENT

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Example 9.5

- Let $\mathrm{x} = (x_1, \ldots, x_n)$, where the x_i are non-negative real numbers. Set

$$M_r(\mathrm{x}) = \left(\frac{x_1^r + x_2^r + \cdots + x_n^r}{n}\right)^{1/r} \quad r \in \mathrm{R} \setminus \{0\},$$

and

$$M_0(\mathrm{x}) = (x_1 x_2 \cdots x_n)^{1/n}.$$

We call $M_r(\mathrm{x})$ the *rth power mean* of x.
Claim:

$$\lim_{r \to 0} M_r(\mathrm{x}) = M_0(\mathrm{x}).$$

Continued Example 9.5

- Define

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 & \dots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1} \end{bmatrix}$$

We call V_n the *Vandermonde matrix* of order n.
Claim:

$$\det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i).$$

Question 4

- $$3^3 + 4^3 + 5^3 = 6^3$$
- $$\sqrt{100} = 10$$
- $$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
- $$\sum_{k=1}^{n} = \frac{n(n+1)}{2}$$
- $$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots$$
- $$\cos\theta = \sin(90 - \theta)$$

Continued Question 4

- $$e^{i\theta} = \cos\theta + i\sin\theta$$
- $$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$$
- $$\lim_{x \to \infty} \frac{\pi(x)}{x/\log x} = 1$$
- $$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$$

Question 5

- Positive numbers a, b, and c are the side lengths of a triangle if and only if $a + b > c$, $b + c > a$, and $c + a > b$.
- The area of a triangle with side lengths a, b, c is given by *Heron's Formula*:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

  , where s is the semi perimeter $\frac{(a+b+c)}{2}$.
- The volume of a regular tetrahedron of edge length 1 is $\frac{\sqrt{2}}{12}$.
- The quadratic equation $ax2 + bx + c = 0$ has roots

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Continued Question 5

- The *derivative* of a function f, denoted f' is defined by :

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- A real-valued function f is *convex* on an interval I if

$$f(\lambda x + (1-\lambda) y) \leq \lambda f(x) + (1-\lambda) f(y),$$

  for all $x,\ y \in \quad I \quad$ and $0 \leq \lambda \leq 1$.

- The general solution to the differential equation

$$y'' - 3y' + 2y = 0$$

  is

$$y = C_1 e^{x} + C_2 e^{2x}.$$

Continued Question 5

- The Fermat number F_n is defined as

$$F_n = 2^{2^n}, \; n \geq 0$$

Question 6

- $$\frac{d}{dx}\left(\frac{x}{x+1}\right) = \frac{1}{(x+1)^2}$$
- $$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = \mathrm{e}$$
- $$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$
- $$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Continued Question 6

- $$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

- $$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

- $$f(x) = \begin{cases} -x^2 \, , \; x < 0 \\ x^2 \, , \; 0 \leq x \leq 2 \\ 4 \quad , \; x > 2 \end{cases}$$

Question 7 Part 1

$$
\begin{aligned}
1+2 &= 3 \\
4+5+6 &= 7+8 \\
9+10+11+12 &= 13+14+15 \\
16+17+18+19+20 &= 21+22+23+24 \\
25+26+27+28+29+30 &= 31+32+33+34+35
\end{aligned}
$$

Question 7 Part 2

$$
\begin{aligned}
(a+b)^2 &= (a+b)(a+b) \\
&= (a+b)a + (a+b)b \\
&= a(a+b) + b(a+b) \\
&= aa + ab + ba + bb \\
&= aa + ab + ab + bb \\
&= a^2 + 2ab + b^2
\end{aligned}
$$

Question 7 Part 3

$$
\begin{aligned}
\tan(\alpha+\beta+\gamma) &= \frac{\tan(\alpha+\beta)+\tan\gamma}{1-\tan(\alpha+\beta)\tan\gamma} \\
&= \frac{\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}+\tan\gamma}{1-\frac{\tan\alpha+\tan\beta}{1-\tan\alpha+\beta}\tan\gamma} \\
&= \frac{\tan\alpha+\tan\beta+\tan\gamma(1-\tan\alpha\tan\beta)}{(1-\tan\alpha\tan\beta)-\tan\gamma(\tan\alpha+\tan\beta)} \\
&= \frac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\beta-\tan\gamma\tan\alpha-\tan\gamma\tan\alpha-\tan\gamma\tan\beta}
\end{aligned}
$$

Question 7 Part 4

$$\begin{aligned}
\prod_p (1 - \frac{1}{p^2}) &= \prod_p (\frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots}) \\
&= (\prod_p (1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots))^{-1} \\
&= (1 + \frac{1}{2^2} + \frac{1}{2^4} + \dots)^{-1} \\
&= \frac{6}{\pi^2}
\end{aligned}$$

END OF SLIDE