~CHETNA MEHRA (MAT/19/9)~

Chapter 3

Section 3.2

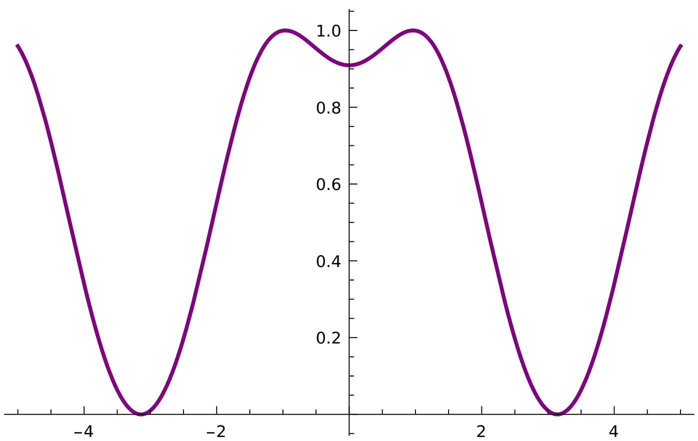
Q 1. Graph each of the functions. Experiment with different domains or viewpoints.

(a) f(x) = x/(1+x^2)

In[3]:=
```
ClearAll[f]
f[x_] := Sin[1 + Cos[x]]
```

In[5]:=
```
Plot[f[x], {x, -5, 5}, PlotStyle → {Purple, Thick}]
```

Out[5]=

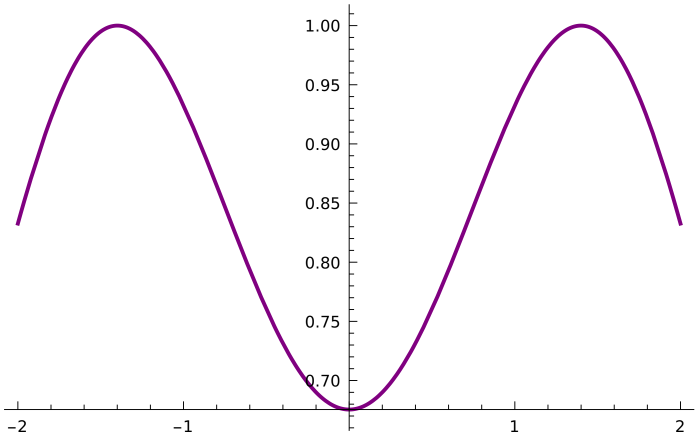


(b) f(x) = x Sin(1/x)

In[6]:=
```
Clear[f]
```

In[7]:=
```
f[x_] := Sin[1.4 + Cos[x]]
```

In[8]:=
```
Plot[f[x], {x, -2, 2}, PlotStyle → {Purple, Thick}]
```

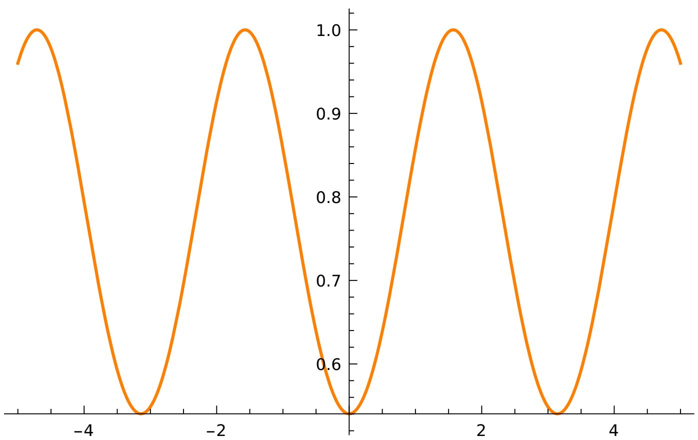
Out[8]=


(c) f(x) = cos(x) + sin(y)

