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1 \documentclass{beamer}
2 \usepackage[utf8]{inputenc}
3 \usepackage{xcolor}
4 \usepackage[most]{tcolorbox}
5 % \setbeamercolor{block body}{bg=cyan, fg=black}
6 \title{\textbf{Assignment 2}}
7 %\author{Richa Devrani }
8 \usepackage{graphicx}
9 \date{}
10 \usefonttheme{serif}
11 \usetheme{AnnArbor}
12
13 \setbeamercolor{frametitle}{fg=black}
14 %\setbeamercolor{blocktitle}{bg=pink, fg=black}
15 \begin{document}
16
17 \begin{frame}{Assignment 2}
18
19 %\titlepage
20 \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
21 opacityback=0.75,opacitybacktitle=0.25,
22 colback=blue!5!white,colframe=blue!75!black,
23 title={}] \vspace{0.6cm}\centering
24 \textbf {\Large {Mata Sundri College For Women}}\ \vspace{0.2cm}
25 \large{University of Delhi} \vspace{0.6cm}
26 \\\large{Richa Devrani}\ \vspace{0.2cm} College Roll no. - MAT/20/136
27 \vspace{0.2cm} \ \
28 University Roll no. - 20044563055} \ \ \vspace{2cm}
29 \end{tcolorbox}
30 \end{frame}
31
32
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34
35
36 \begin{frame}{Example - 9.5 }
37 \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
38 opacityback=0.75,opacitybacktitle=0.25,
39 colback=blue!5!white,colframe=blue!75!black,
40 title=Part a]
41 1 . Let  $\mathbf{x}=(x_1,\dots ,x_n)$ , where the  $x_i$  are nonnegative real
42 numbers.Set
43 
$$M_r(\mathbf{x}) = \left(\frac{x_1^r+x_2^r +\dots +x_n^r}{n} \right)^{1/r} ,\ ;$$

44 
$$\ ; r\in \mathbf{R} \setminus \{0\},\ ;$$

45 and
46 
$$M_0(\mathbf{x}) = (x_1x_2\dots x_n)^{1/n}.\ ;$$

47 We call  $M_r(\mathbf{x})$  the rth power mean of  $x$ .
48 Claim :
49 
$$\lim_{r\rightarrow 0}M_r(x) = M_0(x).\ ;$$

50 \end{tcolorbox}
51 \end{frame}
52
53
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52
53
54
55 * \begin{frame}{Example - 9.5 }
56 * \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
57   opacityback=0.75,opacitybacktitle=0.25,
58   colback=blue!5!white,colframe=blue!75!black,
59   title=Part b]
60 2. Define
61 \[V_n = \left[\begin{array}{cccc}
62   1 & 1 & 1 & \dots & 1 \\
63   x_1 & x_2 & x_3 & \dots & x_n \\
64   x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\
65   \vdots & \vdots & \vdots & \ddots & \vdots \\
66   x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1}
67 \end{array}\right] \]
68 We call  $V_n$  the Vandermonde matrix of order  $n$ .
69 Claim :
70 \[ \det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i) . \]
71 \end{tcolorbox}
72 \end{frame}
73
74
75
76
77 * \begin{frame}{Ques 4 : Make the following equations}
78 * \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
79   opacityback=0.75,opacitybacktitle=0.25,
80   colback=blue!5!white,colframe=blue!75!black,
81   title=(a)] \begin{itemize}
82
83   \item  $3^3 + 4^3 + 5^3 = 6^3$  \\
84   \item  $\sqrt{100} = 10$  \\
85   \item  $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$  \\
86   \item  $\sum_{k=1}^n k = \frac{n(n+1)}{2}$  \\
87   \end{itemize}
88   \end{tcolorbox}
89   \end{frame}
90
91
92
93
94 * \begin{frame}{Ques 4 cont.}
95 * \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
96   opacityback=0.75,opacitybacktitle=0.25,
97   colback=blue!5!white,colframe=blue!75!black,
98   title=(b)]
99 * \begin{itemize}
.00   \item  $\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} +$   

.01   \item  $\frac{1}{9} - \frac{1}{11} + \dots$  \\
.02   \item  $\cos \theta = \sin(90^\circ - \theta)$  \\
.03   \item  $e^{i\theta} = \cos \theta + i \sin \theta$  \\
.04   \item  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$  \\
.05 \end{itemize}
.06 \end{tcolorbox}
.07 \end{frame}

