Assignment 2

latex presentation

Mata Sundri College for Women, University of Delhi

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1. Let $\mathbf{x} = (x_1, \ldots, x_n)$,where the x_i are non-negative real numbers. Set
$$M_r(\mathbf{x}) = \left(\frac{x_1^r + x_2^r + \cdots + x_n^r}{n}\right)^{1/r}, \quad r \in \mathbf{R} \setminus \{0\},$$
and
$$M_0(\mathbf{x}) = (x_1 x_2 \ldots x_n)^{1/n},$$
We call $M_r(\mathbf{x})$the *rth power mean* of $\mathbf{x}$
Claim:
$$\lim_{r \to 0} M_r(\mathbf{x}) = M_0(\mathbf{x}).$$
2. Define

Content of page 69 part (b)

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 & \dots & 7x_n \\ x_1^2 & x_2^2 & x_3^3 & \dots & 7X_n^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1} \end{bmatrix}.$$

We call V_n the *Vandermode matrix* of order n.

Claim:

$$\det V_n = \prod_{1 \le i < j \le n} (x_j - x_i).$$

Question 4 part (a)

- $$3^3 + 4^3 + 5^3 = 6^3$$
- $$\sqrt{100} = 10$$
- $$(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$$
- $$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

Question 4 part (b)

- $$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots$$

- $$\cos\theta = \sin(90^\circ - \theta)$$

- $$^{i\theta} = \cos\theta + i\sin\theta$$

Question 4 part (c)

- $$\lim_{x\to\infty} \frac{\sin\theta}{\theta} = 1$$

- $$\lim_{x\to\infty} \frac{\pi(x)}{x/\log x} = 1$$

- $$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

Question 5 Part (a)

- Positive numbers a,b and c are the side lengths of a triangle if and only if $a + b > c, b + c > a and c + a > b$.
- The area of a triangle with side lengths a,b,c is given by *Heron's formula*:

$$\mathbf{A} = \sqrt{s(s-a)(s-b)(s-c)},$$

  where s is the semiperimeter of (a+b+c)/2.
- The volume of a regular tetrahedron of edge length 1 is $\sqrt{2}/12$.
- The quadratic equation $ax^2 + bx + c$=0 has roots

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

.

Question 5 Part (b)

- The *derivative* of a function f, denotedf', is defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

- A real -valued function f is *convex* on an internal I if

$$f(\lambda x + (1-\lambda)y \leq \lambda f(X) + (1-\lambda) f(y),$$

  for all x,y $\in I$ *and* $0 \leq \lambda \leq 1$
- The general solution to the differential equation

$$y" - 3y' + 2y = 0$$

  is

$$y = C_1 e^x + C_2 e^{2x}.$$

Question 5 part (c)

- The *Fermat number* F_n is defined as

$$F_n = 2^{2^n}, n \geq 0.$$

Question 6 part (a)

- $$\frac{d}{dx}\left(\frac{1}{n}\right) = \frac{1}{(x+1)^2}$$
- $$\lim_{n \to \infty} (1 + \frac{1}{n})^n = e$$
- $$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$
- $$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Question 6 part (b)

- $$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$
- $$f(x) = \begin{cases} -x^2, x < 0 \\ x^2, 0 \leq x \leq 2 \\ 4, x > 2 \end{cases}$$

Question 7 part (a)

$$\begin{gathered} 1 + 2 = 3 \\ 4 + 5 + 6 = 7 + 8 \\ 9 + 10 + 11 + 12 = 13 + 14 + 15 \\ 16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24 \\ 25 + 26 + 27 + 28 + 29 + 30 = 31 + 32 + 33 + 34 + 35 \end{gathered}$$

Question 7 part (b)

$$
\begin{aligned}
(a+b)^2 &= (a+b)(a+b) \\
&= (a+b)a + (a+b)b \\
&= a(a+b) + b(a+b) \\
&= a^2 + ab + ba + b^2 \\
&= a^2 + ab + ab + b^2 \\
&= a^2 + 2ab + b^2
\end{aligned}
$$

Question 7 part (c)

$$
\begin{aligned}
\tan(\alpha+\beta+\gamma) &= \frac{\tan(\alpha+\beta)+\tan\gamma}{1-\tan(\alpha+\beta)\tan\gamma} \\
&= \frac{\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}+\tan\gamma}{1-\left(\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}\right)\tan\gamma} \\
&= \frac{\tan\alpha + +(1-\tan\alpha\tan\beta)\tan\gamma}{1-\tan\alpha\tan\beta-(\tan\alpha+\tan\beta)\tan\gamma} \\
&= \frac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\beta-\tan\alpha\tan\gamma-\tan\beta\tan\gamma}
\end{aligned}
$$