In[9]:=
```
Clear[f]
```

In[10]:=
```
f[x_] := Sin[(Pi / 2) + Cos[x]]
```

In[11]:=
```
Plot[f[x], {x, -5, 5}, PlotStyle → {Orange}]
```

Out[11]=


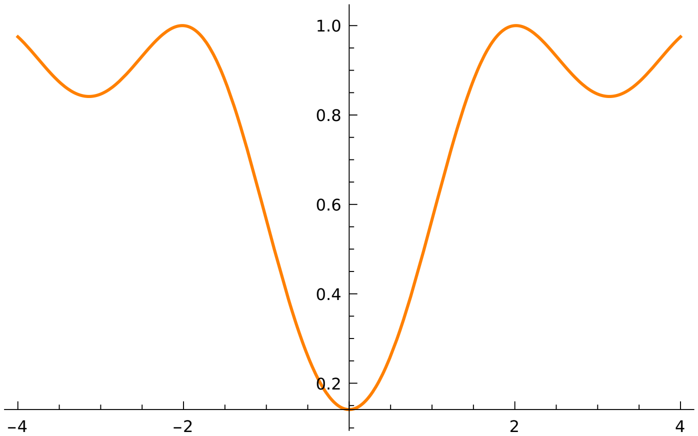
(d) f(x) = x y/(x^2 + y^2)

In[12]:=
```
Clear[f]
```

In[13]:=
```
f[x_] := Sin[2 + Cos[x]]
```

In[15]:=
```
Plot[f[x], {x, -4, 4}, PlotStyle → {Orange}]
```

Out[15]=



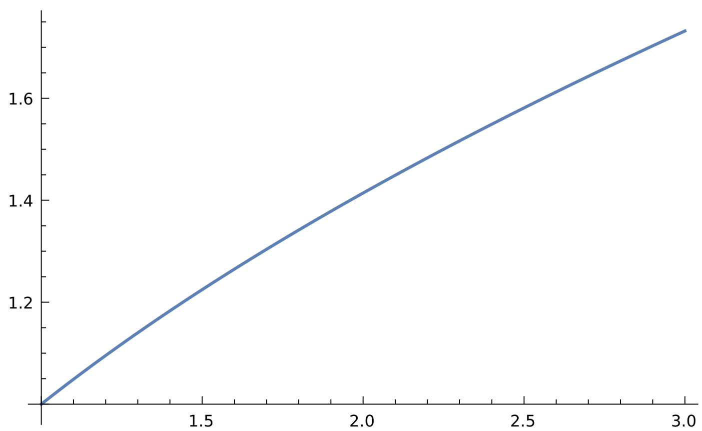
Q 2. Let f(x) = $\sqrt{x}$, x is near 2.

(a) Enter the input to see the graph of f as x goes from 1 to 3.

In[19]:=
```
Clear[f]
```

In[3]:=
```
With[{δ = 10^0}, Plot[√x, {x, 2 - δ, 2 + δ}]]
```

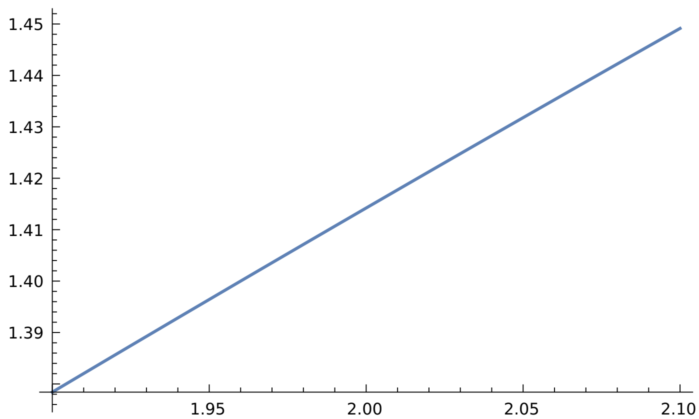
Out[3]=



(b) Now Zoom: Change the value of δ to be 1/10 and re-enter the input above to see the graph of f as x goes from 1.9 to 2.1. Do this again δ= 10^(-2),10^(-3),10^(-4) and 10^(-5).

