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CHAPTER 3 : FUNCTIONS AND THEIR GRAPHS

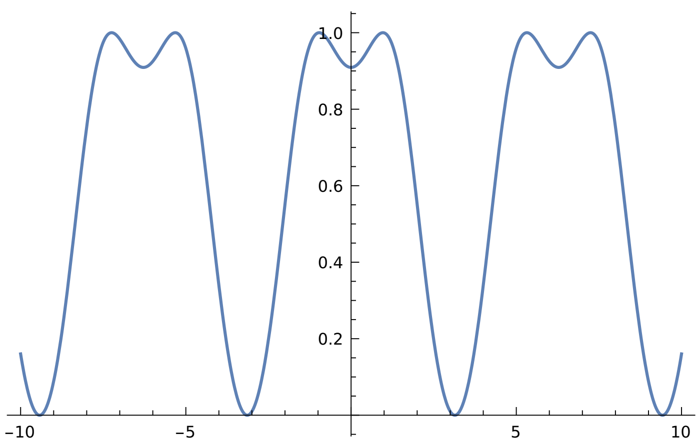
EXERCISES 3.2

1. Plot the following functions on the domain -10 ≤ x ≤ 10.

a. sin(1 + cos(x))

In[1]:=
```
Plot[Sin[1 + Cos[x]], {x, -10, 10}]
```

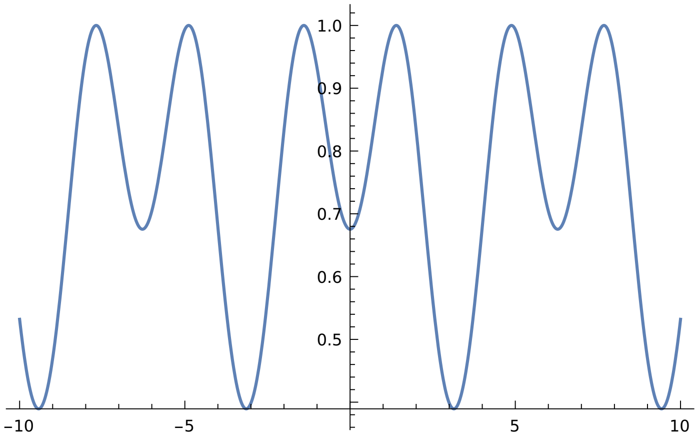
Out[1]=



b. sin(1.4 + cos(x))

In[2]:=
```
Plot[Sin[1.4 + Cos[x]], {x, -10, 10}]
```

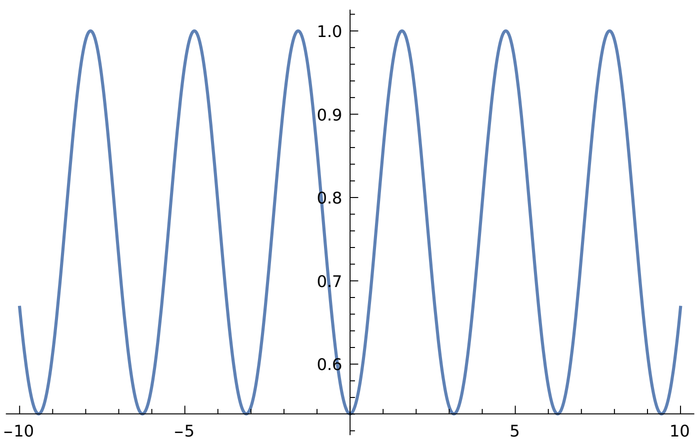
Out[2]=



c. sin(π/2 + cos(x))

In[3]:=
```
Plot[Sin[Pi / 2 + Cos[x]], {x, -10, 10}]
```

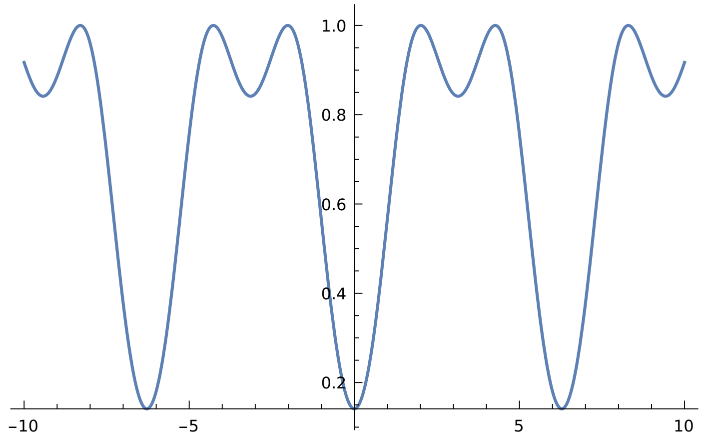
Out[3]=



d. sin(2 + cos(x))

