

CHAPTER 3 - FUNCTIONS AND THEIR PLOT.

EXERCISE

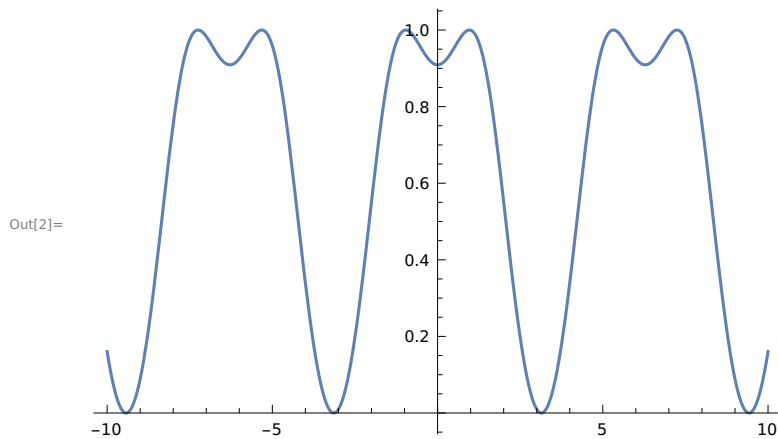
SECTION 3.2 - EXERCISE .

1) Plot the following functions on the domain $10 \leq x \leq 10$.

a. $\sin(1 + \cos(x))$

```
In[2]:= f[x_] := Sin[1 + Cos[x]];
```

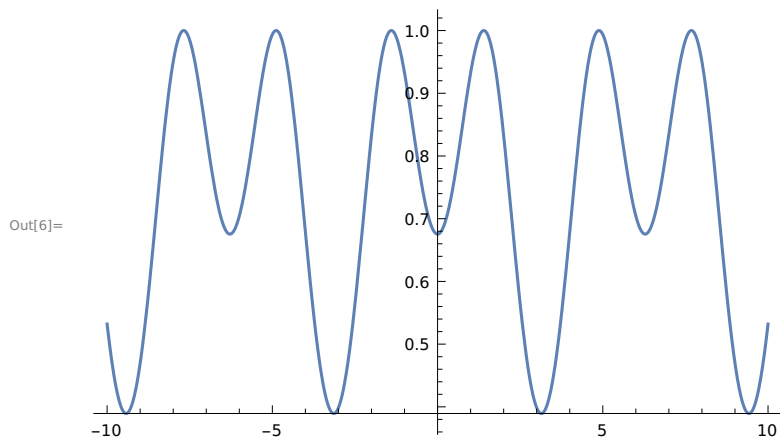
```
In[2]:= Plot[f[x], {x, -10, 10}]
```