Question 7 part (d)

$$\prod_p \left(1 - \frac{1}{p^2}\right) = \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots}$$

$$= \left(\prod_p \left(1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots\right)\right)^{-1}$$

$$= \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\right)^{-1}$$

$$= \frac{6}{\pi^2}$$

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\documentclass[10pt]{beamer}
\usepackage[utf8]{inputenc}
\usepackage{kerkis}
\usepackage{tcolorbox}
\date{}


\title{\textcolor{black}{Assignment 2}}
\subtitle{\textcolor{black}{latex presentation}}
\institute{\huge{Mata Sundri College for Women, \\University of Delhi}}

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\usecolortheme{dolphin}
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\begin{document}
\begin{frame}
\titlepage
\textcolor{red}{\textbf{Name: Anjali Singh}\\\textbf{College Roll No.: MAT/20/83}\\\textbf{University roll no.: 20044563009}}
\end{frame}
%\nextslide
\begin{frame}{Content of page 69}

\begin{enumerate}
\item Let $\mathbf{x}=(x_1,\ldots,x_n)$,where the $x_i$ are non-negative real numbers.
Set
\[
M_r(\mathbf{x})=\left(\frac {x_1^r+x_2^r
+\cdots+x_n^r}{n}\right)^{1/r},
\;\;r\in\mathbf{R}\setminus\{0\},
\]
and
\[
M_0(\mathbf{x})=\left(x_1 x_2\ldots x_n \right)^{1/n},
\]
We call $M_r(\mathbf{x})$the \emph{$r$th power mean}
of $\mathbf{x}$

Claim:
\[
\lim_{r \rightarrow 0} M_r(\mathbf{x})=
M_0(\mathbf{x}).
\]
\item Define
\end{enumerate}
\end{frame}
\begin{frame}{Content of page 69 part (b)}

\item
\[
V_n=
\left[
\begin{array}{ccccc}
1&1&1&\ldots &1\\
x_1 & x_2 & x_3 &\ldots & 7 x_n\\
x_1^2 & x_2^2 & x_3^3 & \ldots & 7 X_n^2\\
\vdots & \vdots & \vdots & \vdots & \vdots\\
x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \ldots & x_n^{n-1}
\end{array}
\right].
\]
We call $V_n$ the \emph{Vandermode matrix} of order $n$.

Claim:
\[\det V_n =\prod_{1\leq i<j\leq n}(x_j-x_i).\]

\end{frame}

%\nextslide
\begin{frame}{Question 4 part (a)}

\begin{itemize}
\begin{tcolorbox}\item \[3^3+4^3+5^3=6^3\]\end{tcolorbox}\\
\begin{tcolorbox}\item \[\sqrt{100}=10\]\end{tcolorbox}\\
\begin{tcolorbox}\item\[\left(a+b \right)^3= a^3+b^3 +3a^2b+3ab^2\]\end{tcolorbox}\\
\begin{tcolorbox}\item\[\sum_{k=1}^{n}k=\frac{n(n+1)}{2}\]\end{tcolorbox}
\end{itemize}

\end{frame}

\begin{frame}{Question 4 part (b)}

\begin{itemize}
\begin{tcolorbox}\item\[\frac{\pi}{4}=\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\dots\]\end{tco
lorbox}
\begin{tcolorbox}\item\[\cos\theta=\sin(90^\circ-\theta)\]\end{tcolorbox}\\
\begin{tcolorbox}\item\[\e^{i\theta}=\cos\theta+i\sin\theta\]\end{tcolorbox}\\
\end{itemize}

\end{frame}

\begin{frame}{Question 4 part (c)}

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\begin{frame}{Question 4 part (c)}

\begin{itemize}
\begin{tcolorbox}\item\[\lim_{x\rightarrow\infty}\frac{\sin\theta}{\theta}=1\]\end{tcolorbox}\\
\begin{tcolorbox}\item\[\lim_{x\rightarrow\infty}\frac{\pi(x)}{x/\log x}=1\]\end{tcolorbox}\\
\begin{tcolorbox}\item\[\int_{-\infty}^{\infty}e^{-x^2}dx=\sqrt{\pi}\]\end{tcolorbox}
\end{itemize}

\end{frame}

\begin{frame}{Question 5 Part (a)}

\begin{itemize}
    \item Positive numbers a,b and c are the side lengths of a triangle if and only if $a+b>c,b+c>a and c+a>b$.\\
    \item The area of a triangle with side lengths a,b,c is given by \emph{Heron's formula}:
    \[\mathbf{A}=\sqrt{s(s-a)(s-b)(s-c)},\]
    where \emph{s} is the semiperimeter of (a+b+c)/2.
   \item The volume of a regular tetrahedron of edge length 1 is $\sqrt{2}/12$.\\
   \item The quadratic equation $ax^2 +bx+c$=0 has roots \[r_1,r_2=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\].