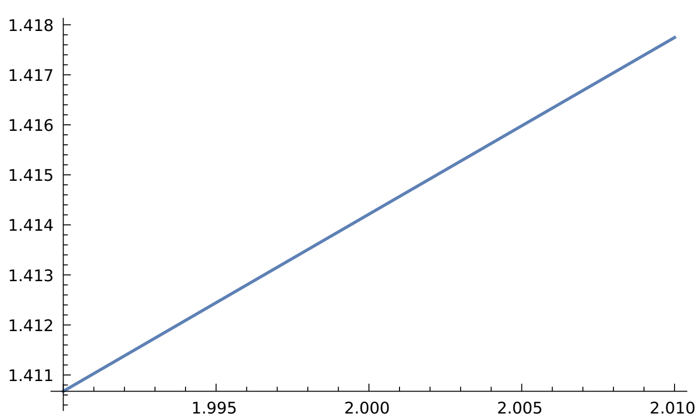
In[4]:= With$\left[\left\{\delta = \frac{1}{10}\right\}, \text{Plot}\left[\sqrt{x}, \{x, 2-\delta, 2+\delta\}\right]\right]$

Out[4]=

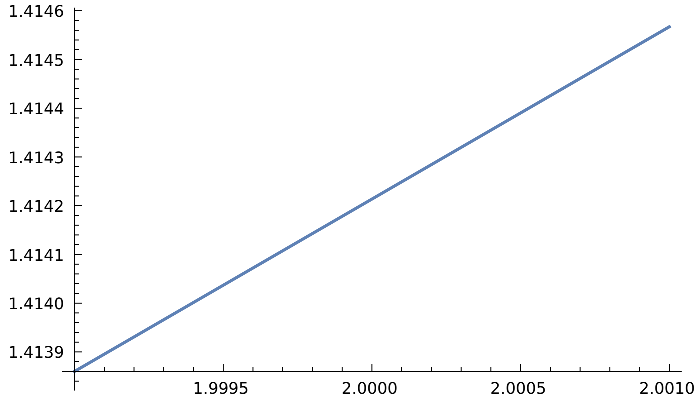


In[5]:= With$\left[\left\{\delta = \frac{1}{10^2}\right\}, \text{Plot}\left[\sqrt{x}, \{x, 2-\delta, 2+\delta\}\right]\right]$

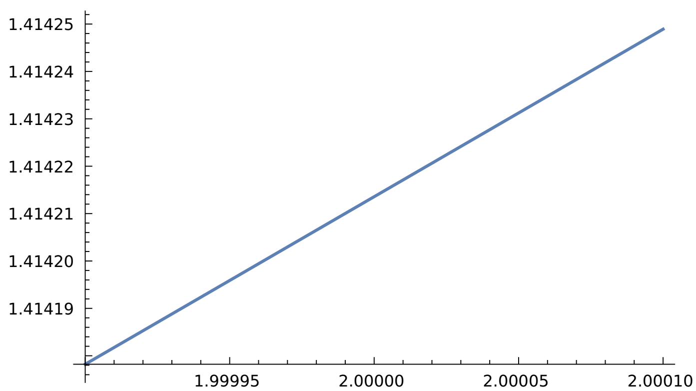
Out[5]=



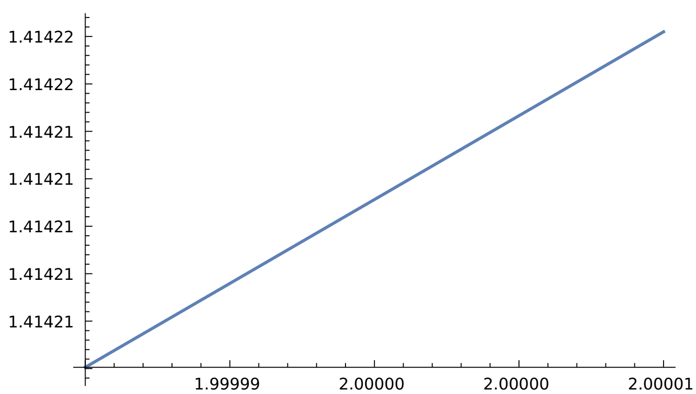
In[7]:= With[{$\delta = \frac{1}{10^3}$}, Plot[$\sqrt{x}$, {x, 2 - δ, 2 + δ}]]

Out[7]=


In[8]:= With[{$\delta = \frac{1}{10^4}$}, Plot[$\sqrt{x}$, {x, 2 - δ, 2 + δ}]]

Out[8]=


In[9]:= With[{$\delta = \frac{1}{10^5}$}, Plot[$\sqrt{x}$, {x, 2 - δ, 2 + δ}]]

Out[9]=


(c) Use the last plot to approximate $\sqrt{2}$ to six significant digits. Check your answers using N.

