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MAT/19/73

CHAPTER-3

Torrence(Ch.3) exercise solutions

EXERCISE: 3.2

Ques 1.Plot the following functions on the domain -10 ≤x ≤10

a. sin(1 +cos(x))

b.sin(1.4 +cos(x))

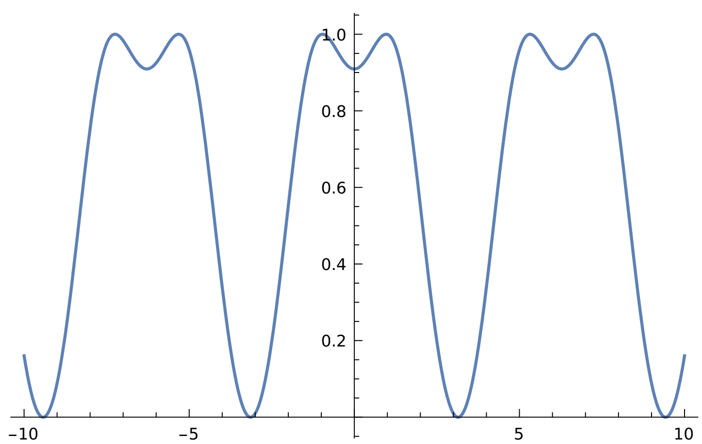
c. sin(π/2 +cos(x))

d. sin(2 +cos(x))

a)

In[1]:=
```
Plot[Sin[1 + Cos[x]], {x, -10, 10}]
```

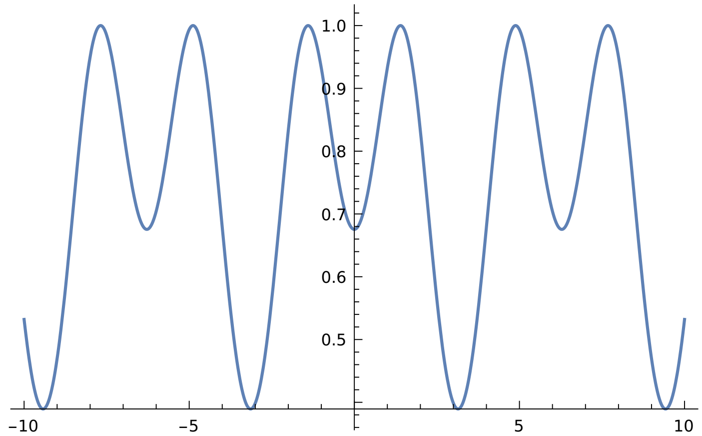
Out[1]=



b)

In[2]:= `Plot[Sin[1.4 + Cos[x]], {x, -10, 10}]`

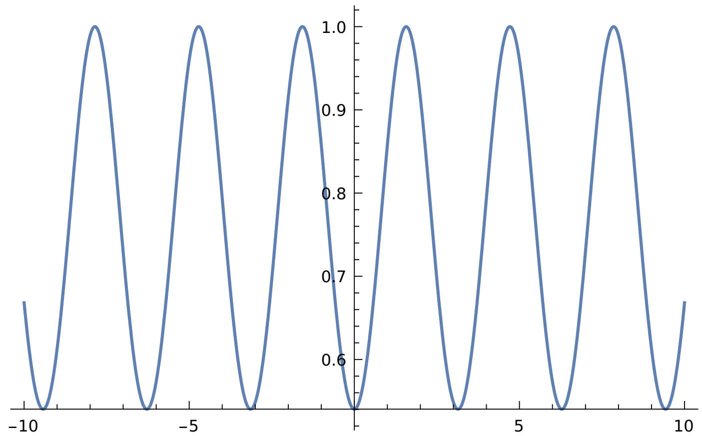
Out[2]=



c)

