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ASSIGNMENT

Page No.- 69 Questions

Q - 1....

- Let $x = (x_1, \ldots, x_n)$,where the x_i are nonnegative real numbers. Set

$$M_r(x) = \left(\frac{x_1^r + x_2^r + \cdots + x_n^r}{n} \right)^{1/r}$$

$$, \ r \in \mathrm{R} \backslash \{0\},$$

and

$$M_0(x) = (x_1 x_2 \ldots x_n)^{1/n}.$$

We call $M_r(x)$ the r th power mean of x.
Claim:

$$\lim_{r \to 0} M_r(x) = M_0(x)$$

.

Q - 2.....

- Define

$$V_n = \begin{matrix} 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 & \dots & {}_n \\ x_1^2 & x_2^2 & x_3^2 & \dots & {}^2_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1} \end{matrix}$$

We call V_n the *Vandermonde matrix* of order n.
Claim:

$$\det V_n = \prod_{1 \le i < j \le n} (x_j - x_i)$$

QUESTION NO. 4

Q - 4.....

- $\boxed{3^3 + 4^3 + 5^3 = 6^3}$
- $\boxed{\sqrt{100} = 10}$
- $\boxed{(\mathrm{a+b})^3 = a^3 + 3a^2b + 3ab^2 + b^3}$

Q - 4.....

- $\boxed{\sum_{k=1}^{n} = \frac{n(n+1)}{2}}$
- $\boxed{\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots}$
- $\boxed{\cos\theta = \sin(90^\circ - \theta)}$

Q - 4.....

- $\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$
- $\boxed{\lim_{\theta\to 0} \frac{sin\theta}{\theta} = 1}$
- $\boxed{\lim_{x\to\infty} \frac{\pi(x)}{x/logx} = 1}$

Q - 4.....

- $\boxed{\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}}$

QUESTION NO. 5

Q - 5.....

Positive numbers a,b, and c are the side lengths of a triangle if and only if $a + b > c$, $b + c > a$ and $c + a > b$.

The area of a triangle with side lengths a, b, c is given by *Heron's formula*:

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where s is the semiperimeter $(a + b + c)/2$.

Q - 5.....

The volume of rectangular tetrahedron of edge length 1 is $\sqrt{2}/12$

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The quadratic equation $ax^2 + bx + c = 0$ has roots

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Q - 5.....

The derivative of the function f, denoted f', is defined by

$$f'(x)=\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

.

A real-valued function f is convex on an interval I if

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda)f(y),$$

for all $x, y \in \text{I} and 0 \leq \lambda \leq 1$.

Q - 5.....

The general solution to the differential equation

$$y'' - 3y'^{+3y=0}$$

is

$$y = C_1 e^x + C_2 e^{2x}.$$

.

The *Fermat number* F_n is defined as

$$F_n = 2^{2^n}, n \geq 0.$$

QUESTION NO. 6

Q - 6.....

- $$\boxed{\frac{d}{dx}\left(\frac{x}{x+1}\right) = \frac{1}{(x+1)^2}}$$
- $$\boxed{\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e}$$
- $$\boxed{\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc}$$

Q - 6.....

- $$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

- $$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

Q - 6....

- $$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

- $$f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & 0 \le x \le 2 \\ 4, & x > 2 \end{cases}$$

QUESTION NO. 7

Q - 7.....

$$1 + 2 = 3$$

$$4 + 5 + 6 = 7 + 8$$

$$9 + 10 + 11 + 12 = 13 + 14 + 15$$

$$16 + 17 + 18 + 19 + 20 = 21 + 23 + 24$$

$$25 + 26 + 27 + 28 + 29 + 30 = 31 + 32 + 33 + 34 + 35$$

Q - 7.....

$$
\begin{aligned}
(a+b)^2 &= (a+b)(a-b) \\
&= (a+b)a + (a+b)b \\
&= a(a+b) + b(a+b) \\
&= a^2 + ab + ba + b^2 \\
&= a^2 + ab + ab + b^2 \\
&= a^2 + 2ab + b^2
\end{aligned}
$$

Q - 7....

$$
\begin{aligned}
\tan(\alpha+\beta+\gamma) &= \frac{\tan(\alpha+\beta)+\tan\gamma}{1-\tan(\alpha+\beta)\tan\gamma} \\
&= \frac{\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}+\tan\gamma}{1-\left(\frac{\tan(\alpha+\beta)}{1-\tan(\alpha+\beta)}\right)\tan\gamma} \\
&= \frac{\tan\alpha+\tan\beta+(1-\tan\alpha\tan\beta)\tan\gamma}{1-\tan\alpha\tan\beta-(\tan\alpha+\tan\beta)\tan\gamma} \\
&= \frac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\beta-\tan\alpha\tan\gamma-\tan\beta\tan\gamma}
\end{aligned}
$$

$$
\begin{aligned}
\prod_p \left(1 - \frac{1}{p^2}\right) &= \prod_p \frac{1}{1+\frac{1}{p^2}+\frac{1}{p^4}+\ldots} \\
&= \left(\prod_p \left(1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots\right)\right)^{-1} \\
&= \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\right)^{-1} \\
&= \frac{6}{\pi^2}
\end{aligned}
$$


Thank You!