By the last plot we can clearly approximate that $\sqrt{2}$=1.41421

In[14]:=
```
N[Sqrt[2], 6]
```
Out[14]=
```
1.41421
```

(d) When making a plot, the lower and upper bounds on the iterator must be distinct when rounded to machine precision. Enter the previous plot command with δ=10^(-20). An error message results. Read the error message and speculate as to what is happening. The bottom line is that zooming has its limits.

In[7]:=
```
With[{δ = 10^(-20)}, Plot[Sqrt[x], {x, 2 - δ, 2 + δ}]]
```

Plot : Endpoints for x in $\left\{x, \frac{199999999999999999999}{100000000000000000000}, \frac{200000000000000000001}{100000000000000000000}\right\}$ must have distinct machine −precision numerical values .

Plot : Endpoints for x in $\left\{x, \frac{199999999999999999999}{100000000000000000000}, \frac{200000000000000000001}{100000000000000000000}\right\}$ must have distinct machine −precision numerical values .

Plot : Endpoints for x in $\left\{x, \frac{199999999999999999999}{100000000000000000000}, \frac{200000000000000000001}{100000000000000000000}\right\}$ must have distinct machine −precision numerical values .

General : Further output of Plot::plld will be suppressed during this calculation .

Plot : Endpoints for x in $\left\{x, \frac{199999999999999999999}{100000000000000000000}, \frac{200000000000000000001}{100000000000000000000}\right\}$ must have distinct machine −precision numerical values .

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Plot : Endpoints for x in $\left\{x, \frac{199999999999999999999}{100000000000000000000}, \frac{200000000000000000001}{100000000000000000000}\right\}$ must have distinct machine −precision numerical values .

General : Further output of Plot::plld will be suppressed during this calculation .

Out[7]= $\text{Plot}\left[\sqrt{x}, \left\{x, 2 - \frac{1}{100\,000\,000\,000\,000\,000\,000}, 2 + \frac{1}{100\,000\,000\,000\,000\,000\,000}\right\}\right]$

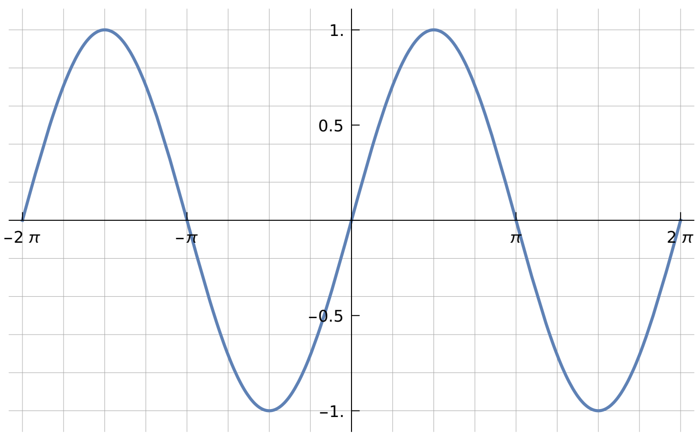
The difference of both the values is very small that it cannot be read by the computer therefore mathematica is showing error.

Section 3.3

Q 1. Use the GridLines **and** Ticks **options, as well as the setting** GridLinesStyle -> Lighter[Gray], **to produce the following** Plot **of the sine function.**

In[14]:=
```
Plot[Sin[x], {x, -2 Pi, 2 Pi}, GridLines → {Range[-2 Pi, 2 Pi, Pi / 4], Range[-1.0, 1.0, 0.2]},
  GridLinesStyle → Lighter[Gray], Ticks → {Range[-2 Pi, 2 Pi, Pi], Range[-1.0, 1.0, 0.5]}]
```

Out[14]=



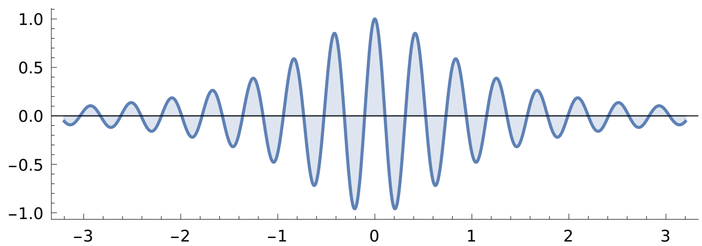
Q 2. Use the Axes,Frame,FrameStyle,PlotRange and AspectRatio options to produce the following Plot of the function y=Cos(15 x)/(1+x^2).

