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- ① Let $\mathbf{x} = (x_1, \dots, x_n)$, where the x_i are non-negative real numbers.
Set

$$M_r(\mathbf{x}) = \left(\frac{x_1^r + x_2^r + \dots + x_n^r}{n} \right)^{1/r}, \quad r \in \mathbf{R} \setminus \{0\},$$

and

$$M_0(\mathbf{x}) = (x_1 x_2 \cdots x_n)^{1/n}.$$

We call $M_r(\mathbf{x})$ the *r*th power mean of \mathbf{x} .

Claim:

$$\lim_{r \rightarrow 0} M_r(\mathbf{x}) = M_0(\mathbf{x}).$$

2 Define

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ x_1 & x_2 & x_3 & \cdots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \cdots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \cdots & x_n^{n-1} \end{bmatrix}$$

We call V_n the *Vandermonde matrix* of order n .

Claim:

$$\det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i).$$

Question. 4

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$$3^3 + 4^3 + 5^3 = 6^3$$

•

$$\sqrt{100} = 10$$

•

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

•

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

•

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots$$

Remaining parts of Ques.4

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$$\cos \theta = \sin(90^\circ - \theta)$$

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$\lim_{\theta \rightarrow \infty} \frac{\sin \theta}{\theta} = 1$$

$$\lim_{x \rightarrow \infty} \frac{\pi(x)}{x / \log x} = 1$$

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

Question 5

- Positive numbers a, b and c are the side lengths of a triangle if and only if $a + b > c, b + c > a,$ and $c + a > b.$
- The area of a triangle with side lengths a, b, c is given by *Heron's formula*:

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where s is the semiperimeter $(a+b+c)/2.$

- The volume of a regular tetrahedron of edge length 1 is $\sqrt{2}/12.$
- The quadratic equation $ax^2 + bx + c = 0$ has roots

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Remaining parts of Ques.5

- The *derivative* of a function f , denoted f' , is defined by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}.$$

- A real-valued function f is *convex* on an interval I if

$$f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y),$$

for all $x, y \in I$ and $0 \leq \lambda \leq 1$.

- The general solution to the differential equation

$$y'' - 3y' + 2y = 0$$

is

$$y = C_1 e^x + C_2 e^{2x}.$$

Remaining parts of Ques.5

- The *Fermat number* F_n is defined as

$$F_n = 2^{2^n}, n \geq 0.$$

Question 6

- $$\frac{d}{dx} \left(\frac{x}{x+1} \right) = \frac{1}{(x+1)^2}$$

- $$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^n = e$$

- $$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

- $$R_\theta = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

Remaining parts of Ques.6

- $$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

- $$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

- $$f(x) = \begin{cases} -x^2 & x < 0 \\ x^2, & 0 \leq x \leq 2 \\ 4, & x > 2 \end{cases}$$

Question 7(1)

$$1 + 2 = 3$$

$$4 + 5 + 6 = 7 + 8$$

$$9 + 10 + 11 + 12 = 13 + 14 + 15$$

$$16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24$$

$$25 + 26 + 27 + 28 + 29 + 30 = 31 + 32 + 33 + 34 + 35$$

Question 7(2)

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) \\ &= (a + b)a + (a + b)b \\ &= a(a + b) + b(a + b) \\ &= a^2 + ab + ba + b^2 \\ &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2\end{aligned}$$

Question 7(3)

$$\begin{aligned}\tan(\alpha + \beta + \gamma) &= \frac{\tan(\alpha + \beta) + \tan \gamma}{1 - \tan(\alpha + \beta) \tan \gamma} \\ &= \frac{\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} + \tan \gamma}{1 - \left(\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \right) \tan \gamma} \\ &= \frac{\tan \alpha + \tan \beta + (1 - \tan \alpha \tan \beta) \tan \gamma}{1 - \tan \alpha \tan \beta - (\tan \alpha + \tan \beta) \tan \gamma} \\ &= \frac{\tan \alpha + \tan \beta + \tan \gamma - \tan \alpha \tan \beta \tan \gamma}{1 - \tan \alpha \tan \beta - \tan \alpha \tan \gamma - \tan \beta \tan \gamma}\end{aligned}$$

Question 7(4)

$$\begin{aligned}\prod_p \left(1 - \frac{1}{p^2}\right) &= \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots} \\ &= \left(\prod_p \left(1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots\right) \right)^{-1} \\ &= \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots\right)^{-1} \\ &= \frac{6}{\pi^2}\end{aligned}$$

Thank You!