\end{itemize}

\end{frame}

\begin{frame}{Question 5 Part (b)}

\begin{itemize}
\item The \emph{derivative} of a function \emph{f} , denoted\emph{f'}, is defined by \[f'(x)=\lim_{h\rightarrow0}\frac{f(x+h)-f(x)}{h}.\]\\
\item A real -valued function \emph{f }is \emph{convex} on an internal \emph{I} if \[f(\lambda x+(1-\lambda)y\le\lambda\:
f(X)+(1-\lambda)\:f(y),\]
for all x,y \(\in I\: and \: 0\le\lambda\le 1\)\\
\item The general solution to the differential equation \[y"-3y'+2y=0\] is \[y=C_1e^x+C_2e^{2x}.\]
\end{itemize}

\end{frame}

\begin{frame}{Question 5 part (c)}

\begin{itemize}
    \item The \emph{Fermat number} $F_n$ is defined as \[F_n=2^{2^n},n\ge0.\]
\end{itemize}
\end{frame}
\begin{frame}{Question 6 part (a)}

\begin{itemize}
    \item \[\frac{d}{dx}\left(\frac{1}{n}\right)=\frac{1}{(x+1)^2}\]
    \item\[\lim_{n\rightarrow\infty} {(1+\frac{1}{n})^n}=e\]
    \item\[\begin{vmatrix}a&b\\c&d\end{vmatrix}=ad-bc\]
    \item\[R_\theta=\begin{bmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{bmatrix}\]
\end{itemize}

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\begin{frame}{Question 6 part (b)}

\begin{itemize}
    \item \[\begin{vmatrix}\textbf{i}&\textbf{j}&\textbf{k}\\a_1&a_2&a_3\\b_1&b_2&b_3\end{vmatrix}=\begin{vmatrix}a_2&a_3\\b_2&b_3\end{vmatrix}\textbf{i}-\begin{vmatrix}a_1&a_3\\b_1&b_3\end{vmatrix}\textbf{j}+\begin{vmatrix}a_1&a_2\\b_1&b_2\end{vmatrix}\textbf{k}\]
    \item \[f(x)=\begin{cases}
-x^2,x<0\\x^2,0\le x\le 2\\4,x>2\end{cases}\]
\end{itemize}

\end{frame}

\begin{frame}{Question 7 part (a)}

\begin{align*}
    1+2&=3\\
    4+5+6&=7+8\\
    9+10+11+12&=13+14+15\\
    16+17+18+19+20&=21+22+23+24\\
    25+26+27+28+29+30&=31+32+33+34+35\\
    \end{align*}

    \end{frame}

    \begin{frame}{Question 7 part (b)}

    \begin{align*}
        (a+b)^2&=(a+b)(a+b)\\
        &=(a+b)a+(a+b)b\\
        &=a(a+b)+b(a+b)\\
        &=a^2+ab+ba+b^2\\
        &=a^2+ab+ab+b^2\\
        &=a^2+2ab+b^2\\
    \end{align*}

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 \begin{frame}{Question 7 part (c)}

 \begin{align*}
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    \end{align*}

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 \begin{frame}{Question 7 part (c)}

 \begin{align*}
     \tan(\alpha+\beta+\gamma)&=\frac{\tan(\alpha+\beta)+\tan\gamma}{1-\tan(\alpha+\beta)\tan\gamma}\\
     &=\frac{\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}+\tan\gamma}{1-\left(\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} \right)\tan\gamma}\\
     &=\frac{\tan\alpha+\tanbeta+(1-\tan\alpha\tan\beta)\tan\gamma}{1-\tan\alpha\tan\beta-(\tan\alpha+\tan\beta)\tan\gamma}\\
     &=\frac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\beta-\tan\alpha\tan\gamma-\tan\beta\tan\gamma}\\
 \end{align*}

 \end{frame}

 \begin{frame}{Question 7 part (d)}

 \begin {align*}
 \prod_p\left(1-\frac{1}{p^2}\right)&=\prod_p\frac{1}{1+\frac{1}{p^2}+\frac{1}{p^4}+\cdots}\\
 &=\left(\prod_p\left(1+\frac{1}{p^2}+\frac{1}{p^4}+\cdots\right)\right)^{-1}\\
 &=\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots\right)^{-1}\\
 &=\frac{6}{\pi^2}
 \end{align*}

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