b. $\sin(1.4 + \cos(x))$

```
In[3]:= g[x_] := Sin[1.4 + Cos[x]];
```

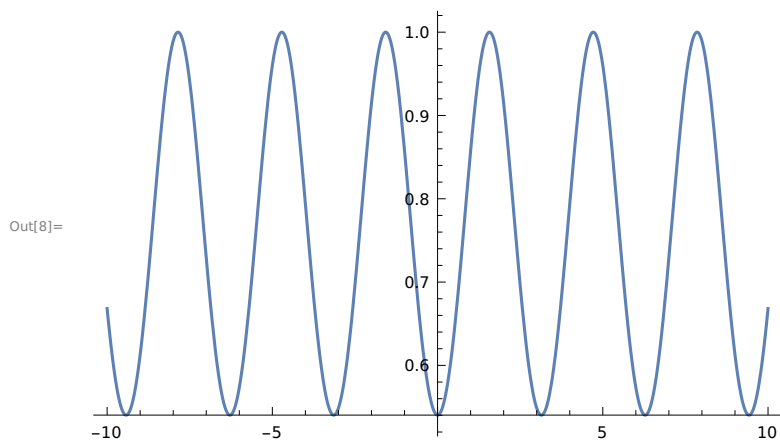
In[6]:= `Plot[g[x], {x, -10, 10}]`



c. $\sin(\pi/2 + \cos(x))$

In[7]:= `h[x_] := Sin[Pi / 2 + Cos[x]];`

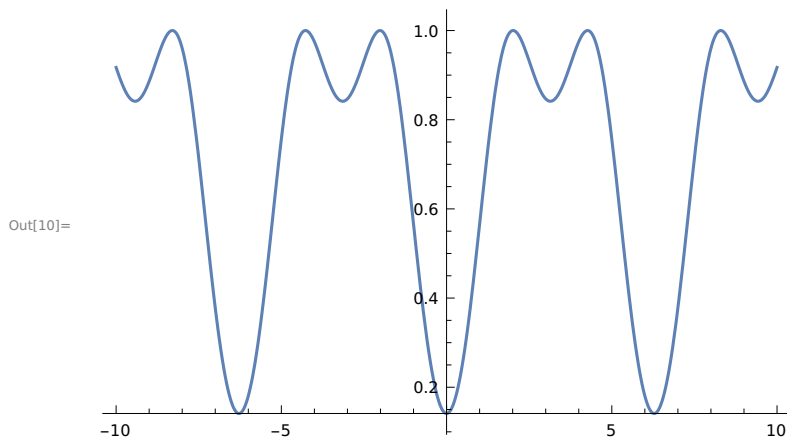
In[8]:= `Plot[h[x], {x, -10, 10}]`



d. $\sin(2 + \cos(x))$

In[9]:= `s[x_] := Sin[2 + Cos[x]];`

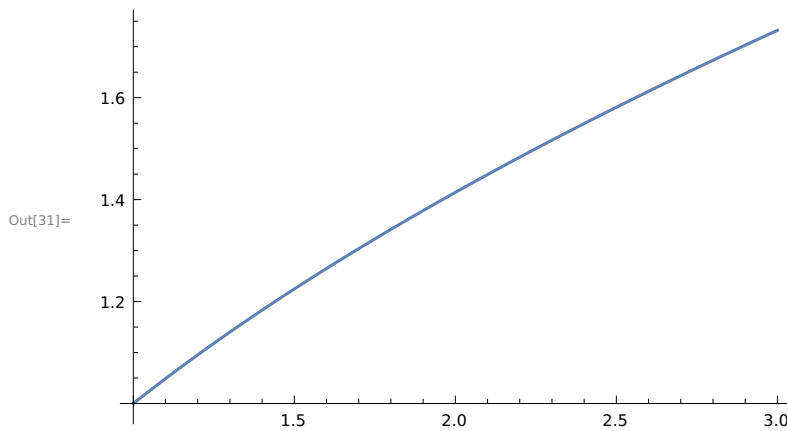
In[10]:= `Plot[s[x], {x, -10, 10}]`



2) One can zoom in toward a particular point in the domain of a function and see how the graph appears at different zoom levels. For instance, consider the square root function $f(x) = \sqrt{x}$ when x is near 2.

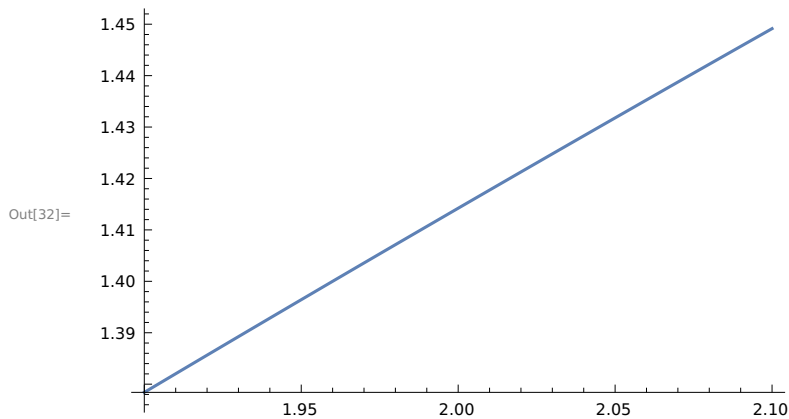
a. Graph of f as x goes from 1 to 3.

In[31]:= `With[{ $\delta = 10^{-6}$ }, Plot[Sqrt[x], {x, 2 - δ , 2 + δ }]`

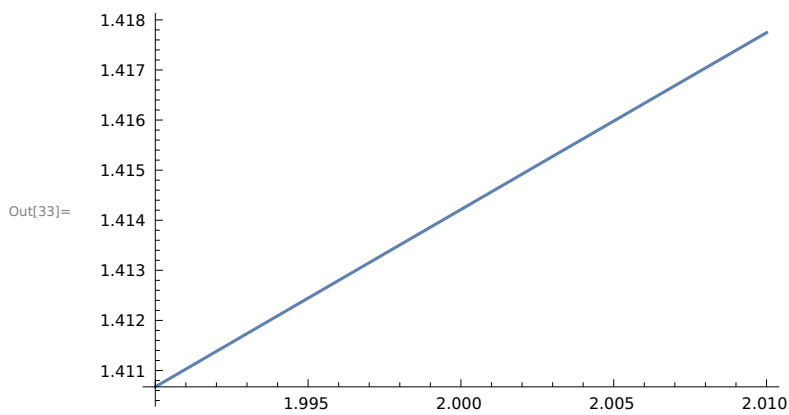


b. Change the value of δ to be 10^{-1} , 10^{-2} , 10^{-3} , 10^{-4} , 10^{-5} and see the graph of f as x goes from 1.9 to 2.1 .

