Source Rich Text

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\documentclass{beamer}
\usepackage[utf8]{inputenc}
\usepackage{xcolor}
\usepackage[most]{tcolorbox}
% \setbeamercolor{block body}{bg=cyan, fg=black}
\title{\textbf{Assignment 2}}
%\author{Richa Devrani }
\usepackage{graphicx}
\date{}
\usefonttheme{serif}
\usetheme{AnnArbor}

\setbeamercolor{frametitle}{fg=black}
%\setbeamercolor{blocktitle}{bg=pink,fg=black}
\begin{document}

\begin{frame}{Assignment 2}

%\titlepage
\begin{tcolorbox}[enhanced,frame style image=blueshade.png,
  opacityback=0.75,opacitybacktitle=0.25,
  colback=blue!5!white,colframe=blue!75!black,
  title={}] \vspace{0.6cm}\centering
  \textbf {\Large {Mata Sundri College For Women}\\ \vspace{0.2cm}
  \large{University of Delhi} \vspace{0.6cm}
  \\\large{Richa Devrani\\ \vspace{0.2cm} College Roll no. - MAT/20/136 \vspace{0.2cm} \\
 University Roll no. - 20044563055}} \\  \vspace{2cm}
    \end{tcolorbox}

\end{frame}





\begin{frame}{Example - 9.5 }
\begin{tcolorbox}[enhanced,frame style image=blueshade.png,
  opacityback=0.75,opacitybacktitle=0.25,
  colback=blue!5!white,colframe=blue!75!black,
  title=Part a]
1 . Let $\mathbf{x}=(x_1,\ldots ,x_n)$, where the $x_i $ are nonnegative real numbers.Set
    \[ M_r(\mathbf{x}) = \left(\frac{x_1^r+x_2^r +\cdots +x_n^r}{n} \right)^{1/r} ,\; \; r\in \mathbf{R} \setminus \{0\},\]
    and
    \[M_0(\mathbf{x}) = (x_1x_2\ldots x_n)^{1/n}.\]
    We call $ M_r(\mathbf{x})$ \emph{the rth power mean of} $x$.\\
    Claim :
    \[\lim_{r\rightarrow 0}M_r(x) = M_0(x).\]
    \end{tcolorbox}
 \end{frame}
```

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\begin{frame}{Example - 9.5 }
\begin{tcolorbox}[enhanced,frame style image=blueshade.png,
 opacityback=0.75,opacitybacktitle=0.25,
 colback=blue!5!white,colframe=blue!75!black,
 title=Part b]
2. Define
\[V_n = \left[\begin{array}{ccccc}
   1 & 1 & 1 & \ldots & 1\\
  x_1 & x_2 & x_3 & \ldots  &x_n \\
  x_1^2 & x_2^2 & x_3^2 &\ldots &x_n^2\\
  \vdots & \vdots & \vdots & \ddots & \vdots\\
  x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \ldots &x_n^{n-1}
\end{array}\right] \]
We call $V_n$ \;the \emph{Vandermonde \;matrix} \;of\; order\; n .
Claim :
\[ \det V_n = \prod_{1 \leq i < j \leq n}(x_j-x_i).\]
\end{tcolorbox}
\end{frame}




 \begin{frame}{Ques 4 : Make the following equations}
 \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
 opacityback=0.75,opacitybacktitle=0.25,
 colback=blue!5!white,colframe=blue!75!black,
 title=(a)] \begin{itemize}

 \item $$3^3 +4^3 +5^3 = 6^3$$ \\
    \item $$\sqrt{100} = 10$$ \\
    \item $$(a+b)^3 =a^3 +3a^2b +3ab^2 +b^3$$ \\
    \item $$\sum_{k=1}^n k = \frac {n(n+1)}{2}$$ \\
     \end{itemize}
     \end{tcolorbox}
     \end{frame}




     \begin{frame}{Ques 4 cont.}
      \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
 opacityback=0.75,opacitybacktitle=0.25,
 colback=blue!5!white,colframe=blue!75!black,
 title=(b)]
 \begin{itemize}
    \item $$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} +\frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots $$ \\
  \item  $$\cos \theta =\sin(90^o - \theta)$$ \\
   \item  $$e^{i \theta} = \cos \theta + i \;\sin \theta$$ \\
   \item $$\lim_{\theta \rightarrow 0} \;\frac{\sin \theta }{\theta} = 1 $$
\end{itemize}
   \end{tcolorbox}
 \end{frame}
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  \end{frame}





\begin{frame}{Ques 4 cont.}
       \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
  opacityback=0.75,opacitybacktitle=0.25,
  colback=blue!5!white,colframe=blue!75!black,
  title=(c)]
  \begin{itemize}
  \item  \[\lim_{x \rightarrow \infty} \;\frac{\pi (x)}{x/log x}\] \\
  \item  \[\int_{- \infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}\]

     \vspace{1.5cm}
        \end{itemize}
     \end{tcolorbox}
\end{frame}






%\section{Introduction}
\begin{frame}{Ques 5 : Typeset the following sentences.}
 \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
  opacityback=0.75,opacitybacktitle=0.25,
  colback=blue!5!white,colframe=blue!75!black,
  title=(a)]
\begin{flushright}
    \begin{itemize}
        \item Positive numbers a,b,and c are the side lengths of a triangle if and only if a+b \(>\) c, b+c \(>\)a , and  c+a \(>\) b.
