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DERIVATIVES

What is a Derivative?

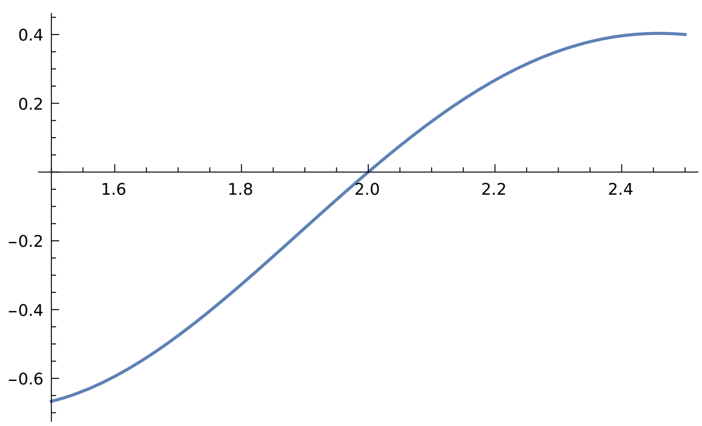
- The derivative of a function of a real variable measures the sensitivity to change of the function value .
- The derivative of a function of a single variable at a chosen input value, when it exists, is the slope of the tangent line to the graph of the function at that point. The tangent line is the best linear approximation of the function near that input value. For this reason, the derivative is often described as the "instantaneous rate of change", the ratio of the instantaneous change in the dependent variable to that of the independent variable.
- The process of finding a derivative is called differentiation.

Let us define a function f(x) .

In[1]:=
```
f[x_] := (Sin[π x]) / x
```

In[2]:=
```
Plot[f[x], {x, 1.5, 2.5}]
```

Out[2]=



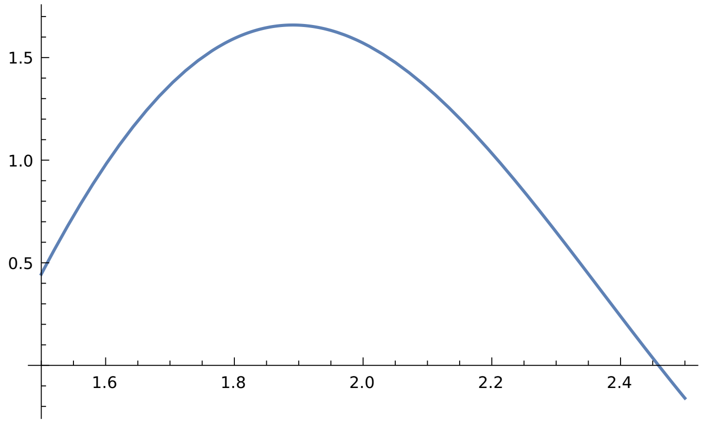
Derivative of the function f, is also the slope of the function f .

In[3]:=
```
f '[x]
```

Out[3]=

$$\frac{\pi \, \mathrm{Cos}[\pi\, x]}{x} - \frac{\mathrm{Sin}[\pi\, x]}{x^2}$$

In[4]:=
```
Plot[f '[x], {x, 1.5, 2.5}]
```

Out[4]=


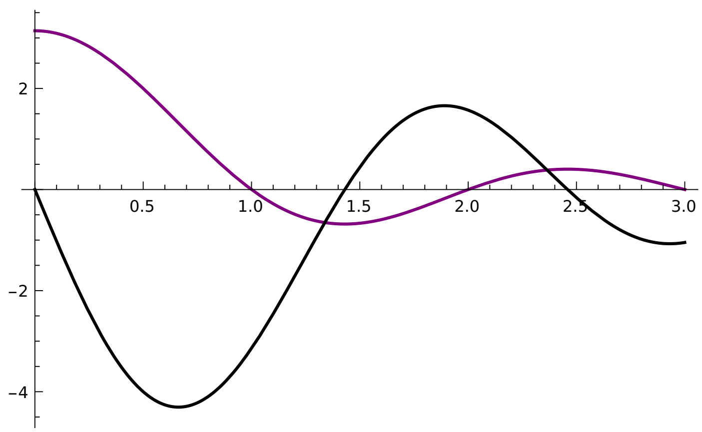
We can calculate the derivative at any point.

At x=2

In[5]:=
```
f '[2]
```

Out[5]= $\frac{\pi}{2}$

In[6]:=
```
Plot[{f[x], f '[x]}, {x, 0, 3}, PlotStyle → {Purple, Black}]
```

Out[6]=


Another way to calculate the derivatives.

In[7]:=
```
D[f[x], x]
```

Out[7]= $\frac{\pi\,\mathrm{Cos}[\pi\,x]}{x} - \frac{\mathrm{Sin}[\pi\,x]}{x^2}$

Higher Order Derivatives

In[8]:=
```
f[x_] := (Sin[π x]) / x
```

Second order derivative.

In[9]:=
```
f''[x]
```

Out[9]=
$$-\frac{2\,\pi\,\mathrm{Cos}[\pi\,x]}{x^2}+\frac{2\,\mathrm{Sin}[\pi\,x]}{x^3}-\frac{\pi^2\,\mathrm{Sin}[\pi\,x]}{x}$$

Third order derivative

In[10]:=
```
f'''[x]
```

Out[10]=
$$\frac{6\,\pi\,\mathrm{Cos}[\pi\,x]}{x^3}-\frac{\pi^3\,\mathrm{Cos}[\pi\,x]}{x}-\frac{6\,\mathrm{Sin}[\pi\,x]}{x^4}+\frac{3\,\pi^2\,\mathrm{Sin}[\pi\,x]}{x^2}$$

Another way of calculating third order derivative

In[11]:=
```
D[f[x], {x, 3}]
```

Out[11]=
$$\frac{6\,\pi\,\mathrm{Cos}[\pi\,x]}{x^3}-\frac{\pi^3\,\mathrm{Cos}[\pi\,x]}{x}-\frac{6\,\mathrm{Sin}[\pi\,x]}{x^4}+\frac{3\,\pi^2\,\mathrm{Sin}[\pi\,x]}{x^2}$$

In[12]:=
```
ClearAll[f]
```

DIFFERENCIATION OF THE PRODUCT OF FUNCTIONS

The derivative of the product of two functions is the derivative of the first one multiplied by the second one plus the first one multiplied by the derivative of the second one.