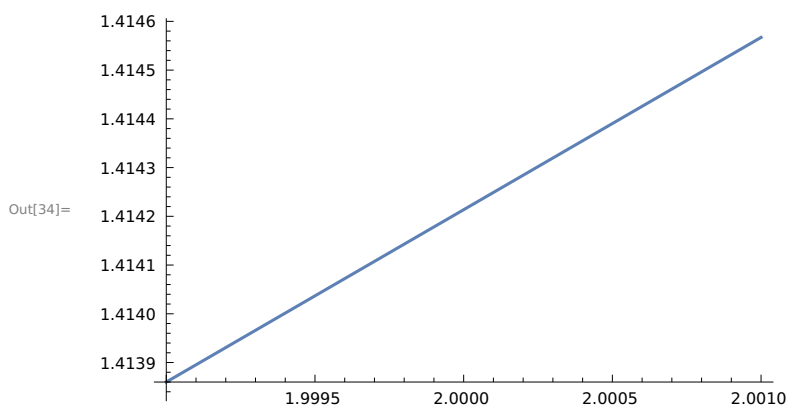
In[32]:= `With[{ $\delta = 10^{-1}$ }, Plot[Sqrt[x], {x, 2 - δ , 2 + δ }]`



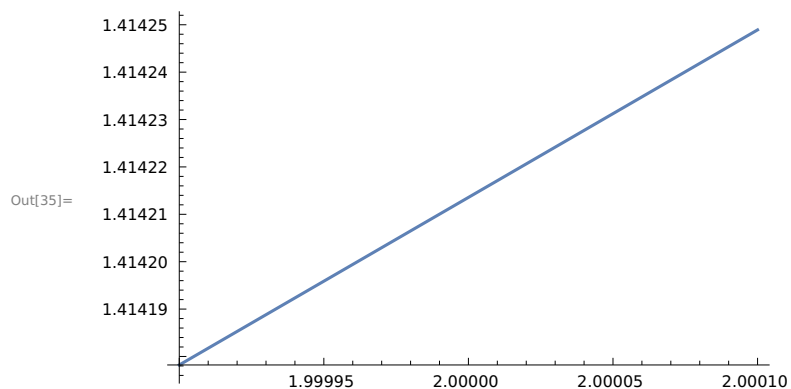
In[33]:= `With[{ $\delta = 10^{-2}$ }, Plot[Sqrt[x], {x, 2 - δ , 2 + δ }]`



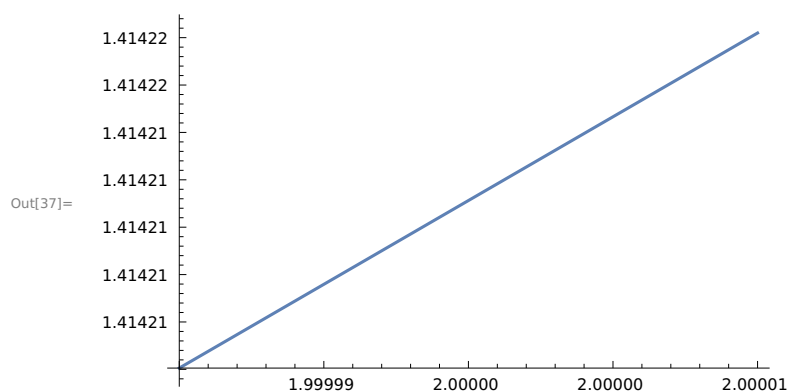
In[34]:= `With[{ $\delta = 10^{-3}$ }, Plot[Sqrt[x], {x, 2 - δ , 2 + δ }]`



In[35]:= `With[{δ = 10^-4}, Plot[Sqrt[x], {x, 2 - δ, 2 + δ}]]`



In[37]:= `With[{δ = 10^-5}, Plot[Sqrt[x], {x, 2 - δ, 2 + δ}]]`



c) Use the last plot to approximate $\sqrt{2}$ to six significant digits . check your answer using N.

