ASSIGNMENT-2

MATA SUNDRI COLLEGE FOR WOMEN

$\mathcal{NAME}$:-SHRISHTI KANSAL

$\mathcal{COLLEGE\ ROLLNO}$:-MAT/20/86

$\mathcal{UNIVERSITY\ ROLLNO}$:-20044563020

$\mathcal{UNIVERSITY\ NAME}$:-DELHI UNIVERSITY

- Let $x = (x_1, \ldots, x_n)$, where the x_i are non-negative real numbers . Set

$$M_r(x) = \left(\frac{x_1^r + x_2^r + \cdots + x_n^r}{n} \right)^{1/r}, \quad r \in \mathsf{R} \setminus \{0\},$$

and

$$M_0(x) = (x_1 x_2 \ldots x_n)^{1/n}$$

we call $M_r(x)$ the *rth power mean* of x
Claim:

$$\lim_{r \to 0} M_r(x) = M_0(x).$$

- Define

$$
V_n = \begin{bmatrix}
1 & 1 & 1 & \dots & 1 \\
x_1 & x_2 & x_3 & \dots & x_n^2 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1}
\end{bmatrix}.
$$

we call V_n the *Vandermonde matrix* of order n. Claim:

$$
\det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i).
$$

Question - 4 (EQUATIONS)

$$3^3 + 4^3 + 5^3 = 6^3$$
$$\sqrt{100} = 10$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$
$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots$$
$$Cos\theta = Sin(90 - \theta)$$
$$e^{i\theta} = Cos\theta + iSin\theta$$
$$\lim_{\theta \to 0} \frac{Sin\theta}{\theta} = 1$$

$$\lim_{x \to \infty} \frac{\pi(x)}{x/log(x)} = 1$$

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

Question -5 (statements)

- Positive numbers a,b and c are the side lengths of a triangle if and only if $a + b > c, b + c > a, c + a > b$
- The area of a triangle with side lengths a,b,c is given by *Heron'S formula*:
$$A = \sqrt{s(s-a)(s-b)(s-c)},$$
where s is the semiperimeter $(a+b+c)/2$.
- The volume of a regular tetrahedron of edge length 1 is $\sqrt{2}/12$.

- The quadratic equation $ax^2 + bx + c = 0$ has roots

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The derivative of a function f,denoted f',is defined by ,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- A real-valued function f is *convex* on an interval I if

$$f(\lambda x + (1-\lambda)y) \le \lambda f(x) + (1-\lambda)f(y)$$

, for all $x, y \in I$ and $0 \le \lambda \le 1$.

- The general solution to the differential equation

$$y'' - 3y' + 2y = 0$$

  is

$$y = C_1 e^x + C_2 e^{2x}$$

- The *fermat* number Fn is defined as

$$Fn = 2^{2^n}, n \geq 0$$

Question - 6 (EQUATIONS)

- $$\frac{d}{dx}\left(\frac{x}{x+1}\right) = \frac{1}{(x+1)^2}$$

- $$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$

- $$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

- $$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

- $$\begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\mathbf{k}$$

-
$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

-
$$f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & 0 \le x \le 2 \\ 4, & x > 2 \end{cases}$$

Question -7 (MULTI-LINE EQUATIONS)

1st part

1+2 =3
4+5+6=7+8
9+10+11+12=13+14+15
16+17+18+19+20=21+22+23+24
25+26+27+28+29+30=31+32+33+34+35

2nd part

$$
\begin{aligned}
(a+b)^2 &= (a+b)(a+b) \\
&= (a+b)a + (a+b)b \\
&= a(a+b) + b(a+b) \\
&= a^2 + ab + ba + b^2 \\
&= a^2 + ab + ab + b^2 \\
&= a^2 + 2ab + b^2
\end{aligned}
$$

3rd part

$tan(\alpha + \beta + \gamma) = \frac{tan(\alpha+\beta)+tan\gamma}{1-tan(\alpha+\beta)tan\gamma}$

$=$

$$\frac{\frac{tan\alpha+tan\beta}{1-tan\alpha tan\beta} + tan\gamma}{1 - (\frac{tan\alpha+tan\beta}{1-tan\alpha tan\beta})tan\gamma}$$

$=$

$$\frac{tan\alpha + tan\beta + (1 - tan\alpha tan\beta)tan\gamma}{1 - tan\alpha tan\beta - (tan\alpha + tan\beta)tan\gamma}$$

$=$

$$\frac{tan\alpha + tan\beta + tan\gamma - tan\alpha tan\beta tan\gamma}{1 - tan\alpha tan\beta - tan\alpha tan\gamma - tan\beta tan\gamma}$$

4th part

$$
\begin{aligned}
\prod_p \left(1 - \frac{1}{p^2}\right) &= \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots} \\
&= \left(\left(\prod_p (1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots)\right)\right)^{-1} \\
&= \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\right)^{-1} \\
&= \frac{6}{\pi^2}
\end{aligned}
$$

thank you