In[3]:= `Plot[Sin[Pi / 2 + Cos[x]], {x, -10, 10}]`

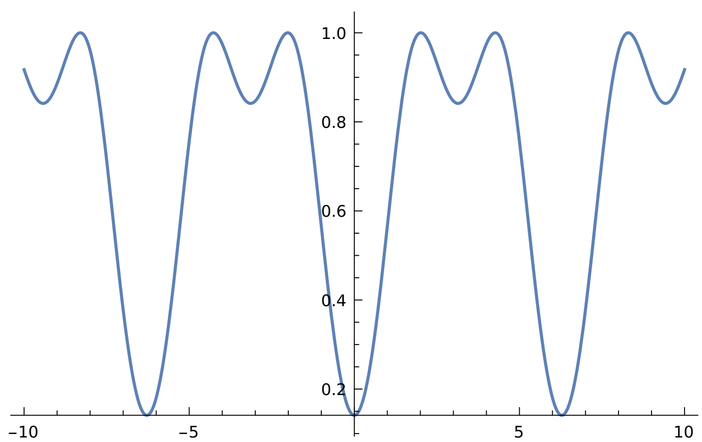
Out[3]=



d)

In[4]:= `Plot[Sin[2 + Cos[x]], {x, -10, 10}]`

Out[4]=



Ques 2. One can zoom in toward a particular point in the domain of a function and see how the graph appears at different zoom levels. For instance, consider the square root function f(x)= $\sqrt{x}$ ***when***

x is near 2.

a. Enter the input below to see the graph of f as x goes from 1 to 3.

With [{δ=10^(0), Plot[$\sqrt{x}$,{x,2-δ,2+δ}]

b. Now zoom; change the value of δ to be 10^(-1)and re-enter the input above to see the graph of f as x goes from 1.9 to 2.1. Do this again for δ=10^-2,10^-3,10^-4 and 10^-5

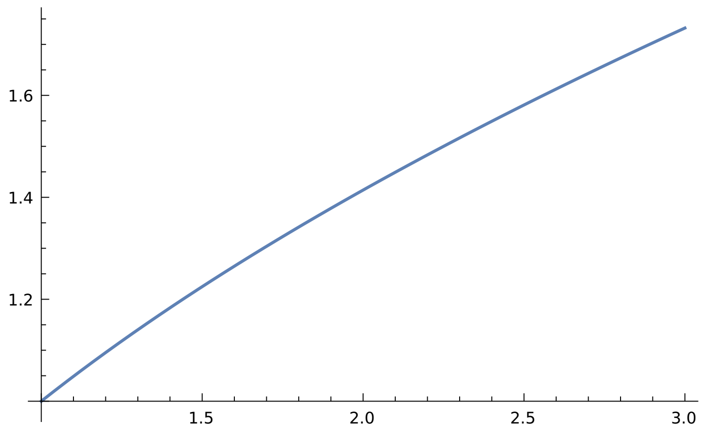
c. Use the last plot to approximate $\sqrt{2}$ to six significant digits. Check your answer using N.

d. When making a Plot, the lower and upper bounds on the iterator must be distinct when rounded to machine precision. Enter the previous Plot command with δ=10^(-20). An error message results. Read the error message and speculate as to what is happening. The bottom line is that zooming has its limits.

a)

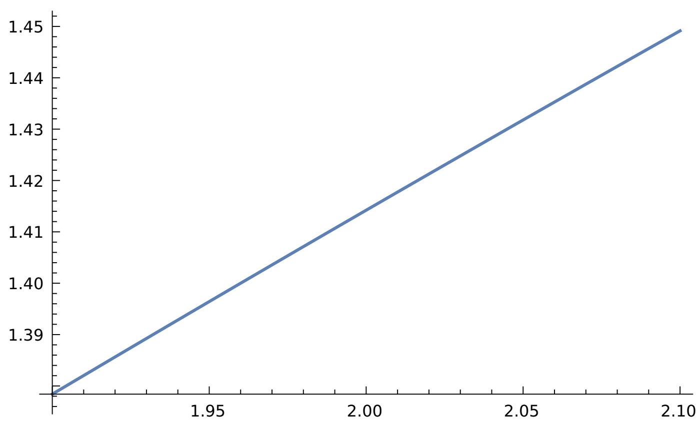
In[5]:=
```
With[{δ = 10 ^ (0)}, Plot[Sqrt[x], {x, 2 - δ, 2 + δ}]]
```

