Assignment 2

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Exercise 9.5

- Letx $= (x_1, \ldots, x_n)$,where the x_i are nonnegative real numbers.Set

$$M_r(\mathrm{x}) = \left(\frac{x_1^r + x_2^r + \cdots + x_n^r}{n}\right)^{1/r}, \quad r \in \mathrm{R} \setminus \{0\},$$

and

$$M_0(\mathrm{x}) = (x_1 x_2 \ldots x_n)^{1/n}$$

We cal $M_r(\mathrm{x})$ the *rth power mean* of x
Claim:

$$\lim_{r \to 0} M_r(\mathrm{x}) = M_0(\mathrm{x}).$$

Exercise 9.5

- Define

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 & \dots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \dots & x_n^{n-1} \end{bmatrix}.$$

We call V_n the *Vandermonde matrix* of order n.
Claim:

$$\det V_n = \prod_{1 \le i < j \le n} (x_j - x_i).$$

Question 4

Make the following equations

- $$3^3 + 4^3 + 5^3 = 6^3$$
- $$\sqrt{100} = 10$$
- $$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
- $$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

Remaining part of Question 4

- $$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11}$$
- $$\cos\theta = \sin(90 - \theta)$$
- $$e^{\imath\theta} = \cos\theta + i\sin\theta$$
- $$\lim_{\theta \to \ 0} \frac{sin\theta}{\theta} = 1$$
- $$\lim_{x \to \infty} \frac{\pi(x)}{x/logx} = 1$$
- $$\int_{-\infty}^{\infty} e^{x^2}, dx = \sqrt{\pi}$$

Question 5

Typeset the following Sentence

- Positive numbers a, b, and c are the side lengths of a triangle if and only of $a + b \geq c$, $b + c \geq a$ and $c + a \geq b$
- The area of a triangle with side length a, b c is given by *Heron's Formula*:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

,
where s is the semiperimeter (a+b+c)/2

Remaining part of Question 5

- The volume of a regular tetrahedron of edge length 1 is $\sqrt{2}/12$
- The quadratic equation $ax^2 + bx + c = 0$ has roots

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The derivative of the functions f, is defined by

$$f^{'}(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Remaining part of Question 5

- A real - valued functions f is *convex* on a an interval I if

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda)f(y)$$

  for all $x,\ y \in I$ and $0 \leq 1$

- The general solution to the differential equation

$$y^{''} - 3y^{'} + 2y = 0$$

  is

$$y = C_1 e^{x} + C_2 e^{2x}$$

- The *Fermat Number* Fn is defined as

$$F_n = 2^{2^n}, n \geq 0$$

Question 6

Make the following equations. Notice the large delimiters.

- $$\frac{d}{dx}\left(\frac{x}{x+1}\right) = \frac{1}{(x+1)^2}$$
- $$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$
- $$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$
- $$R_\theta = \begin{bmatrix} \cos\theta & -sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Remaining Part of Question 6

- $$\begin{vmatrix} i & j & k \\ a_1 & b_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{j} - \begin{vmatrix} a_1 & b_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

- $$\begin{bmatrix} a_{11} & a_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

- $$f(x) = \begin{cases} -x^2 & x < 0 \\ x^2 & 0 \leq x \leq 2 \\ 4 & x > 2 \end{cases}$$

Question 7 Make the following multi-line equations

Part 1

$$
\begin{aligned}
1+2 &= 3 \\
4+5+6 &= 7+8 \\
9+10+11+12 &= 13+14+15 \\
16+17+18+19+20 &= 21+22+23+24 \\
25+26+27+28+30 &= 31+32+33+34+35
\end{aligned}
$$

Remaining part of Question 7

Part 2

$$
\begin{aligned}
(a+b)^2 &= (a+b)(a+b) \\
&= (a+b)a + (a+b)b \\
&= a(a+b) + b(a+b) \\
&= a^2 + ab + ba + b^2 \\
&= a^2 + ab + ab + b^2
\end{aligned}
$$

Remaining part of Question 7

Part 3

$$\begin{aligned} tan(\alpha+\beta+\gamma) &= \frac{tan(\alpha+\beta)+tan\gamma}{1-tan(\alpha+\beta)tan\gamma} \\ &= \frac{\frac{tan\alpha+tan\beta}{1-tan\alpha tan\beta}+tan\gamma}{1-(\frac{tan\alpha+tan\beta}{1-tan\alpha tan\beta})tan\gamma} \\ &= \frac{tan\alpha+tan\beta+(1-tan\alpha tan\beta)tan\gamma}{1-tan\alpha tan\beta-(tan\alpha+tan\beta)tan\gamma} \\ &= \frac{tan\alpha+tan\beta+tan\gamma-tan\alpha tan\beta tan\gamma}{1-tan\alpha tan\beta-tan\alpha tan\gamma-tan\beta tan\gamma} \end{aligned}$$

Remaining part of Question 7

Part 4

$$\begin{aligned}
\prod_p \left(1 - \frac{1}{p^2}\right) &= \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots} \\
&= \left(\prod_p \left(1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots\right)\right)^{-1} \\
&= \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\right)^{-1} \\
&= \frac{6}{\pi^2}
\end{aligned}$$

Thankyou