

## Assignment 2

latex presentation

# Mata Sundri College for Women, University of Delhi

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**College Roll No.: MAT/20/83**

**University roll no.: 20044563009**



Assignment 2

### Content of page 69

- ① Let  $\mathbf{x} = (x_1, \dots, x_n)$ , where the  $x_i$  are non-negative real numbers.  
Set

$$M_r(\mathbf{x}) = \left( \frac{x_1^r + x_2^r + \dots + x_n^r}{n} \right)^{1/r}, \quad r \in \mathbf{R} \setminus \{0\},$$

and

$$M_0(\mathbf{x}) = (x_1 x_2 \dots x_n)^{1/n},$$

We call  $M_r(\mathbf{x})$  the  $r$ th power mean of  $\mathbf{x}$

Claim:

$$\lim_{r \rightarrow 0} M_r(\mathbf{x}) = M_0(\mathbf{x}).$$

- ② Define



Assignment 2

## Content of page 69 part (b)

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 & \dots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1} \end{bmatrix}.$$

We call  $V_n$  the *Vandermonde matrix* of order  $n$ .

Claim:

$$\det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i).$$



Assignment 2

## Question 4 part (a)

•

$$3^3 + 4^3 + 5^3 = 6^3$$

•

$$\sqrt{100} = 10$$

•

$$(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$$

•

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

Assignment 2

## Question 4 part (b)

•

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots$$

•

$$\cos \theta = \sin(90^\circ - \theta)$$

•

$$e^{i\theta} = \cos \theta + i \sin \theta$$

Assignment 2

## Question 4 part (c)

•

$$\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0$$

•

$$\lim_{x \rightarrow \infty} \frac{\pi(x)}{x / \log x} = 1$$

•

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

Assignment 2

## Question 5 Part (a)

- Positive numbers  $a, b$  and  $c$  are the side lengths of a triangle if and only if  $a + b > c$ ,  $b + c > a$  and  $c + a > b$ .
- The area of a triangle with side lengths  $a, b, c$  is given by *Heron's formula*:

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where  $s$  is the semiperimeter of  $(a+b+c)/2$ .

- The volume of a regular tetrahedron of edge length 1 is  $\sqrt{2}/12$ .
- The quadratic equation  $ax^2 + bx + c = 0$  has roots

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



Assignment 2

## Question 5 Part (b)

- The *derivative* of a function  $f$ , denoted  $f'$ , is defined by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}.$$

- A real-valued function  $f$  is *convex* on an interval  $I$  if

$$f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y),$$

for all  $x, y \in I$  and  $0 \leq \lambda \leq 1$

- The general solution to the differential equation

$$y'' - 3y' + 2y = 0$$

is

$$y = C_1 e^x + C_2 e^{2x}.$$



Assignment 2



## Question 5 part (c)

- The *Fermat number*  $F_n$  is defined as

$$F_n = 2^{2^n}, n \geq 0.$$



Assignment 2

## Question 6 part (a)

- 
- 
- 
- 

$$\frac{d}{dx} \left( \frac{1}{n} \right) = \frac{1}{(x+1)^2}$$

$$\lim_{n \rightarrow \infty} \left( 1 + \frac{1}{n} \right)^n = e$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$R_\theta = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$



Assignment 2

## Question 6 part (b)

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

$$f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & 0 \leq x \leq 2 \\ 4, & x > 2 \end{cases}$$



Assignment 2

## Question 7 part (a)

$$1 + 2 = 3$$

$$4 + 5 + 6 = 7 + 8$$

$$9 + 10 + 11 + 12 = 13 + 14 + 15$$

$$16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24$$

$$25 + 26 + 27 + 28 + 29 + 30 = 31 + 32 + 33 + 34 + 35$$



Assignment 2

## Question 7 part (b)

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) \\ &= (a + b)a + (a + b)b \\ &= a(a + b) + b(a + b) \\ &= a^2 + ab + ba + b^2 \\ &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2\end{aligned}$$



Assignment 2

## Question 7 part (c)

$$\begin{aligned}\tan(\alpha + \beta + \gamma) &= \frac{\tan(\alpha + \beta) + \tan \gamma}{1 - \tan(\alpha + \beta) \tan \gamma} \\ &= \frac{\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} + \tan \gamma}{1 - \left( \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \right) \tan \gamma} \\ &= \frac{\tan \alpha + (1 - \tan \alpha \tan \beta) \tan \gamma}{1 - \tan \alpha \tan \beta - (\tan \alpha + \tan \beta) \tan \gamma} \\ &= \frac{\tan \alpha + \tan \beta + \tan \gamma - \tan \alpha \tan \beta \tan \gamma}{1 - \tan \alpha \tan \beta - \tan \alpha \tan \gamma - \tan \beta \tan \gamma}\end{aligned}$$