In[2]:=
```
ClearAll[f]
```

In[3]:=
$$f[x\_] := \frac{\text{Cos}[15\ x]}{1 + x^2}$$

In[6]:=
```
Plot[f[x], {x, -3.2, 3.2}, Axes → {True, False}, Frame → {{True, False}, {True, False}},
 Filling → Axis, PlotRange → Full, AspectRatio → Automatic, FrameStyle → Black]
```

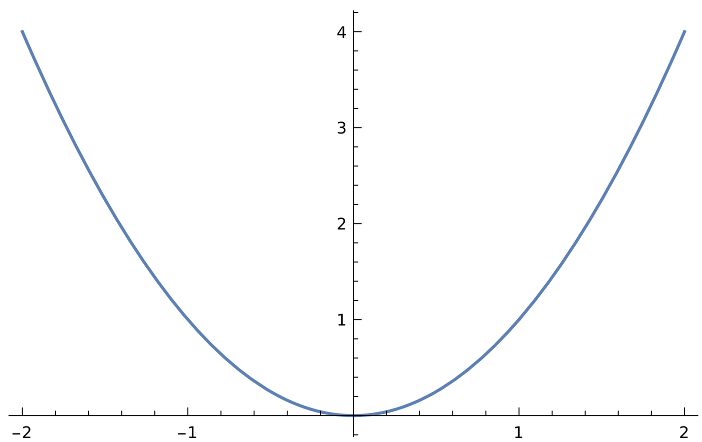
Out[6]=



Q 4. Plot the function f(x)=x^2 on domain -2≤x≤2, and set Exclusions to {x==1}. Note that f has no vertical asymptote at x=1.

In[35]:=
```
Plot[x^2, {x, -2, 2}, Exclusions → {x == 1}]
```

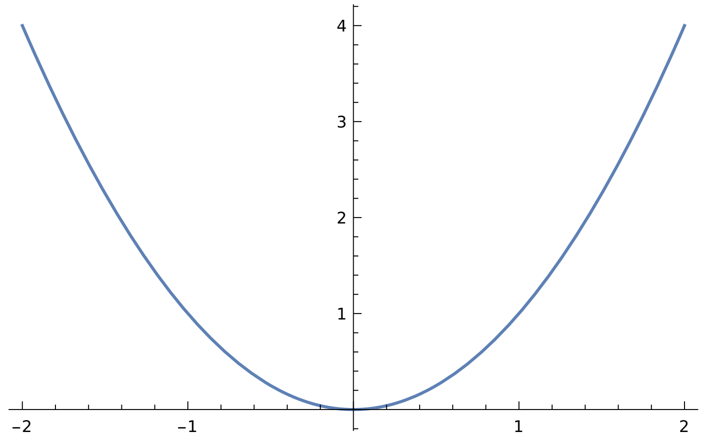
Out[35]=



Since f has no vertical asymptote at x=1 therefore the graph will be same without exclusion command also.

In[36]:= `Plot[x^2, {x, -2, 2}]`

Out[36]=

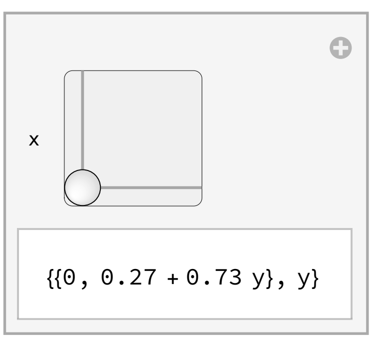


Section 3.4

Q 1. The following simple Manipulate has two sliders: one for x and one for y. Make a Manipulate that also has output [x,y] but that has a single Slider2D controller.