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106   \end{frame}
107
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109
110
111
112 * \begin{frame}{Ques 4 cont.}
113 *   \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
114   opacityback=0.75,opacitybacktitle=0.25,
115   colback=blue!5!white,colframe=blue!75!black,
116   title=(c)]
117 *   \begin{itemize}
118   \item [\(\lim_{x \rightarrow \infty} \frac{\pi(x)}{x/\log x}\)] \(\)
119   \item [\(\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}\)]
120
121   \vspace{1.5cm}
122   \end{itemize}
123   \end{tcolorbox}
124 \end{frame}
125
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129
130
131 * %\section{Introduction}
132 * \begin{frame}{Ques 5 : Typeset the following sentences.}
133 *   \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
134   opacityback=0.75,opacitybacktitle=0.25,
135   colback=blue!5!white,colframe=blue!75!black,
136   title=(a)]
137 *   \begin{flushright}
138 *     \begin{itemize}
139       \item Positive numbers a,b,and c are the side lengths of a triangle if and
140         only if  $a+b > c$ ,  $b+c > a$ , and  $c+a > b$ .
141       \item The area of a triangle side lengths a,b,c is given Heron's
142         formula :
143       \[A = \sqrt{s(s-a)(s-b)(s-c)},\]
144       where s is the semiperimeter  $(a+b+c)/2$ .
145
146       \item The volume of a regular tetrahedron of edge length 1 is  $\sqrt{2}/12$ .
147     \end{itemize}
148     \vspace{1cm}
149   \end{flushright}
150   \end{tcolorbox}
151 \end{frame}
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157 * \begin{frame}
158 *   \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
159   opacityback=0.75,opacitybacktitle=0.25,
160   colback=blue!5!white,colframe=blue!75!black,
161   title=(b)]

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160   colback=blue!5!white,colframe=blue!75!black,
161   title=(b)]
162 * \begin{flushright}
163 * \begin{itemize}
164     \item The quadratic equation  $ax^2 + bx + c = 0$  has roots
165      $[r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}]$ 
166     \item The derivative of a function  $f$ , denoted  $f'$  is defined by
167      $[f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}]$ 
168
169     \item A real - valued function  $f$  is convex on an interval  $I$  if
170      $[f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y),]$ 
171      $[ \text{for } \lambda; \text{ all } x, y \in I \text{ and } 0 \leq \lambda \leq 1.]$ 
172     \end{itemize}
173 \end{flushright}
174 \end{tcolorbox}
175 \end{frame}
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180 * \begin{frame}
181 * \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
182   opacityback=0.75,opacitybacktitle=0.25,
183   colback=blue!5!white,colframe=blue!75!black,
184   title=(c)]
185 *   \begin{flushright}
186 *   \begin{itemize}
187     \item The general solution to the differential equation
188      $[y'' - 3y' + 2y = 0]$ 
189      $[y = C_1 e^x + C_2 e^{2x}]$ 
190     \item The Fermat number  $F_n$  is defined as
191      $[F_n = 2^{2^n}, n \geq 0]$ 
192     \end{itemize}
193     \vspace{1.2cm}
194   \end{flushright}
195   \end{tcolorbox}
196 \end{frame}
197
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201
202 * \begin{frame}{Question 6 }
203 * \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
204   opacityback=0.75,opacitybacktitle=0.25,
205   colback=blue!5!white,colframe=blue!75!black,
206   title=(a)]
207 *   \begin{itemize}
208     \item  $\frac{d}{dx} \left( \frac{x}{x+1} \right) = \frac{1}{(x+1)^2}$ 
209     \item  $\lim_{n \rightarrow \infty} \left( 1 + \frac{1}{n} \right)^n$ 
210     \item  $\left| \begin{array}{cc} a & b \\ c & d \end{array} \right|$ 
211 *   \begin{array}{cc}
212     a & b \\
213     c & d \end{array} \right|
214   \end{array} \right|
215   =ad-bc
216   \item  $R_{\theta} =$ 
217      $\left[ \begin{array}{lr} \end{array} \right]$ 