        \item The area of a triangle side lengths a,b,c is given \emph{Heron's formula} :
  \[A = \sqrt{s(s-a)(s-b)(s-c)},\]
  where s is the semiperimeter (a+b+c)/2.

        \item The volume of a regular tetrahedron of edge length 1 is $\sqrt{2}/12.$

        \end{itemize}
        \vspace{1cm}
        \end{flushright}
        \end{tcolorbox}
 \end{frame}






 \begin{frame}
  \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
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  colback=blue!5!white,colframe=blue!75!black,
  title=(b)]
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 colback=blue!5!white,colframe=blue!75!black,
 title=(b)]
\begin{flushright}
\begin{itemize}
       \item The quadratic equation $ax^2 +bx +c = 0$ has roots
 \[r_1,r_2 = \frac{-b \pm \sqrt{b^2 -4ac}}{2a}\]
     \item The derivative of a function $f$ , denoted $f'$ is defined by
     \[ f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}\]

        \item A real - valued function $f$ is convex on an interval \emph{I} if
        \[f(\lambda x + (1- \lambda )y) \leq \lambda f(x) + (1- \lambda)f(y),\]
        \[ for \; all\; x,y \in  I \;and \;0 \leq \lambda \leq 1.\]
         \end{itemize}
\end{flushright}
\end{tcolorbox}
\end{frame}




\begin{frame}
\begin{tcolorbox}[enhanced,frame style image=blueshade.png,
 opacityback=0.75,opacitybacktitle=0.25,
 colback=blue!5!white,colframe=blue!75!black,
 title=(c)]
   \begin{flushright}
   \begin{itemize}
       \item The general solution to the differential equation
        \[y'' - 3y' +2y =0 \]
        \[y= C_1 e^x +C_2e^{2x}\]
        \item The \emph{Fermat number} $F_n$ is defined as
        \[F_n = 2^{2^n} , n\geq 0\]
   \end{itemize}
   \vspace{1.2cm}
   \end{flushright}
   \end{tcolorbox}
\end{frame}





\begin{frame}{Question 6 }
\begin{tcolorbox}[enhanced,frame style image=blueshade.png,
 opacityback=0.75,opacitybacktitle=0.25,
 colback=blue!5!white,colframe=blue!75!black,
 title=(a)]
 \begin{itemize}
\item $$\frac{d}{dx} \left( \frac{x}{x+1} \right) = \frac{1}{(x+1)^2}$$\\
\item $$\lim_{n \rightarrow \infty}\left( 1+ \frac{1}{n} \right)^n$$\\
\item $$\left|
\begin{array}{cc}
   a & b \\
    c & d
\end{array} \right|
=ad-bc$$\\
 \item  $$R_\theta =
   \left[ \begin{array}{lr}
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 \item  $$R_\theta =
   \left[ \begin{array}{lr}
       \cos \theta  & - \sin \theta \\
       \sin \theta  & \cos \theta
    \end{array} \right] $$

   \end{itemize}
     \end{tcolorbox}
\end{frame}

\begin{frame}
 \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
  opacityback=0.75,opacitybacktitle=0.25,
  colback=blue!5!white,colframe=blue!75!black,
  title=(b)]
\begin{flushright}
    $$\left| \begin{array}{ccc}
        \textbf{i} & \textbf{j} & \textbf{ k} \\
       a_1  & a_2 &a_3\\
       b_1 & b_2 & b_3
    \end{array} \right|
    =\left| \begin{array}{cc}
       a_2  & a_3  \\
         b_2& b_3
    \end{array} \right| \textbf{i} -
    \left| \begin{array}{cc}
        a_1 & a_3  \\
         b_1& b_3
    \end{array} \right| \textbf{j} +
   \left| \begin{array}{cc}
        a_1 & a_2  \\
         b_1& b_2
    \end{array} \right| \textbf{k} $$

    $$ \left[ \begin{array}{cc}
       a_11  & a_12  \\
         a_21& a_22
    \end{array}\right]    \left[ \begin{array}{cc}
       b_11  & b_12  \\
         b_21& b_22
    \end{array}\right]
    = \left[ \begin{array}{cc}
       a_{11} b_{11} + a_{12} b_{12}  & a_{11}b_{12} + a_{12}b_{22} \\