In[25]:= `Manipulate[{x, y}, {x, y, {0, 1}}]`

Out[25]=

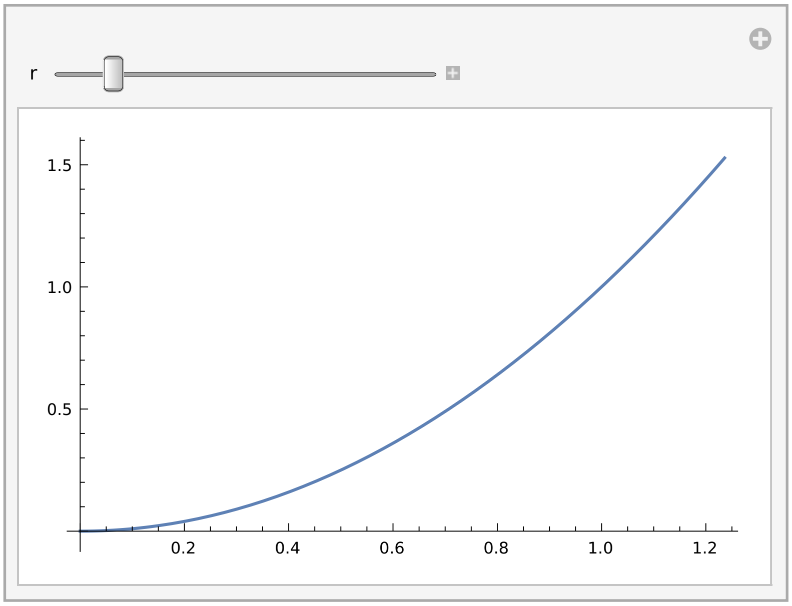


Q 2. Make a manipulate of a plot where the user can adjust the aspectratio in real time from starting value of 1/5 to an ending value of 5. Set image size to

{Automatic128} so the height remains constant as the slider is moved.

In[30]:=
```
Manipulate[Plot[x^2, {x, 0, r}], {r, 1, 3}, ImageSize → {Automatic128}, AspectRatio → 5/6]
```

Out[30]=



Section 3.5

Q 1. The partition command is used to break a single list into sublists of equal length. It is useful for breaking up a list into rows for displays within a grid.

(a) Enter the following inputs and discuss the outputs.

In[31]:=
```
Range[100]
```

Out[31]=
```
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22,
 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41,
 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61,
 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81,
 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100}
```

In[39]:=
```
Partition[Range[100], 10]
```

Out[39]=
```
{{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, {11, 12, 13, 14, 15, 16, 17, 18, 19, 20},
 {21, 22, 23, 24, 25, 26, 27, 28, 29, 30}, {31, 32, 33, 34, 35, 36, 37, 38, 39, 40},
 {41, 42, 43, 44, 45, 46, 47, 48, 49, 50}, {51, 52, 53, 54, 55, 56, 57, 58, 59, 60},
 {61, 62, 63, 64, 65, 66, 67, 68, 69, 70}, {71, 72, 73, 74, 75, 76, 77, 78, 79, 80},
 {81, 82, 83, 84, 85, 86, 87, 88, 89, 90}, {91, 92, 93, 94, 95, 96, 97, 98, 99, 100}}
```

(b) Form a table of the first 100 integers, with twenty digits in a row.

In[40]:=
```
data = Table[x, {x, 1, 100}]
```

Out[40]=
```
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22,
 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41,
 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61,
 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81,
 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100}
```

In[44]:=
```
data1 = Partition[data, 20]
```

Out[44]=
```
{{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20},
 {21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40},
 {41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60},
 {61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80},
 {81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100}}
```

In[45]:=
```
Grid[data1]
```

Out[45]=

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

(c) Make the same table as above, but use only the table and Range command.

In[1]:=
```
Grid[Table[Range[x, x + 19], {x, {1, 21, 41, 61, 81}}]]
```

Out[1]=

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

(d) Make the same table as above, but use only the table command twice. Do not use partition or range.

```
In[85]:= sol = Table[Table[x, {x, r, r + 19}], {r, 1, 100, 20}]

Out[85]= {{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20},
  {21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40},
  {41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60},
  {61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80},
  {81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100}}

In[86]:= Grid[sol]
```

Out[86]=

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

Q 4. The Sum command has a syntax similar to that of table. Solve the following questions using given commands.