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216 \item $$R_{\theta} =
217 \left[ \begin{array}{lr}
218 \cos \theta & - \sin \theta \\
219 \sin \theta & \cos \theta
220 \end{array} \right] $$
221
222 \end{itemize}
223 \end{tcolorbox}
224 \end{frame}
225
226
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230 \begin{frame}
231 \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
232 opacityback=0.75,opacitybacktitle=0.25,
233 colback=blue!5!white,colframe=blue!75!black,
234 title=(b)]
235 \begin{flushright}
236 $$\left| \begin{array}{ccc}
237 \textbf{i} & \textbf{j} & \textbf{k} \\
238 a_1 & a_2 & a_3 \\
239 b_1 & b_2 & b_3
240 \end{array} \right|
241 = \left| \begin{array}{cc}
242 a_2 & a_3 \\
243 b_2 & b_3
244 \end{array} \right| \textbf{i} -
245 \left| \begin{array}{cc}
246 a_1 & a_3 \\
247 b_1 & b_3
248 \end{array} \right| \textbf{j} +
249 \left| \begin{array}{cc}
250 a_1 & a_2 \\
251 b_1 & b_2
252 \end{array} \right| \textbf{k} $$
253
254 $$ \left[ \begin{array}{cc}
255 a_{11} & a_{12} \\
256 a_{21} & a_{22}
257 \end{array} \right] \left[ \begin{array}{cc}
258 b_{11} & b_{12} \\
259 b_{21} & b_{22}
260 \end{array} \right]
261 = \left[ \begin{array}{cc}
262 a_{11} b_{11} + a_{12} b_{21} & a_{11} b_{12} + a_{12} b_{22} \\
263 a_{21} b_{11} + a_{22} b_{21} & a_{21} b_{12} + a_{22} b_{22}
264 \end{array} \right] $$
265
266 $$ f(x) = \left. \begin{array}{ccc}
267 -x^2, & & x < 0 \\
268 x^2, & & 0 \leq x \leq 2 \\
269 4, & & x > 2
270 \end{array} \right\}
271 \end{flushright}
272 \end{tcolorbox}
273 \end{frame}

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273 \end{frame}
274
275
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277
278
279 * \begin{frame}{Question 7 }
280 * \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
281   opacityback=0.75,opacitybacktitle=0.25,
282   colback=blue!5!white,colframe=blue!75!black,
283   title=Part (a)]
284 \centering \vspace{0.8cm}
285     1 + 2 = 3 \\
286     4 + 5 + 6 = 7 + 8 \\
287     9 + 10 + 11 + 12 = 13 + 14 + 15 \\
288     16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24 \\
289     25 + 26 + 27 + 28 + 29 + 30 = 31 + 32 + 33 + 34 + 35
290     \vspace{1.8cm} \end{tcolorbox}
291 \end{frame}
292 * \begin{frame} {Question 7 }
293 * \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
294   opacityback=0.75,opacitybacktitle=0.25,
295   colback=blue!5!white,colframe=blue!75!black,
296   title=Part (b)]
297 * \begin{eqnarray*}
298     (a+b)^2 = (a+b)(a+b) \\
299     = (a+b)a + (a+b)b \\
300     = a(a+b) + b(a+b) \\
301     = a^2 + ab + ba + b^2 \\
302     = a^2 + ab + ab + b^2 \\
303     = a^2 + 2ab + b^2
304 \end{eqnarray*}
305 \vspace{0.8cm}
306 \end{tcolorbox}
307 \end{frame}
308
309
310
311
312
313 * \begin{frame}{Question 7 }
314 * \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
315   opacityback=0.75,opacitybacktitle=0.25,
316   colback=blue!5!white,colframe=blue!75!black,
317   title=Part (c)]
318 * \begin{eqnarray*}
319     \tan(\alpha + \beta + \gamma) = \frac{\tan(\alpha + \beta)
320     + \tan \gamma}{1 - \tan(\alpha + \beta)\tan \gamma} \\
321     = \frac{\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} + \tan
322     \gamma}{1 - (\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta})\tan
323     \gamma} \\
324     = \frac{\tan \alpha + \tan \beta + (1 - \tan \alpha
325     \tan \beta)\tan \gamma}{1 - \tan \alpha \tan \beta - (\tan \alpha +

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323 &=&\frac{\tan\alpha + \tan\beta + (1-\tan\alpha
\tan\beta)\tan\gamma}{1-\tan\alpha\tan\beta - (\tan\alpha+
\tan\beta)\tan\gamma} \\
324 \\
325 &=&\frac{\tan\alpha + \tan\beta + \tan\gamma - \tan\alpha\tan\beta
\tan\gamma}{1-\tan\alpha\tan\beta - \tan\alpha\tan\gamma - \tan\beta\tan\gamma}
326 \end{eqnarray*}
327 \end{tcolorbox}
328 \end{frame}
329
330
331
332
333 * \begin{frame}{Question 7}
334
335 * \begin{tcolorbox}[enhanced,frame style image=blueshade.png,
336 opacityback=0.75,opacitybacktitle=0.25,
337 colback=blue!5!white,colframe=blue!75!black,
338 title=Part (d)]
339 * \begin{eqnarray*}
340 \prod_p \left(1 - \frac{1}{p^2}\right) &=& \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p
^4} + \cdots} \\
341 &=& \left(\prod_p \left(1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots\right)\right)^{-1} \\
342 &=& \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\right)^{-1} \\
343 &=& \frac{6}{\pi^2}
344 \end{eqnarray*}
345 \end{tcolorbox}
346 \end{frame}
347
348
349
350
351 * \begin{frame}
352 \includegraphics[width=13cm,height=8.5cm]{pic.jpg}
353 \end{frame}
354
355 \end{document}
356 |