In[4]:=
```
Plot[Sin[2 + Cos[x]], {x, -10, 10}]
```

Out[4]=



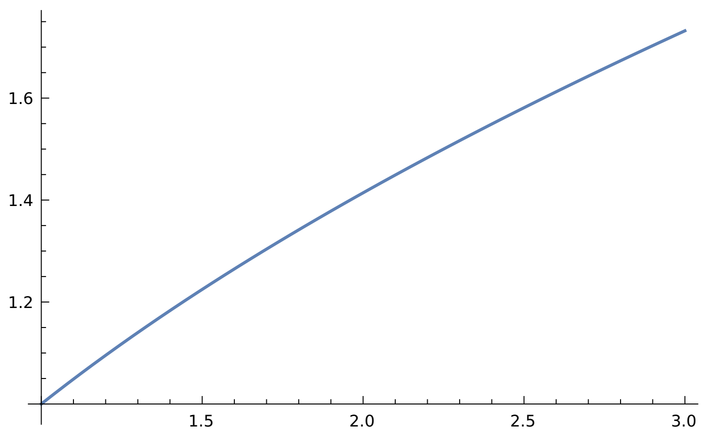
2. One can zoom in toward a particular point in the domain of a function and see how the graph appears at different zoom levels. For instance, consider the square root of a function f(x) = $\sqrt{x}$ when x is near 2.

a. Enter the input below to see the graph of f as x goes from 1 to 3.

With[{δ = 10⁰}, Plot[$\sqrt{x}$, {x, 2 - δ , 2 + δ }]]

In[5]:=
```
With[{δ = 10^0}, Plot[Sqrt[x] , {x, 2 - δ , 2 + δ }]]
```

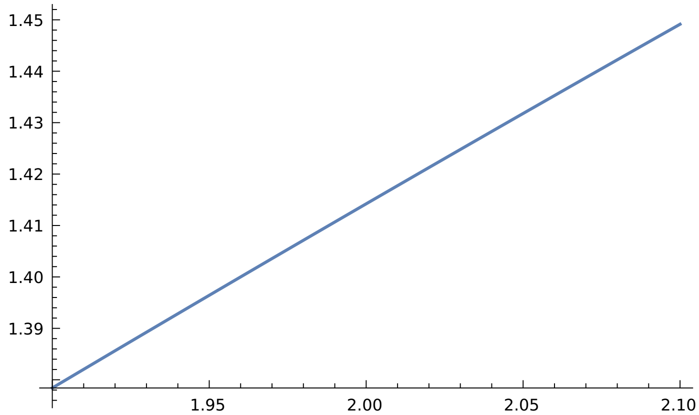
Out[5]=



b. Now zoom; change the value of δ to be $\frac{1}{10}$ and re-enter the input above to see the graph of f as x goes from 1.9 to 2.1. Do this again for $\delta = \frac{1}{10^2}$, $\frac{1}{10^3}$, $\frac{1}{10^4}$, and $\frac{1}{10^5}$.

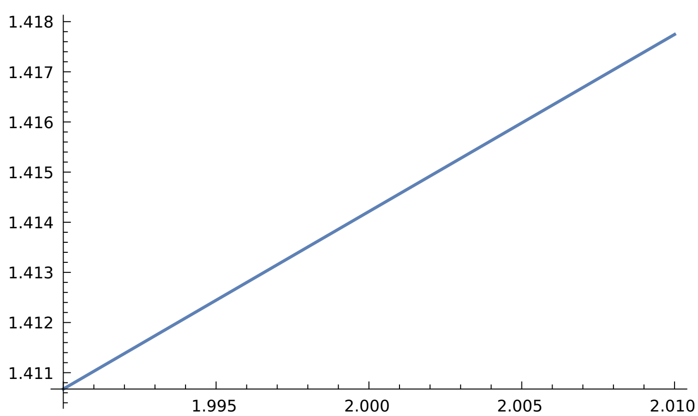
In[6]:= With[{$\delta = \frac{1}{10}$}, Plot[$\sqrt{x}$, {x, 2 - δ, 2 + δ}]]