Assignment 2



Question 7 part (d)

$$\begin{aligned}\prod_p \left(1 - \frac{1}{p^2}\right) &= \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots} \\ &= \left( \prod_p \left(1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots\right) \right)^{-1} \\ &= \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots\right)^{-1} \\ &= \frac{6}{\pi^2}\end{aligned}$$



Assignment 2



Assignment 2



```

1 \documentclass[10pt]{beamer}
2 \usepackage[utf8]{inputenc}
3 \usepackage{kerkis}
4 \usepackage{tcolorbox}
5 \date{}
6
7
8 \title{\textcolor{black}{Assignment 2}}
9 \subtitle{\textcolor{black}{latex presentation}}
10 \institute{\huge{Mata Sundri College for Women, \\\University of Delhi}}
11
12 \usetheme{Warsaw}
13 \usecolortheme{dolphin}
14 \useinnertheme{circles}
15 \usefonttheme{serif}
16 \setbeamertemplate{background}{\includegraphics[width=\paperwidth,height=\paperwidth]{bg.jpg}}
17
18
19
20 \begin{document}
21 \begin{frame}
22 \titlepage
23 \textcolor{red}{\textbf{Name: Anjali Singh}}\textbf{College Roll No.: MAT/20/83}\textbf{University roll no.: 20044563009}
24 \end{frame}
25 %\nextslide
26 \begin{frame}{Content of page 69}
27
28 \begin{enumerate}
29 \item Let  $\mathbf{x}=(x_1,\dots,x_n)$ , where the  $x_i$  are non-negative real numbers.
30 Set
31 \[
32 M_r(\mathbf{x})=\left(\frac{x_1^r+x_2^r
33 +\dots+x_n^r}{n}\right)^{1/r},
34 \];  $r\in\mathbf{R}\setminus\{0\}$ ,
35 \]
36 and
37 \[
38 M_0(\mathbf{x})=\left(x_1 x_2 \dots x_n \right)^{1/n},
39 \]
40 We call  $M_r(\mathbf{x})$  the rth power mean
41 of  $\mathbf{x}$ 
42
43 Claim:
44 \[
45 \lim_{r \rightarrow 0} M_r(\mathbf{x})=
46 M_0(\mathbf{x}).
47 \]
48 \item Define
49 \end{enumerate}
50 \end{frame}
51 \begin{frame}{Content of page 69 part (b)}
52
53 \item
54 \[
55 V_n=
56 \left[
57 \begin{array}{cccc}
58 1&1&1&\dots&1\\
59 x_1 & x_2 & x_3 & \dots & x_n \\
60 x_1^2 & x_2^2 & x_3^3 & \dots & x_n^2 \\
61 \vdots & \vdots & \vdots & \vdots & \vdots \\
62 x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1}
63 \end{array}
64 \right].
65 \]
66 We call  $V_n$  the Vandermonde matrix of order  $n$ .
67
68 Claim:
69 
$$\det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i).$$

70
71 \end{frame}
72
73 %\nextslide
74 \begin{frame}{Question 4 part (a)}
75
76 \begin{itemize}
77 \begin{tcolorbox}\item  $3^3+4^3+5^3=6^3$ \end{tcolorbox}
78 \begin{tcolorbox}\item  $\sqrt{100}=10$ \end{tcolorbox}
79 \begin{tcolorbox}\item  $(a+b)^3 = a^3+b^3 +3a^2b+3ab^2$ \end{tcolorbox}
80 \begin{tcolorbox}\item  $\sum_{k=1}^n k = \frac{n(n+1)}{2}$ \end{tcolorbox}
81 \end{itemize}
82
83 \end{frame}
84
85 \begin{frame}{Question 4 part (b)}
86
87 \begin{itemize}
88 \begin{tcolorbox}\item  $\frac{1}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots$ \end{tcolorbox}
89 \begin{tcolorbox}\item  $\cos \theta = \sin(90^\circ - \theta)$ \end{tcolorbox}
90 \begin{tcolorbox}\item  $e^{i\theta} = \cos \theta + i \sin \theta$ \end{tcolorbox}
91 \end{itemize}
92
93 \end{frame}
94
95 \begin{frame}{Question 4 part (c)}
96

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94
95 \begin{frame}{Question 4 part (c)}
96
97 \begin{itemize}
98 \begin{tcolorbox}\item\[\lim_{x\rightarrow\infty}\frac{\sin\theta}{\theta}=1\]\end{tcolorbox}\
99 \begin{tcolorbox}\item\[\lim_{x\rightarrow\infty}\frac{\pi(x)}{x/\log x}=1\]\end{tcolorbox}\
100 \begin{tcolorbox}\item\[\int_{-\infty}^{\infty}e^{-x^2}dx=\sqrt{\pi}\]\end{tcolorbox}
101 \end{itemize}
102
103 \end{frame}
104
105 \begin{frame}{Question 5 Part (a)}
106
107 \begin{itemize}
108 \item Positive numbers a,b and c are the side lengths of a triangle if and only if  $a+b>c, b+c>a$  and  $c+a>b$ .\
109 \item The area of a triangle with side lengths a,b,c is given by Heron's formula:
110 