Out[5]=



In[6]:=
```
With[{δ = 10 ^ (- 1)}, Plot[Sqrt[x], {x, 2 - δ, 2 + δ}]]
```

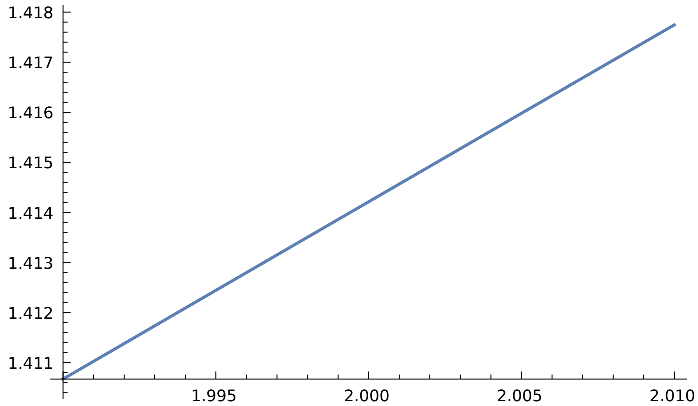
Out[6]=



b)

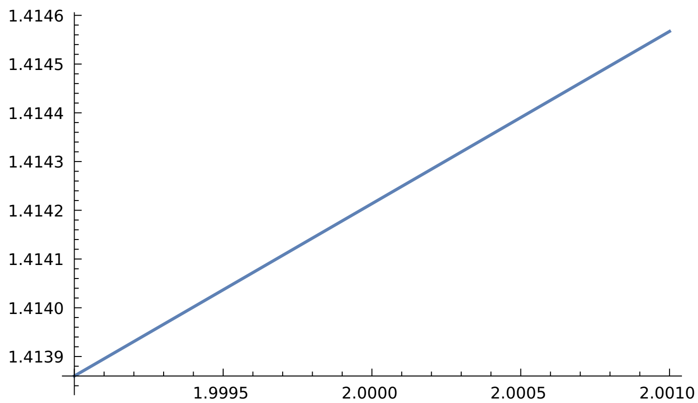
In[7]:=
```
With[{δ = 10^(-2)}, Plot[Sqrt[x], {x, 2 - δ, 2 + δ}]]
```

Out[7]=



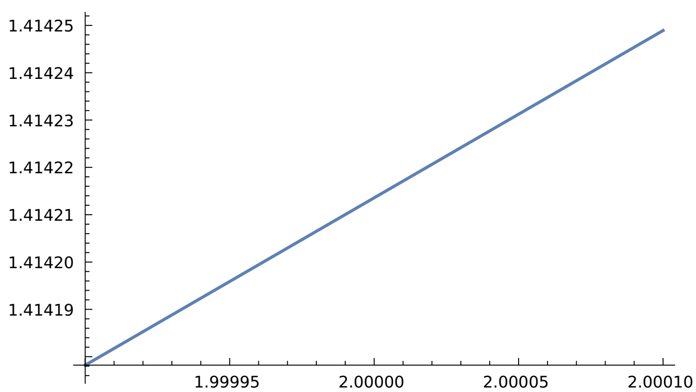
In[8]:=
```
With[{δ = 10^(-3)}, Plot[Sqrt[x], {x, 2 - δ, 2 + δ}]]
```

Out[8]=



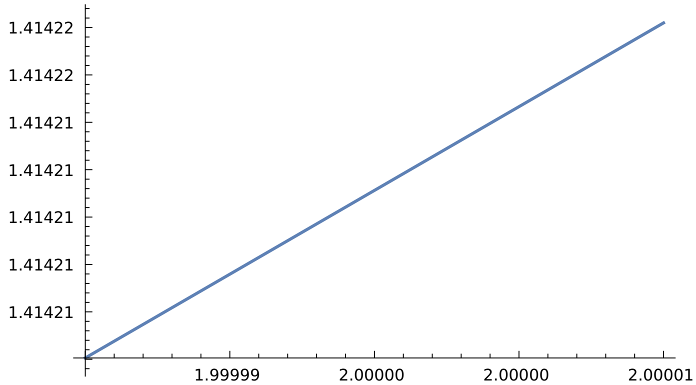
In[9]:=
```
With[{δ = 10^(-4)}, Plot[Sqrt[x], {x, 2 - δ, 2 + δ}]]
```