- By the above plots we can approximate that $\sqrt{2} = 1.41421$

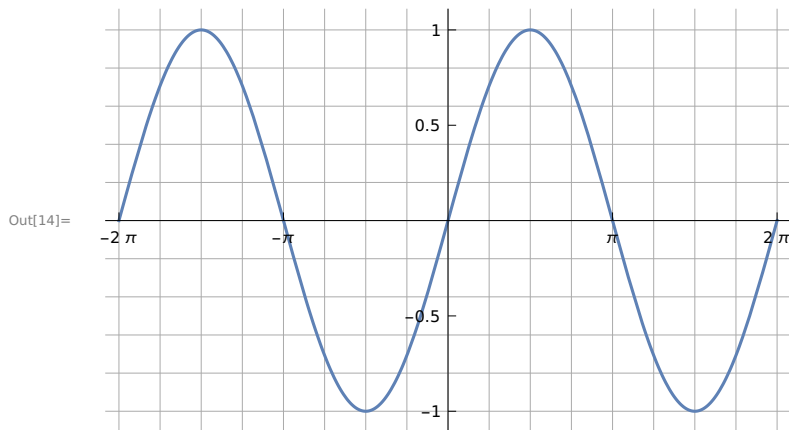
In[36]:= `N[Sqrt[2], 6]`

Out[36]= 1.41421

SECTION 3.3 - EXERCISE

1) Use the `GridLines` and `Ticks` options, as well as the setting `GridLineStyle Lighter$Gray`, to produce the following Plot of the sine function:

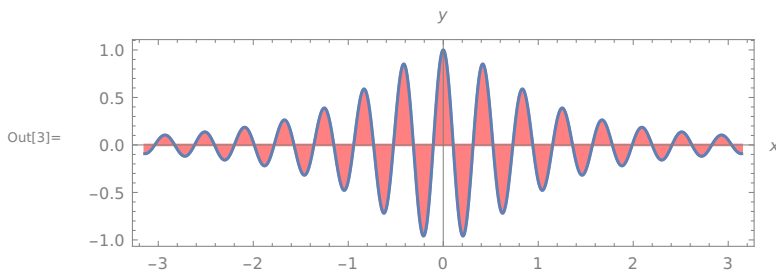
```
In[14]:= Plot[Sin[x], {x, -2 Pi, 2 Pi}, GridLines -> {Range[-2 Pi, 2 Pi, Pi/4], Range[-1, 1, 0.2]},
  Ticks -> {{-2 Pi, -Pi, Pi, 2 Pi}, {-0.5, -1, 0.5, 1}}, GridLineStyle -> Lighter[Gray]]
```



2) Use the `Axes`, `Frame`, `Filling`, `FrameStyle`, `PlotRange`, and `AspectRatio` options to produce the following plot of the function $y = \cos + 15x$

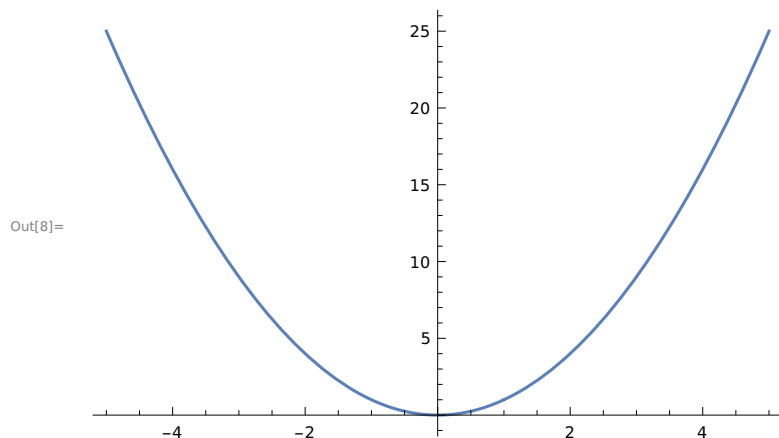
```
In[1]:= f[x_] := Cos[15 * x] / (1 + x ^ 2);
```

```
In[3]:= Plot[ f[x], {x, -Pi, Pi}, Axes -> True , AxesLabel -> {x, y},
  AxesStyle -> Directive[Gray], Frame -> True, Filling -> Axis, FillingStyle -> Pink,
  FrameStyle -> Gray, PlotRange -> Full, AspectRatio -> Automatic]
```



4) Plot the function $f(x) = x^2$ on the Domain $-2 \leq x \leq 2$ and set exclusions to $x = 1$. Note that f has no vertical asymptote at $x = 1$. What happens ?