(a)Use the sum command to evaluate sum of x^3 where x = 1,2,3.........20.

```
In[62]:= ClearAll[f]

In[63]:= f[x_] := x ^ 3

In[64]:= Sum[f[x], {x, 1, 20}]

Out[64]= 44 100
```

(b)Make a table of values x = 1,2,3........10 for the function f(x)=1+2^x+3^x+........20^x.

```
In[69]:= ClearAll[f]

In[70]:= f[x_] := i ^ x

In[71]:= findsum = Sum[f[x], {i, 1, 20}]
```

Out[71]= $1 + 2^x + 3^x + 4^x + 5^x + 6^x + 7^x + 8^x + 9^x + 10^x + 11^x + 12^x + 13^x + 14^x + 15^x + 16^x + 17^x + 18^x + 19^x + 20^x$

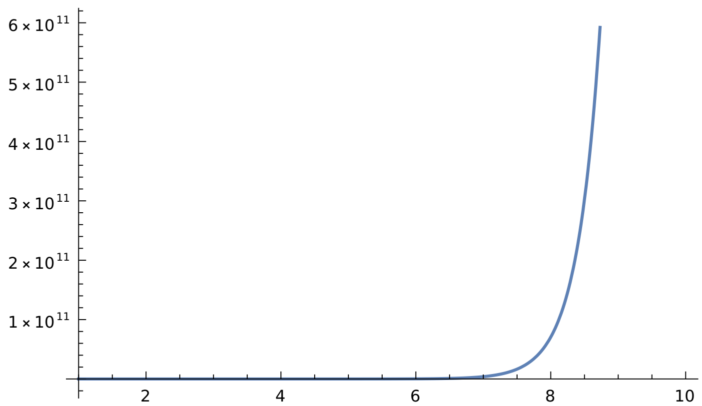
In[72]:=
```
Table[findsum, {x, 1, 10}]
```

Out[72]= {210, 2870, 44 100, 722 666, 12 333 300, 216 455 810,
3 877 286 700, 70 540 730 666, 1 299 155 279 940, 24 163 571 680 850}

(c)Plot f(x) on the domain 1≤x≤10.

In[73]:=
```
Plot[findsum, {x, 1, 10}]
```

Out[73]=



Section 3.6

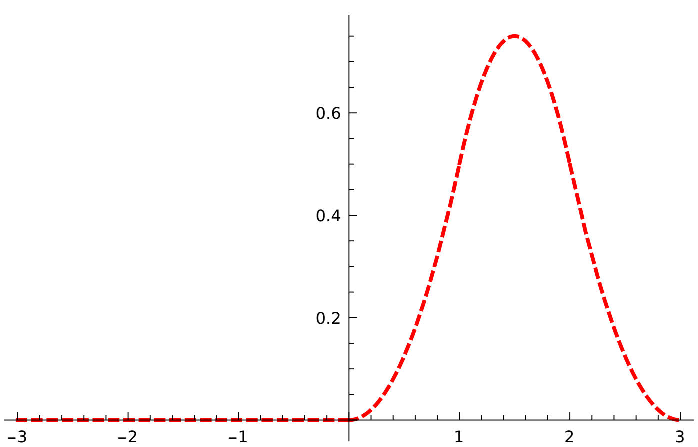
Q 2. Make a plot of a piecewise function below.

In[88]:=
```
ClearAll[f]
```

In[90]:=
```
f[x_] := Piecewise[{{0, x < 0}, {x^2/2, 0 ≤ x ≤ 1},
   {-x^2 + 3 x - 3/2, 1 ≤ x ≤ 2}, {(1/2) (3 - x)^2, 2 ≤ x ≤ 3}, {0, x ≤ 3}}]
```

In[94]:=
```
Plot[f[x], {x, -3, 3}, PlotStyle → {Red, Dashed, Thick}]
```

Out[94]=



Q 3. A step function assumes a constant value

between consecutive integers n and n+1. Make a plot of the step function f(x) whose value is n^2 when n≤x≤ n+1. Use the domain 0≤x≤20.

In[7]:=
```
ClearAll[f]
```

In[19]:=
```
h[x_] := Piecewise[{{n^2, n ≤ x ≤ n + 1}, {1, n ≤ x ≤ n + 1}}]
```

In[20]:=
```
Plot[h[x], {x, 0, 20}]
```

Out[20]=