Out[9]=



In[49]:=
```
With[{δ = 10^(-5)}, Plot[Sqrt[x], {x, 2 - δ, 2 + δ}]]
```

Out[49]=



c)

According to the last plot Sqrt[2] is 1.41421

In[11]:=
```
N[Sqrt[2]]
```

Out[11]= 1.41421

In[50]:=
```
With[{δ = 10^(-20)}, Plot[Sqrt[x], {x, 2 - δ, 2 + δ}]]
```

Plot : Endpoints for x in $\left\{x, \frac{199999999999999999999}{100000000000000000000}, \frac{200000000000000000001}{100000000000000000000}\right\}$ must have distinct machine −precision numerical values .

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General : Further output of Plot::plld will be suppressed during this calculation .

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General : Further output of Plot::plld will be suppressed during this calculation .

Out[50]= $\text{Plot}\left[\sqrt{x}, \left\{x, 2 - \frac{1}{100\,000\,000\,000\,000\,000\,000}, 2 + \frac{1}{100\,000\,000\,000\,000\,000\,000}\right\}\right]$

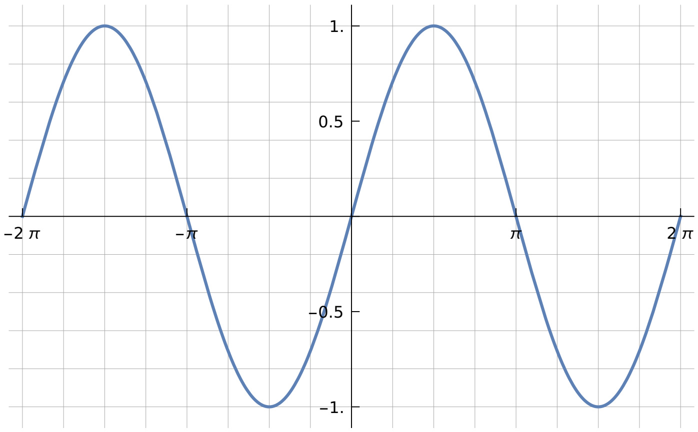
(***Zooming has its limits)***

EXERCISE: 3.3

Ques 1. Use the GridLines and Ticks options, as well as the setting GridLinesStyle→ Lighter[Gray],to produce the following Plot of the sine function

In[13]:=
```
Plot[Sin[x], {x, -2 Pi, 2 Pi}, GridLines → {Range[-2 Pi, 2 Pi, Pi / 4],
Range[-1, 1, 0.2]}, GridLinesStyle → Lighter[Gray],
Ticks → {Range[-2 Pi, 2 Pi, Pi], Range[-1, 1, 0.5]}]
```

Out[13]=



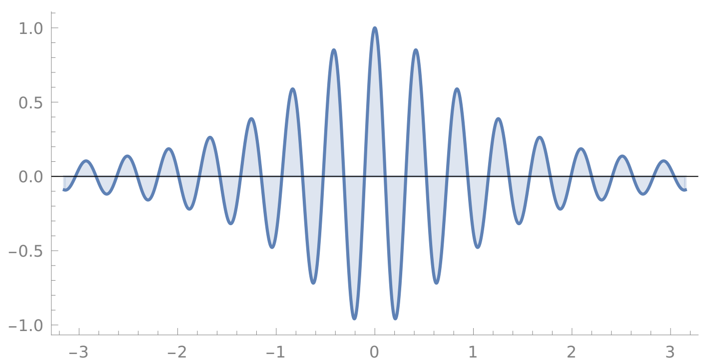
Ques 2. Use the Axes, Frame, Filling, FrameStyle, PlotRange, and AspectRatio options to produce the following plot of the function
y = Cos[15x]/(1+x^2)