$$A=\sqrt{s(s-a)(s-b)(s-c)},$$

111 where  $s$  is the semiperimeter of  $(a+b+c)/2$ .
112 \item The volume of a regular tetrahedron of edge length 1 is  $\frac{\sqrt{2}}{12}$ .\
113 \item The quadratic equation  $ax^2+bx+c=0$  has roots  $r_1, r_2=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ .\
114
115 \end{itemize}
116
117 \end{frame}
118
119 \begin{frame}{Question 5 Part (b)}
120
121 \begin{itemize}
122 \item The derivative of a function  $f$ , denoted  $f'$ , is defined by
123 
$$f'(x)=\lim_{h\rightarrow 0}\frac{f(x+h)-f(x)}{h}.$$

124 \item A real-valued function  $f$  is convex on an interval  $I$  if  $f(\lambda x+(1-\lambda)y)\leq\lambda f(x)+(1-\lambda)f(y)$ ,
125 for all  $x,y\in I$  and  $\lambda\in[0,1]$ .
126 \item The general solution to the differential equation  $y''-3y'+2y=0$  is  $y=C_1e^x+C_2e^{2x}$ .\
127 \end{itemize}
128
129 \end{frame}
130
131 \begin{frame}{Question 5 part (c)}
132
133 \begin{itemize}
134 \item The Fermat number  $F_n$  is defined as  $F_n=2^{2^n}+1, n\geq 0$ .\
135 \end{itemize}
136 \end{frame}
137 \begin{frame}{Question 6 part (a)}
138
139 \begin{itemize}
140 \item  $\frac{d}{dx}\left(\frac{1}{n}\right)=\frac{1}{(x+1)^2}$ 
141 \item  $\lim_{n\rightarrow\infty}\left(1+\frac{1}{n}\right)^n=e$ 
142 \item  $\begin{vmatrix} a & b \\ c & d \end{vmatrix}=ad-bc$ 
143 \item  $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ 
144 \end{itemize}
145
146 \end{frame}
147
148 \begin{frame}{Question 6 part (b)}
149
150 \begin{itemize}
151 \item  $\begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}=\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}-\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}+\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$ 
152 \item  $f(x)=\begin{cases} -x^2, & x<0 \\ x^2, & 0\leq x\leq 2 \\ 4, & x>2 \end{cases}$ 
153 \end{itemize}
154
155 \end{frame}
156
157
158 \begin{frame}{Question 7 part (a)}
159
160 \begin{align*}
161 1+2&=3 \\
162 4+5+6&=7+8 \\
163 9+10+11+12&=13+14+15 \\
164 16+17+18+19+20&=21+22+23+24 \\
165 25+26+27+28+29+30&=31+32+33+34+35 \\
166 \end{align*}
167
168 \end{frame}
169
170 \begin{frame}{Question 7 part (b)}
171
172 \begin{align*}
173 (a+b)^2&=(a+b)(a+b) \\
174 &=a(a+b)+b(a+b) \\
175 &=a^2+ab+ba+b^2 \\
176 &=a^2+ab+ab+b^2 \\
177 &=a^2+2ab+b^2 \\
178 \end{align*}
179
180 \end{frame}
181
182
183 \begin{frame}{Question 7 part (c)}
184
185 \begin{align*}

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179 \end{align*}
180
181 \end{frame}
182
183 \begin{frame}{Question 7 part (c)}
184
185 \begin{align*}
186 \tan(\alpha+\beta+\gamma)&=\frac{\tan(\alpha+\beta)+\tan\gamma}{1-\tan(\alpha+\beta)\tan\gamma} \\
187 &=\frac{\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}+\tan\gamma}{1-\left(\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}\right)\tan\gamma} \\
188 &=\frac{\tan\alpha+\tan\beta+(1-\tan\alpha\tan\beta)\tan\gamma}{1-\tan\alpha\tan\beta-(\tan\alpha+\tan\beta)\tan\gamma} \\
189 &=\frac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\beta-\tan\alpha\tan\gamma-\tan\beta\tan\gamma} \\
190 \end{align*}
191
192 \end{frame}
193
194 \begin{frame}{Question 7 part (d)}
195
196 \begin{align*}
197 \prod_p \left(1-\frac{1}{p^2}\right) &= \prod_p \frac{1}{1+\frac{1}{p^2}+\frac{1}{p^4}+\cdots} \\
198 &= \left(\prod_p \left(1+\frac{1}{p^2}+\frac{1}{p^4}+\cdots\right)\right)^{-1} \\
199 &= \left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots\right)^{-1} \\
200 &= \frac{6}{\pi^2} \\
201 \end{align*}
202
203 \end{frame}
204
205 \setbeamertemplate{background}{\includegraphics[width=\paperwidth,height=\paperwidth]{Ty2.jpg}}
206 \begin{frame}
207
208 \end{frame}
209
210 \end{document}

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