In[8]:= `Plot[x^2, {x, -5, 5}, Exclusions -> {x == 1}]`

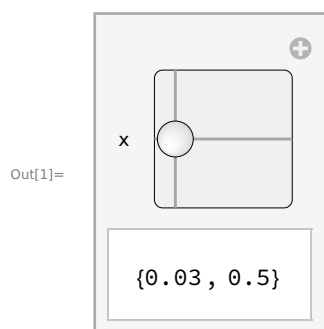


f has no vertical asymptote at $x = 1$ which shows that f is continuous.

SECTION 3.4 - EXERCISE

1) The following simple Manipulate has two sliders: one for x and one for y . Make a Manipulate that also has output $\{x, y\}$, but that has a single Slider2D controller.

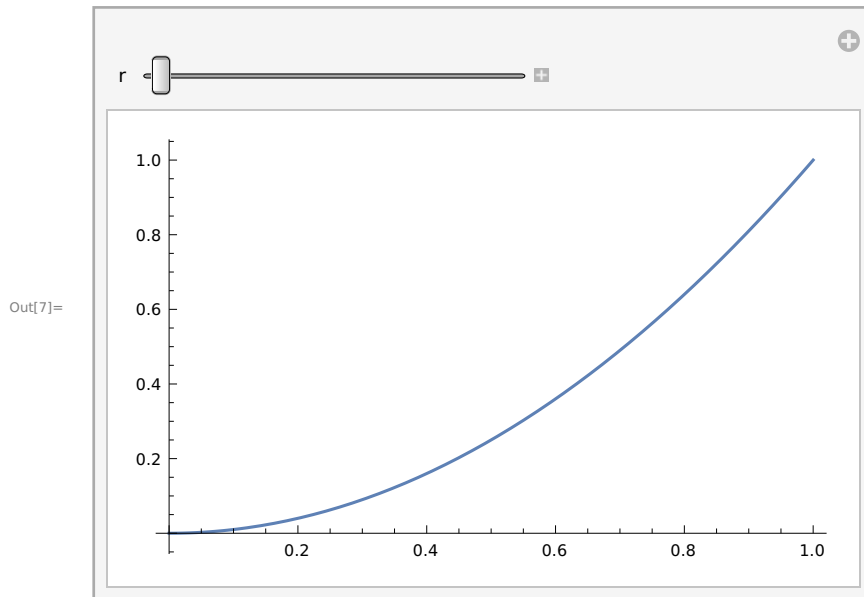
In[1]:= `Manipulate [x, {{x, y}, {0, 0}, {1, 1}}]`



2) Make a Manipulate of a Plot where the user can adjust the AspectRatio in real time, from a starting value of $1/5$ (five times as wide as it is tall) to an ending value of 5

(five times as tall as it is wide). Set `ImageSize` to `Automatic, 128` so the height remains constant as the slider is moved.

```
In[7]:= Manipulate[Plot[x^2, {x, 0, r}], {r, 1, 3},
  AspectRatio -> 5/6, ImageSize -> {Automatic, 128}]
```



SECTION 3.5 - EXERCISE

1)The Partition command is used to break a single list into sublists of equal length. It is useful for breaking up a list into rows for display within a Grid.

a. Enter the following inputs and discuss the outputs.

```
In[4]:= Range[100]
```

```
Out[4]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22,
  23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41,
  42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61,
  62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81,
  82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100}
```


In[5]:= **Partition [Range[100], 10]**

Out[5]= {{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, {11, 12, 13, 14, 15, 16, 17, 18, 19, 20},
 {21, 22, 23, 24, 25, 26, 27, 28, 29, 30}, {31, 32, 33, 34, 35, 36, 37, 38, 39, 40},
 {41, 42, 43, 44, 45, 46, 47, 48, 49, 50}, {51, 52, 53, 54, 55, 56, 57, 58, 59, 60},
 {61, 62, 63, 64, 65, 66, 67, 68, 69, 70}, {71, 72, 73, 74, 75, 76, 77, 78, 79, 80},
 {81, 82, 83, 84, 85, 86, 87, 88, 89, 90}, {91, 92, 93, 94, 95, 96, 97, 98, 99, 100}}

b. Form a table of the first 100 integers , with twenty digits per row. he first two

rows , for example , should look like this :

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40

In[6]:= **Table[x, {x, 1, 100}]**

Out[6]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22,
 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41,
 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61,
 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81,
 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100}

In[7]:= **Partition [Table[x, {x, 1, 100}], 20]**

Out[7]= {{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20},
 {21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40},
 {41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60},
 {61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80},
 {81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100}}

c. Make the same table as above , but use only the table and range command.