         a_{21}b_{11} + a_{22}b_{21}& a_{21}b_{12} + a_{22}b_{22}
    \end{array}\right]$$

   $$ f(x) = \left.\begin{array}{ccc}
       -x^2,  & x<0 \\
     x^2,  & 0\leq x \leq 2 \\
    4,  & x>2
    \end{array} \right\}$$
    \end{flushright}
    \end{tcolorbox}
\end{frame}
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\end{frame}

\begin{frame}{Question 7 }
 \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
  opacityback=0.75,opacitybacktitle=0.25,
  colback=blue!5!white,colframe=blue!75!black,
  title=Part (a)]
\centering \vspace{0.8cm}
    1 + 2 = 3 \\
    4 + 5 + 6 = 7 + 8\\
    9 + 10 + 11 + 12 = 13 + 14 + 15\\
    16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24\\
    25 + 26 + 27 + 28 + 29 + 30 = 31 + 32 + 33 + 34 + 35
      \vspace{1.8cm}    \end{tcolorbox}
\end{frame}
\begin{frame} {Question 7 }
 \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
  opacityback=0.75,opacitybacktitle=0.25,
  colback=blue!5!white,colframe=blue!75!black,
  title=Part (b)]
\begin{eqnarray*}
    (a+b)^2& = &(a+b)(a+b) \\
    &=&(a+b)a+(a+b)b\\
    &=&a(a+b)+b(a+b)\\
    &=&a^2 +ab+ba +b^2\\
    &=&a^2+ab+ab+b^2\\
    &=&a^2 +2ab +b^2
    \end{eqnarray*}
    \vspace{0.8cm}
    \end{tcolorbox}
\end{frame}

\begin{frame}{Question 7 }
 \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
  opacityback=0.75,opacitybacktitle=0.25,
  colback=blue!5!white,colframe=blue!75!black,
  title=Part (c)]
    \begin{eqnarray*}
    \tan(\alpha + \beta+ \gamma) &=& \frac{\tan(\alpha +\beta )+\tan\gamma}{1-\tan(\alpha +\gamma)\tan\gamma}\\
    \\
    &=& \frac{\frac{\tan\alpha +\tan \beta}{1-\tan\alpha \tan\beta}+\tan \gamma}{1-(\frac{\tan \alpha + \tan \beta}{1- \tan \alpha \tan \beta })\tan \gamma}\\
    \\
     &=&\frac{\tan \alpha + \tan \beta +(1-\tan\alpha \tan\beta)\tan\gamma}{1-\tan\alpha \tan\beta -(\tan\alpha+ \tan\beta)\tan\gamma}\\
     \\
     &=&\frac{\tan\alpha+\tan\beta +\tan\gamma-\tan\alpha \tan\beta
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      &=&\frac{\tan \alpha + \tan \beta +(1-\tan\alpha \tan\beta)\tan\gamma}{1-\tan\alpha \tan\beta -(\tan\alpha+ \tan\beta)\tan\gamma}\\
      \\
      &=&\frac{\tan\alpha+\tan\beta +\tan\gamma-\tan\alpha \tan\beta \tan\gamma}{1-\tan\alpha \tan\beta -\tan\alpha \tan\gamma-\tan\beta \tan\gamma}
     \end{eqnarray*}
     \end{tcolorbox}
\end{frame}




\begin{frame}{Question 7}

 \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
  opacityback=0.75,opacitybacktitle=0.25,
  colback=blue!5!white,colframe=blue!75!black,
  title=Part (d)]
     \begin{eqnarray*}
     \prod_p \left(1-\frac{1}{p^2}\right)&=&\prod_p\frac{1}{1+\frac{1}{p^2}+\frac{1}{p^4}+\cdots}\\
     &=&\left(\prod_p\left((1+\frac{1}{p^2}+\frac{1}{p^4}+\cdots\right)\right)^{-1}\\
     &=& \left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots \right)^{-1}\\
     &=&\frac{6}{\pi^2}
     \end{eqnarray*}
\end{tcolorbox}
\end{frame}




\begin{frame}
     \includegraphics[width=13cm,height=8.5cm]{pic.jpg}
\end{frame}

\end{document}
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Mata Sundri College For Women

University of Delhi

Richa Devrani

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Example - 9.5

Part a

1 . Let $\mathbf{x} = (x_1, \ldots, x_n)$, where the x_i are nonnegative real numbers.Set

$$M_r(\mathbf{x}) = \left(\frac{x_1^r + x_2^r + \cdots + x_n^r}{n} \right)^{1/r}, \quad r \in \mathbf{R} \setminus \{0\},$$

and

$$M_0(\mathbf{x}) = (x_1 x_2 \ldots x_n)^{1/n}.$$

We call $M_r(\mathbf{x})$ *the rth power mean of* x.