Out[6]=



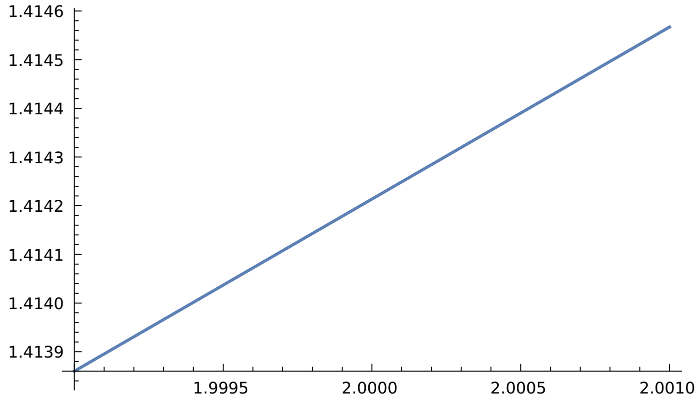
In[7]:= With[{$\delta = \frac{1}{10^2}$}, Plot[$\sqrt{x}$, {x, 2 - δ, 2 + δ}]]

Out[7]=



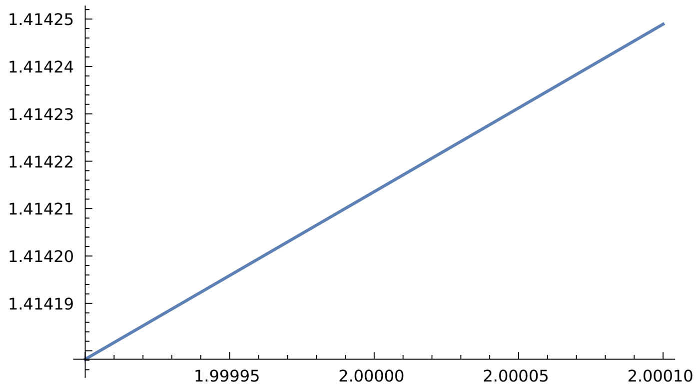
In[8]:= With$\left[\left\{\delta = \frac{1}{10^3}\right\}, \text{Plot}\left[\sqrt{x}, \{x, 2-\delta, 2+\delta\}\right]\right]$

Out[8]=



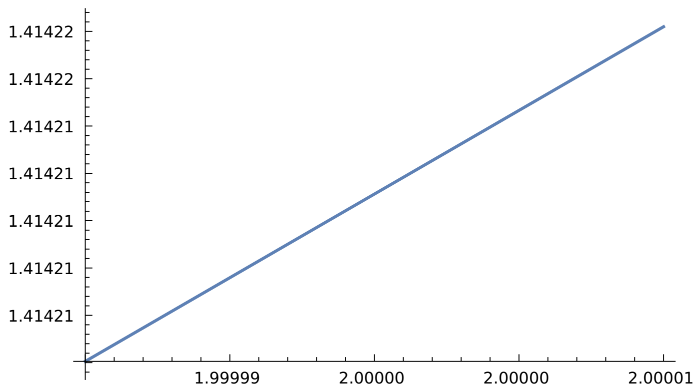
In[9]:= With$\left[\left\{\delta = \frac{1}{10^4}\right\}, \text{Plot}\left[\sqrt{x}, \{x, 2-\delta, 2+\delta\}\right]\right]$

Out[9]=



In[10]:= With$\left[\left\{\delta = \frac{1}{10^5}\right\}, \text{Plot}\left[\sqrt{x}, \{x, 2-\delta, 2+\delta\}\right]\right]$

Out[10]=

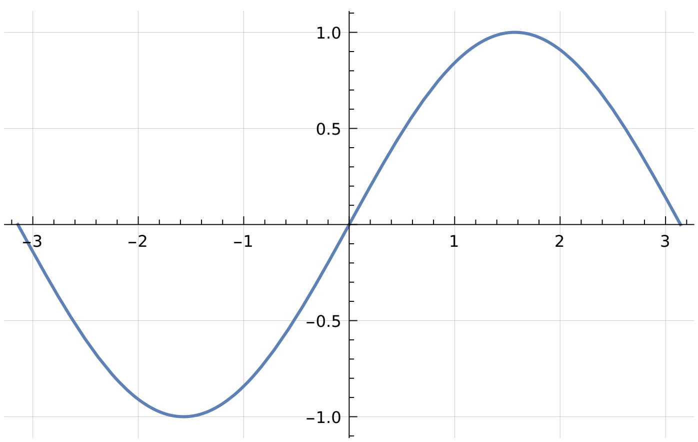


c. Use the last two plots to approximate $\sqrt{2}$ to six significant digits. Check your answer using N.