In[14]:=
```
y = Cos[15 x] / (1 + x ^ 2)
```

Out[14]= $\frac{\text{Cos}[15\ x]}{1 + x^2}$

In[15]:=
```
Plot[ y, {x, -3.15, 3.15}, PlotRange → Full, Axes → {True, False},
Frame → {True, True, False, False}, FrameStyle → Gray, Filling → Axis,
AspectRatio → 0.5]
```

Out[15]=

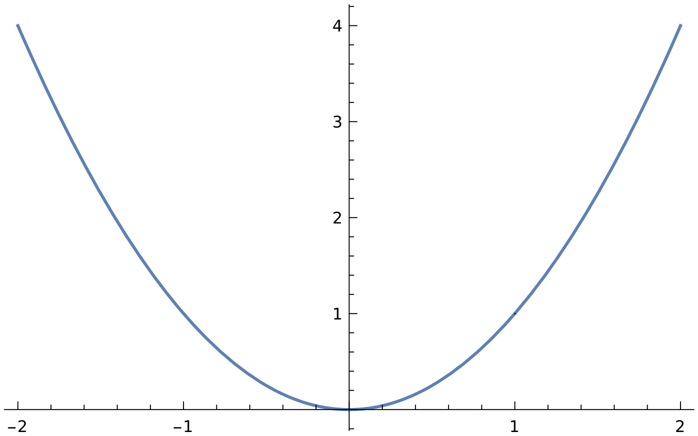


Ques 4. Plot the function f (x)=x^2 on the domain -2≤ x≤2 , and set Exclusions to [x=1]. Note that f has no vertical asymptote at x = 1. What happens?

In[62]:=
```
f[x_] := x ^ 2
```

In[63]:=
```
Plot[f[x], {x, -2, 2}, Exclusions → {x == 1}, ExclusionsStyle → True]
```

Out[63]=



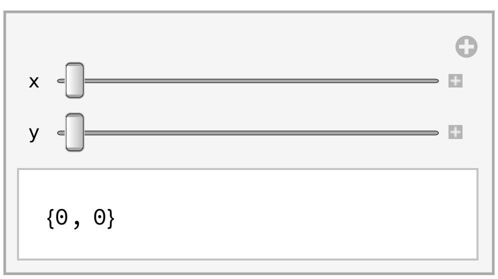
Yes, f has no vertical asymptotes at x=1 because Exclusions has little visible effect at a point unless there is an essential discontinuity there

EXERCISE: 3.4

Ques 1. The following simple Manipulate has two sliders: one for x and one for y. Make a Manipulate that also has output {x,y}, but that has a single Slider2D controller.

In[16]:=
```
Manipulate [{x, y}, {x, 0, 1}, {y, 0, 1}]
```

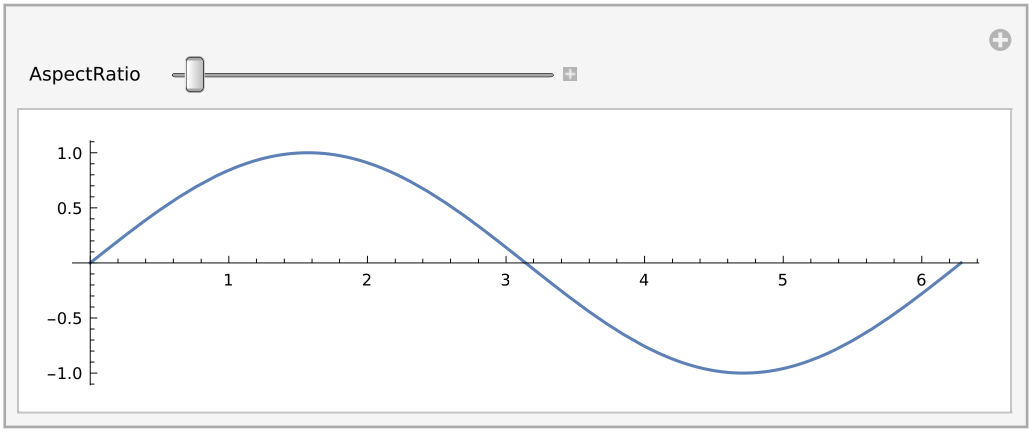
Out[16]=



Ques 2. Make a Manipulate of a Plot where the user can adjust the AspectRatio in real time, from a starting value of 1 / 5 (five times as wide as it is tall) to an ending value of 5 (five times as tall as it is wide). Set ImageSize to {Automatic, 128} so the height remains constant as the slider is moved.