Claim :

$$\lim_{r \to 0} M_r(x) = M_0(x).$$

Example - 9.5

Part b

2. Define

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 & \dots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1} \end{bmatrix}$$

We call V_n the *Vandermonde matrix* of order n . Claim :

$$\det V_n = \prod_{1 \le i < j \le n} (x_j - x_i).$$

Ques 4 : Make the following equations

(a)

- $$3^3 + 4^3 + 5^3 = 6^3$$
- $$\sqrt{100} = 10$$
- $$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
- $$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

Ques 4 cont.

(b)

- $$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots$$
- $$\cos\theta = \sin(90^o - \theta)$$
- $$e^{i\theta} = \cos\theta + i\ \sin\theta$$
- $$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$$

Ques 4 cont.

(c)

- $$\lim_{x \to \infty} \frac{\pi(x)}{x/logx}$$

- $$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

Ques 5 : Typeset the following sentences.

(a)

- Positive numbers a,b,and c are the side lengths of a triangle if and only if a+b $>$ c, b+c $>$a , and c+a $>$ b.
- The area of a triangle side lengths a,b,c is given *Heron's formula* :

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

  where s is the semiperimeter (a+b+c)/2.
- The volume of a regular tetrahedron of edge length 1 is $\sqrt{2}/12$.

(b)

- The quadratic equation $ax^2 + bx + c = 0$ has roots

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The derivative of a function f , denoted f' is defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- A real - valued function f is convex on an interval I if

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda) f(y),$$

$$for\ all\ x, y \in I\ and\ 0 \leq \lambda \leq 1.$$

(c)

- The general solution to the differential equation

$$y'' - 3y' + 2y = 0$$

$$y = C_1e^x + C_2e^{2x}$$

- The *Fermat number* F_n is defined as

$$F_n = 2^{2^n}, n \geq 0$$

Question 6

(a)

- $$\frac{d}{dx}\left(\frac{x}{x+1}\right) = \frac{1}{(x+1)^2}$$
- $$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$
- $$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$
- $$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

(b)

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

$$\begin{bmatrix} a_1 1 & a_1 2 \\ a_2 1 & a_2 2 \end{bmatrix} \begin{bmatrix} b_1 1 & b_1 2 \\ b_2 1 & b_2 2 \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{12} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

$$f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & 0 \leq x \leq 2 \\ 4, & x > 2 \end{cases}$$

Question 7

Part (a)

$$
\begin{gathered}
1 + 2 = 3 \\
4 + 5 + 6 = 7 + 8 \\
9 + 10 + 11 + 12 = 13 + 14 + 15 \\
16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24 \\
25 + 26 + 27 + 28 + 29 + 30 = 31 + 32 + 33 + 34 + 35
\end{gathered}
$$

Question 7

Part (b)

$$
\begin{aligned}
(a+b)^2 &= (a+b)(a+b) \\
&= (a+b)a + (a+b)b \\
&= a(a+b) + b(a+b) \\
&= a^2 + ab + ba + b^2 \\
&= a^2 + ab + ab + b^2 \\
&= a^2 + 2ab + b^2
\end{aligned}
$$

Question 7

Part (c)

$$
\begin{aligned}
\tan(\alpha+\beta+\gamma) &= \frac{\tan(\alpha+\beta)+\tan\gamma}{1-\tan(\alpha+\gamma)\tan\gamma} \\
&= \frac{\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}+\tan\gamma}{1-(\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta})\tan\gamma} \\
&= \frac{\tan\alpha+\tan\beta+(1-\tan\alpha\tan\beta)\tan\gamma}{1-\tan\alpha\tan\beta-(\tan\alpha+\tan\beta)\tan\gamma} \\
&= \frac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\beta-\tan\alpha\tan\gamma-\tan\beta\tan\gamma}
\end{aligned}
$$

Question 7

Part (d)

$$
\begin{aligned}
\prod_p \left(1 - \frac{1}{p^2}\right) &= \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots} \\
&= \left(\prod_p \left((1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots\right)\right)^{-1} \\
&= \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\right)^{-1} \\
&= \frac{6}{\pi^2}
\end{aligned}
$$


Thank you