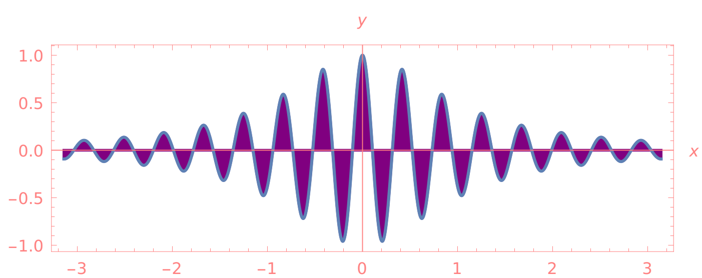
In[11]:=
```
N[√2 , 6]
```
Out[11]=
```
1.41421
```

EXERCISES 3.3

1. Use the **GridLines** and **Ticks** options, as well as the setting **GridLines → Lighter[Gray]**, to produce the **Plot** of the sine function.

In[12]:=
```
Plot[Sin[x], {x, -Pi, Pi}, GridLines → Automatic,
 GridLines → Lighter[Gray], Ticks → Automatic]
```
Out[12]=


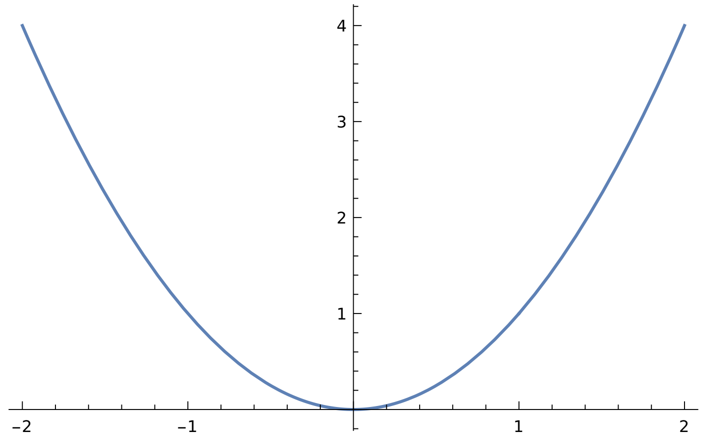
2. Use the **Axes**, **Frame**, **Filling**, **FrameStyle**, **PlotRange**, and **AspectRatio** options to produce the plot of the the function $y = \frac{\cos(15x)}{1+x^2}$.

In[13]:=
```
Plot[Cos[15 x]/(1 + x^2), {x, -Pi, Pi}, Axes → True, AxesLabel → {x, y},
 AxesStyle → Directive[Pink], Frame → True, Filling → Axis, FillingStyle → Purple,
 FrameStyle → Pink, PlotRange → Full, AspectRatio → Automatic]
```
Out[13]=


4. Plot the function f(x) = x^2 on the domain -2 ≤ x ≤ 2, and set **Exclusions** to {x==1}. Note that f has no vertical asymptote at x = 1. What happens ?

In[14]:=
```
Plot[x^2, {x, -2, 2}, Exclusions → {x == 1}]
```

Out[14]=



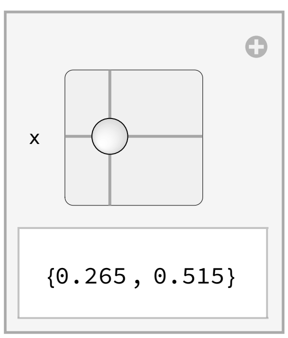
f has no vertical asymptote at x = 1 which shows that f is continuous.