In[57]:=
```
Manipulate[Plot[Sin[x], {x, 0, 2 Pi}, ImageSize → {Automatic, 128}, AspectRatio → a],
 {{a, 1, "AspectRatio"}, 0.2, 5}]
```

Out[57]=



EXERCISE: 3.5

Ques 1. The Partition command is used to break a single list into sublists of equal length. It is useful for

breaking up a list into rows for display within a Grid.

a. Enter the following inputs and discuss the outputs.

Range[100]

Partition[Range[100],10]

b. Format a table of the first 100 integers, with twenty digits per row. The first two rows, for example, should look like this:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40

c. Make the same table as above, but use only the Table and Range commands. Do not use Partition.

d. Make the same table as above, but use only the Table command (twice). Do not use Partition or Range.

a)

In[18]:=
```
Range[100]
```

Out[18]=
```
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22,
 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41,
 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61,
 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81,
 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100}
```

In[19]:=
```
Partition[Range[100], 10]
```

Out[19]=
```
{{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, {11, 12, 13, 14, 15, 16, 17, 18, 19, 20},
 {21, 22, 23, 24, 25, 26, 27, 28, 29, 30}, {31, 32, 33, 34, 35, 36, 37, 38, 39, 40},
 {41, 42, 43, 44, 45, 46, 47, 48, 49, 50}, {51, 52, 53, 54, 55, 56, 57, 58, 59, 60},
 {61, 62, 63, 64, 65, 66, 67, 68, 69, 70}, {71, 72, 73, 74, 75, 76, 77, 78, 79, 80},
 {81, 82, 83, 84, 85, 86, 87, 88, 89, 90}, {91, 92, 93, 94, 95, 96, 97, 98, 99, 100}}
```

Range[100] generates list of all numbers from 1 to 100 and the Partition[Range[100],10] generates sublists of length 10

b)

In[20]:=
```
Grid[Partition[Range[100], 20]]
```

Out[20]=

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

c)

In[21]:=
```
data = Range[x, x + 19]
```

Out[21]=
```
{x, 1 + x, 2 + x, 3 + x, 4 + x, 5 + x, 6 + x, 7 + x, 8 + x, 9 + x,
 10 + x, 11 + x, 12 + x, 13 + x, 14 + x, 15 + x, 16 + x, 17 + x, 18 + x, 19 + x}
```

In[22]:=
```
Grid[Table[data, {x, 1, 100, 20}]]
```

Out[22]=

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

d)

In[23]:=
```
data = Table[x, {x, n, n + 19}]
```

Out[23]=
```
{n, 1 + n, 2 + n, 3 + n, 4 + n, 5 + n, 6 + n, 7 + n, 8 + n, 9 + n,
 10 + n, 11 + n, 12 + n, 13 + n, 14 + n, 15 + n, 16 + n, 17 + n, 18 + n, 19 + n}
```

In[24]:=
```
Grid[Table[data, {n, 1, 100, 20}]]
```

Out[24]=

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

Ques 4. The Sum command has a syntax similar to that of Table.

a. Use the Sum command to evaluate the following expression:

1^3+2^3+3^3+4^3+5^3+6^3+7^3+8^3+9^3+10^3+11^3+12^3+13^3+14^3+15^3+16^3+17^3+18^3+19^3

+20^3

b. Make a table of values for x=1, 2, ... ,10 for the function

f (x)=

1^x+2^x+3^x+4^x+5^x+6^x+7^x+8^x+9^x+10^x+11^x+12^x+13^x+14^x+15^x+16^x+17^x+18^x+19^x +20^x

c. Plot f (x)on the domain 1 ≤x ≤10.

a)

In[25]:=
```
f[x_] := x^3
```

In[26]:=
```
Sum[f[x], {x, 1, 20}]
```

Out[26]= 44 100

b)