In[8]:= **Table[Range[10], 10]**

Out[8]= {{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10},
 {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10},
 {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10},
 {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}}

d. Make the same table as above but use only the table command twice . do not use partition or range .

```
In[3]:= Table[x, {x, 1, 100}]
```

```
Out[3]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22,
23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41,
42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61,
62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81,
82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100}
```

4) The Sum command has a syntax similar to that of Table.

**a). Use the sum command to evaluate the following expression:
 $1^3+2^3+3^3+4^3+5^3+6^3+7^3+8^3+9^3+10^3+11^3+12^3+13^3+14^3+15^3$
 $+16^3+17^3+18^3+19^3+20^3$**

```
In[11]:= f[x_] := x^3
```

```
Sum[f[x], {x, 1, 20}]
```

```
Out[12]= 44100
```

**b. Make a table of values for $x=1, 2, \dots, 10$ for the function
 $f(x)=1+2^x+3^x+4^x+5^x+6^x+7^x+8^x+9^x+10^x+11^x+12^x+13^x+14^x+15^x$
 $10+16^x+17^x+18^x+19^x+20^x$**

```
In[13]:= f[x_] := 1 + 2^x + 3^x + 4^x + 5^x + 6^x + 7^x + 8^x + 9^x + 10^x +
```

```
11^x + 12^x + 13^x + 14^x + 15^x + 16^x + 17^x + 18^x + 19^x + 20^x
```

```
In[14]:= Table[f[x], {x, 1, 10}]
```

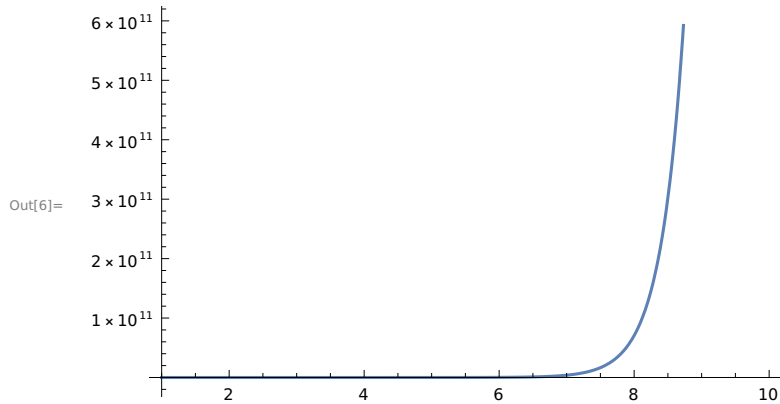
```
Out[14]= {210, 2870, 44100, 722666, 12333300, 216455810,
3877286700, 70540730666, 1299155279940, 24163571680850 }
```

c. Plot $f(x)$ on the domain $1 \leq x \leq 10$.

```
In[5]:= f[x_] := 1 + 2^x + 3^x + 4^x + 5^x + 6^x + 7^x + 8^x + 9^x + 10^x +
```

```
11^x + 12^x + 13^x + 14^x + 15^x + 16^x + 17^x + 18^x + 19^x + 20^x
```

In[6]:= `Plot[f[x], {x, 1, 10}]`



SECTION 3.6 - EXERCISE

2) Make a plot of the piecewise function below, and comment on its shape.

$f(x) = \begin{cases} 0, & x < 0 \\ \end{cases}$

$\begin{cases} x^2/2, & 0 \leq x < 1 \\ \end{cases}$

$\begin{cases} -x^2 + 3x - 3/2, & 1 \leq x < 2 \\ \end{cases}$

$\begin{cases} (1/2)(3-x)^2, & 2 \leq x < 3 \\ \end{cases}$

$\begin{cases} 0, & 3 \leq x \end{cases}$

In[1]:= `f[x_] := Piecewise [{{0, x < 0}, {x^2 / 2, 0 ≤ x < 1},
{- x^2 + 3 x - 3 / 2, 1 ≤ x < 2}, {(1 / 2) (3 - x)^2, 2 ≤ x < 3}, {0, x ≤ 3}}]`

In[2]:= `Plot[f[x], {x, -3, 3}]`