EXERCISES 3.4

1. Make a **Manipulate** that has output {x,y}, and has a single **Slider2D** controller.

In[15]:=
```
Manipulate[x, {{x, y}, {0, 0}, {1, 1}}]
```

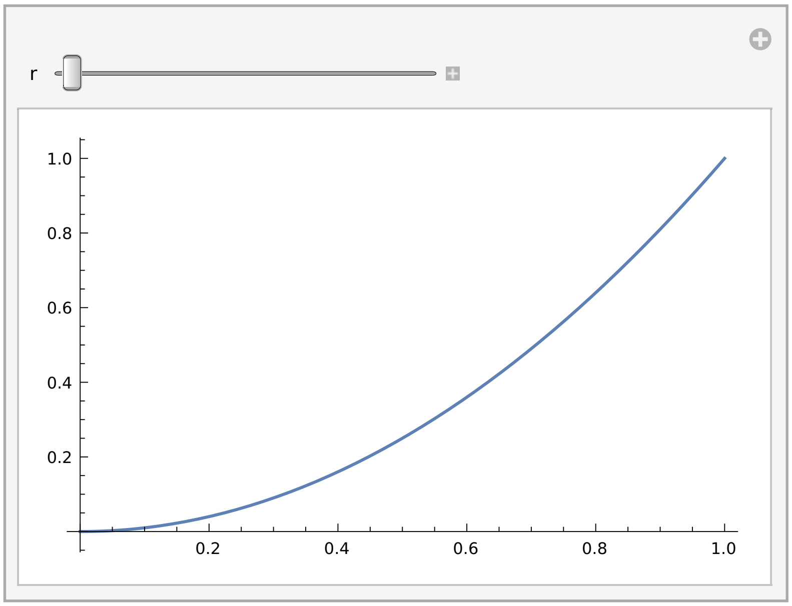
Out[15]=



2. Make a **Manipulate** of a **Plot** where the user can adjust the **AspectRatio** in real time, from a starting value of 1/5 (five times as wide as it is tall) to an ending value of 5 (five times as tall as it is wide) . Set **ImageSize** to **{Automatic, 128}** so that height remains constant as the slider is moved.

In[16]:=
```
Manipulate[Plot[x^2, {x, 0, r}], {r, 1, 3},
 AspectRatio → 5/6, ImageSize → {Automatic, 128}]
```

Out[16]=



EXERCISES 3.5

1. The **Partition** command is used to break a single list into sublists of equal length. It is useful for breaking up a list into rows for display within a **Grid**.

a. Enter the following inputs and discuss the outputs.

Range[100]
Partition[Range[100],10]

In[17]:=
```
Range[100]
```

Out[17]=
```
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22,
 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41,
 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61,
 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81,
 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100}
```

Range[100] display numbers from 1 to 100.

In[18]:=
```
Partition[Range[100], 10]
```

Out[18]=
```
{{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, {11, 12, 13, 14, 15, 16, 17, 18, 19, 20},
 {21, 22, 23, 24, 25, 26, 27, 28, 29, 30}, {31, 32, 33, 34, 35, 36, 37, 38, 39, 40},
 {41, 42, 43, 44, 45, 46, 47, 48, 49, 50}, {51, 52, 53, 54, 55, 56, 57, 58, 59, 60},
 {61, 62, 63, 64, 65, 66, 67, 68, 69, 70}, {71, 72, 73, 74, 75, 76, 77, 78, 79, 80},
 {81, 82, 83, 84, 85, 86, 87, 88, 89, 90}, {91, 92, 93, 94, 95, 96, 97, 98, 99, 100}}
```

Partition[Range[100],10] display the numbers from 1 to 100 while simultaneously segregating them in a list 8 of 10 numbers

b. Format a table for the first 100 integers, with twenty digits per row.

In[19]:=
```
Grid[Partition[Table[x, {x, 1, 100}], 20]]
```

Out[19]=

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

c. Make the same table as above, but use only the **Table** and **Range** commands. Do not use Partition.

In[20]:=
```
Grid[Table[Range[x, x + 19], {x, {1, 21, 41, 61, 81}}]]
```

Out[20]=

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

d. Make the same table as above, but use only the **Table** command (twice). Do not use **Partition** or **Range**.