In[27]:=
```
f[x_] := Sum[n^x, {n, 1, 20}]
```

In[28]:=
```
f[x]
```

Out[28]= $1 + 2^x + 3^x + 4^x + 5^x + 6^x + 7^x + 8^x + 9^x + 10^x + 11^x + 12^x + 13^x + 14^x + 15^x + 16^x + 17^x + 18^x + 19^x + 20^x$

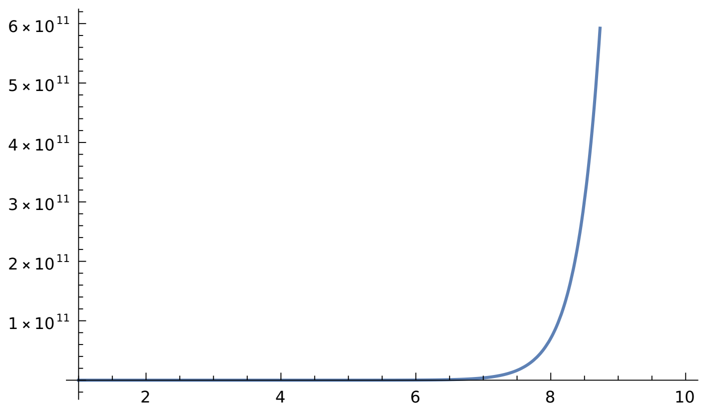
In[29]:=
```
Table[f[x], {x, 1, 10}]
```

Out[29]= {210 , 2870 , 44 100 , 722 666 , 12 333 300 , 216 455 810 , 3 877 286 700 , 70 540 730 666 , 1 299 155 279 940 , 24 163 571 680 850 }

c)

In[30]:=
```
Plot[f[x], {x, 1, 10}]
```

Out[30]=



EXERCISE: 3.6

Ques 2. Make a plot of the piecewise function below, and comment on its shape.

In[31]:=
```
f[x_] := Piecewise[{{0, x < 0}, {(x^2)/2, 0 ≤ x < 1},
    {-x^2 + 3 x - 3/2, 1 ≤ x < 2}, {1/2 (3 - x)^2, 2 ≤ x < 3}, {0, x ≥ 3}}]
```