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Assignment 2

Mata Sundri College For Women
University of Delhi

Richa Devrani

College Roll no. - MAT/20/136

University Roll no. - 20044563055

Example - 9.5

Part a

1 . Let $\mathbf{x} = (x_1, \dots, x_n)$, where the x_i are nonnegative real numbers. Set

$$M_r(\mathbf{x}) = \left(\frac{x_1^r + x_2^r + \dots + x_n^r}{n} \right)^{1/r}, \quad r \in \mathbf{R} \setminus \{0\},$$

and

$$M_0(\mathbf{x}) = (x_1 x_2 \dots x_n)^{1/n}.$$

We call $M_r(\mathbf{x})$ *the rth power mean of x*.

Claim :

$$\lim_{r \rightarrow 0} M_r(x) = M_0(x).$$

Example - 9.5

Part b

2. Define

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 & \dots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1} \end{bmatrix}$$

We call V_n the *Vandermonde matrix* of order n . Claim :

$$\det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i).$$

Ques 4 : Make the following equations

(a)

•

$$3^3 + 4^3 + 5^3 = 6^3$$

•

$$\sqrt{100} = 10$$

•

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

•

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

Ques 4 cont.

(b)

- $$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots$$

- $$\cos \theta = \sin(90^\circ - \theta)$$

- $$e^{i\theta} = \cos \theta + i \sin \theta$$

- $$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$$

Ques 4 cont.

(c)

- $$\lim_{x \rightarrow \infty} \frac{\pi(x)}{x/\log x}$$

- $$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

Ques 5 : Typeset the following sentences.

(a)

- Positive numbers $a, b,$ and c are the side lengths of a triangle if and only if $a+b > c$, $b+c > a$, and $c+a > b$.
- The area of a triangle side lengths a, b, c is given *Heron's formula* :

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where s is the semiperimeter $(a+b+c)/2$.

- The volume of a regular tetrahedron of edge length 1 is $\sqrt{2}/12$.

(b)

- The quadratic equation $ax^2 + bx + c = 0$ has roots

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The derivative of a function f , denoted f' is defined by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- A real - valued function f is convex on an interval I if

$$f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y),$$

for all $x, y \in I$ and $0 \leq \lambda \leq 1$.

(c)

- The general solution to the differential equation

$$y'' - 3y' + 2y = 0$$

$$y = C_1e^x + C_2e^{2x}$$

- The *Fermat number* F_n is defined as

$$F_n = 2^{2^n}, n \geq 0$$

Question 6

(a)

- $$\frac{d}{dx} \left(\frac{x}{x+1} \right) = \frac{1}{(x+1)^2}$$

- $$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^n$$

- $$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

- $$R_\theta = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

(b)

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

$$f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & 0 \leq x \leq 2 \\ 4, & x > 2 \end{cases}$$

Question 7

Part (a)

$$1 + 2 = 3$$

$$4 + 5 + 6 = 7 + 8$$

$$9 + 10 + 11 + 12 = 13 + 14 + 15$$

$$16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24$$

$$25 + 26 + 27 + 28 + 29 + 30 = 31 + 32 + 33 + 34 + 35$$

Question 7

Part (b)

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) \\ &= (a + b)a + (a + b)b \\ &= a(a + b) + b(a + b) \\ &= a^2 + ab + ba + b^2 \\ &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2\end{aligned}$$

Question 7

Part (c)

$$\begin{aligned}\tan(\alpha + \beta + \gamma) &= \frac{\tan(\alpha + \beta) + \tan \gamma}{1 - \tan(\alpha + \beta) \tan \gamma} \\ &= \frac{\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} + \tan \gamma}{1 - \left(\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}\right) \tan \gamma} \\ &= \frac{\tan \alpha + \tan \beta + (1 - \tan \alpha \tan \beta) \tan \gamma}{1 - \tan \alpha \tan \beta - (\tan \alpha + \tan \beta) \tan \gamma} \\ &= \frac{\tan \alpha + \tan \beta + \tan \gamma - \tan \alpha \tan \beta \tan \gamma}{1 - \tan \alpha \tan \beta - \tan \alpha \tan \gamma - \tan \beta \tan \gamma}\end{aligned}$$

Question 7

Part (d)

$$\begin{aligned}\prod_p \left(1 - \frac{1}{p^2}\right) &= \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots} \\ &= \left(\prod_p \left(1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots\right) \right)^{-1} \\ &= \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots\right)^{-1} \\ &= \frac{6}{\pi^2}\end{aligned}$$