In[21]:=
```
Grid[Table[Table[x, {x, x, x + 19}], {x, {1, 21, 41, 61, 81}}]]
```

Out[21]=

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

4. The S**um** command has a syntax similar to that of **Table**.

a. Use the Sum command to evaluate the following expression :

$1^3+2^3+3^3+4^3+5^3+6^3+7^3+8^3+9^3+10^3+11^3+12^3+13^3+14^3+15^3+16^3+17^3+18^3+19^3+20^3$

In[22]:=
```
f[x_] := x^3;
```

In[23]:=
```
Sum[f[x], {x, 1, 20}]
```

Out[23]= 44 100

b. Make a table of values for x = 1, 2, . . . , 10 for the function

$f(x) = 1+2+3+4+5+6+7+8+9+10+11+12+13+14+15+16+17+18+19+20$

In[24]:=
```
f[x_] := 1 + 2^x + 3^x + 4^x + 5^x + 6^x + 7^x + 8^x + 9^x + 10^x +
   11^x + 12^x + 13^x + 14^x + 15^x + 16^x + 17^x + 18^x + 19^x + 20^x
```

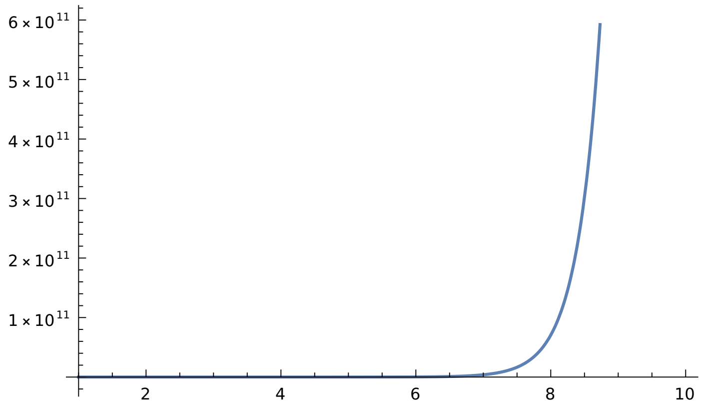
In[25]:=
```
Table[f[x], {x, 1, 10}]
```

Out[25]= {210, 2870, 44 100, 722 666, 12 333 300, 216 455 810,
3 877 286 700, 70 540 730 666, 1 299 155 279 940, 24 163 571 680 850}

c. Plot f (x) on the domain 1 ≤ x ≤ 10 .

In[26]:=
```
Plot[f[x], {x, 1, 10}]
```

Out[26]=



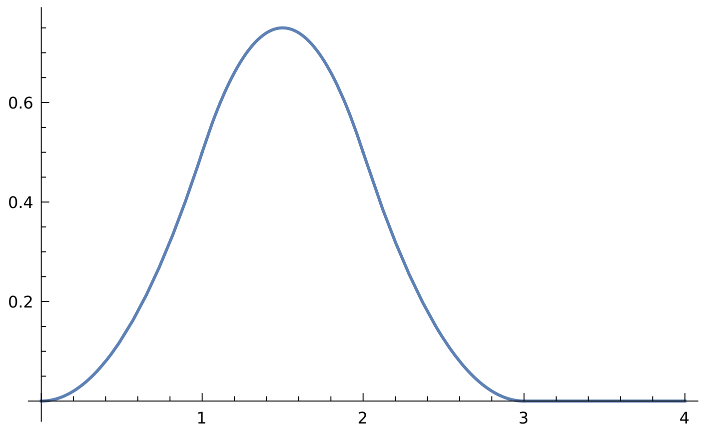
EXERCISES 3.6

1. Make a plot of the piecewise function and comment on its shape .

In[27]:=
```
g[x_] := Piecewise[{{0, x < 0}, {x^2/2, 0 ≤ x ≤ 1},
    {-x^2 + 3 x - 3/2, 1 ≤ x < 2}, {1/2 (3 - x)^2, 2 ≤ x < 3}, {0, 3 ≤ x}}]
```

In[28]:=
```
Plot[g[x], {x, 0, 4}]
```

Out[28]=



2. A step function assumes a constant value between consecutive integers n and n + 1 . Make a plot of the step function f (x) whose value is n^2 when n ≤ x ≤ n + 1 . Use the domain 0 ≤ x ≤ 20 .

In[43]:=
```
f[x_, n_] := Piecewise[{{n^2, n ≤ x ≤ n + 1}, {n, n ≤ x ≤ n + 1}}]
```

In[44]:=
```
Plot[f[x_, n_], {{x, 0, 20}, {n, 0, 19}}]
```

Plot : Range specification {{x, 0, 20}, {n, 0, 19}} is not of the form {x, xmin , xmax }.

Out[44]=
```
Plot[f[x_, n_], {{x, 0, 20}, {n, 0, 19}}]
```