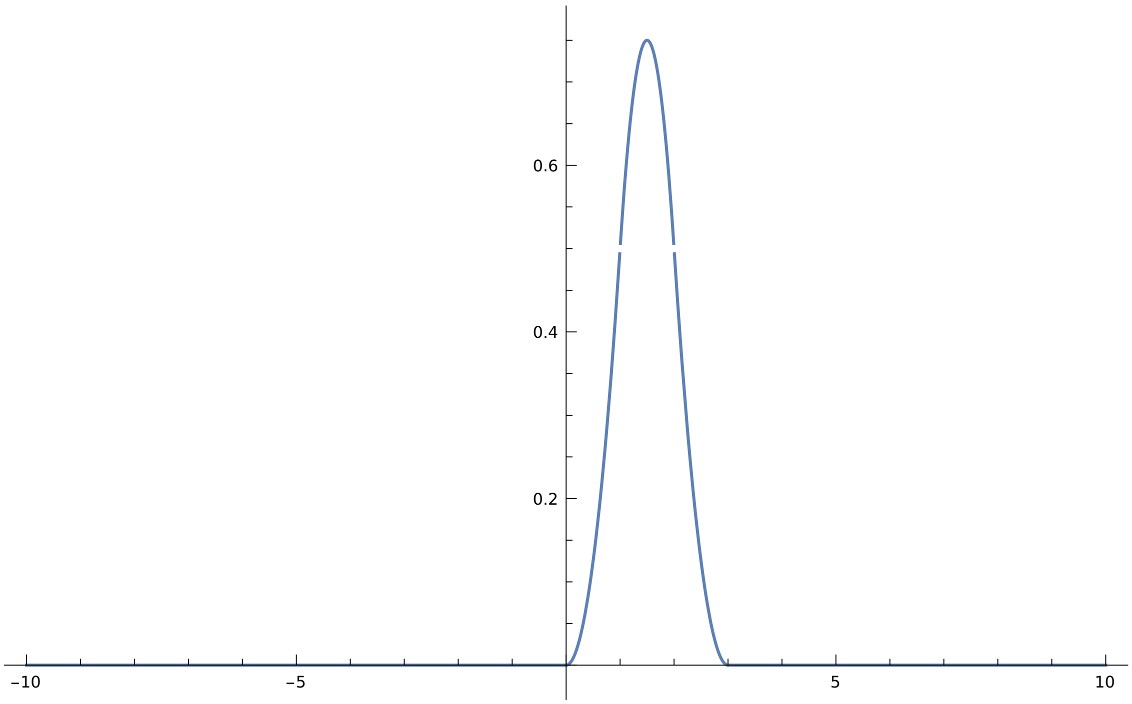
In[32]:=
```
f[x]
```

Out[32]=

$$f(x)=\begin{cases} 0 & x < 0 \\ \frac{x^2}{2} & 0 \le x < 1 \\ -\frac{3}{2} + 3x - x^2 & 1 \le x < 2 \\ \frac{1}{2}(3-x)^2 & 2 \le x < 3 \\ 0 & \text{True} \end{cases}$$

In[33]:=
```
Plot[f[x], {x, -10, 10}]
```

Out[33]=



The piecewise function f[x] is not continuous on x=1 and x=2

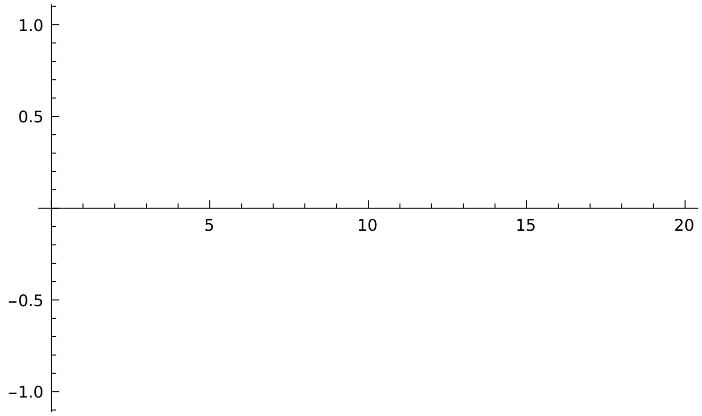
Ques 3. A step function assumes a constant value between consecutive integers n and n+ 1. Make a plot of
the step function f(x) whose value is n^2 when n≤ x < n+ 1. Use the domain 0≤ x < 20.

Tried several times(some mistake)

In[87]:=
```
f[x_] := Piecewise[{{n^2, n ≤ x < n + 1}, {1, n ≤ x ≤ n + 1}}]
```

In[88]:= `Plot[f[x], {x, 0, 20}]`

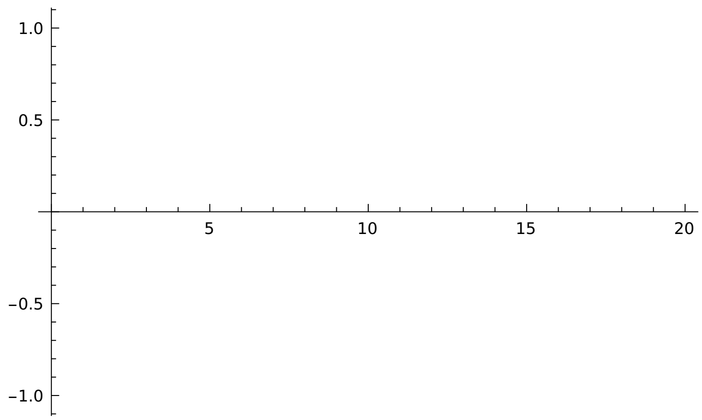
Out[88]=



In[89]:= `f[x_] := Piecewise[{{Floor[n^2], n ≤ x < n + 1}}]`

In[90]:= `Plot[f[x], {x, 0, 20}]`

Out[90]=



In[91]:= `f[x_] := Piecewise[{{Floor[n^2], n ≤ x < n + 1}, {1, n ≤ x ≤ n + 1}}]`

In[92]:= `Plot[f[x], {x, 0, 20}]`

Out[92]=