Second derivative product rule

In[8]:=
```
D[f[x] * g[x], {x, 2}]
```

Out[8]= $2\,f'[x]\,g'[x]+g[x]\,f''[x]+f[x]\,g''[x]$

Third derivative product rule

In[9]:=
```
D[f[x] * g[x], {x, 3}]
```

Out[9]= $3\,g'[x]\,f''[x]+3\,f'[x]\,g''[x]+g[x]\,f^{(3)}[x]+f[x]\,g^{(3)}[x]$

MAXIMA AND MINIMA

- A function can only attain its maximum and minimum value at its critical value, where the graph has horizontal tangents,or where no tangent line exists.
- For a diiferentiable function there is no unique tangent line at each point in the domain , so the critical points are all of the first type.

In[13]:=
```
f[x_] := x^3 - 9 x + 5
```

In[14]:=
```
f'[x]
```

Out[14]= $-9 + 3 x^2$

In[15]:=
```
Reduce[f'[x] == 0, x]
```

Out[15]= $x == -\sqrt{3} \,||\, x == \sqrt{3}$

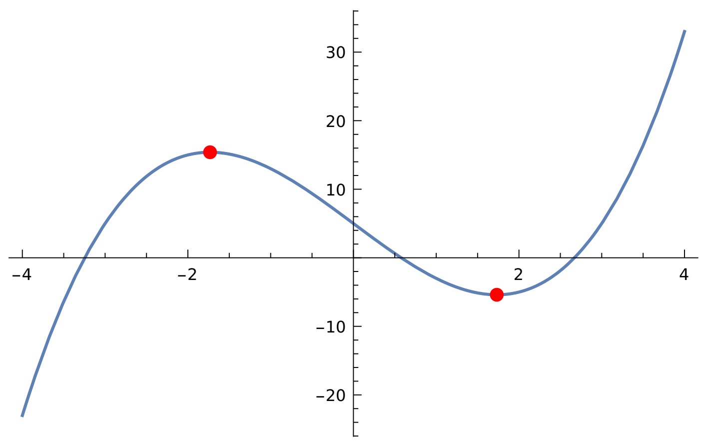
In[16]:=
```
Solve[f'[x] == 0, x]
```

Out[16]= $\{\{x \to -\sqrt{3}\}, \{x \to \sqrt{3}\}\}$

In[17]:=
```
extrema = {x, f[x]} /. %
```

Out[17]= $\{\{-\sqrt{3}, 5 + 6\sqrt{3}\}, \{\sqrt{3}, 5 - 6\sqrt{3}\}\}$

In[18]:=
```
Plot[f[x], {x, -4, 4}, Epilog → {PointSize[0.02], Red, Point[extrema]}]
```

Out[18]=


By using the second derivative we can check the extreme points are minimum/maximum points.

In[19]:=
```
f''[-√3] < 0
```

Out[19]= True

In[20]:=
```
f''[√3] > 0
```

Out[20]= True

So the function f has a **maximum at x= -$\sqrt{3}$** and a **minimum at x= $\sqrt{3}$.**

Using MAXIMIZE and MINIMIZE

Maximize and Minimize return absolute maximas and minimas not local maximas and mimnimas.

```
f[x_] := x^3 - 9 x + 5

In[23]:= Maximize[f[x], x]
Maximize : The maximum is not attained at any point satisfying the given constraints .
Out[23]= {∞, {x → ∞}}
```

Since this returns an indeterminate value we restrain the domain rather than calculating on the full domain.

```
In[24]:= Maximize[{f[x], -4 ≤ x ≤ 0}, x]
```

Out[24]= $\{5 + 6\sqrt{3}, \{x \to -\sqrt{3}\}\}$

The above output is a list of two.
The first gives the maximum value of the function , while the other indicates the point in the domain where the maximum occurs.

```
In[25]:= Minimize[{f[x], -4 ≤ x ≤ 0}, x]
Out[25]= {-23, {x → -4}}
```

INFLECTION POINTS

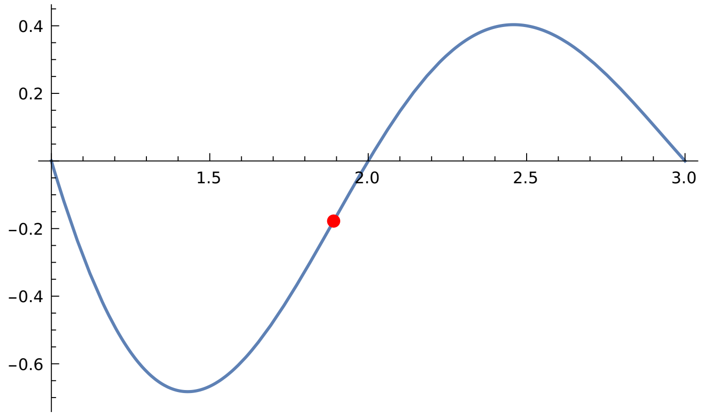
```
In[26]:= g[x_] := (Sin[π x]) / x

In[27]:= FindRoot[g''[x] == 0, {x, 2}]
Out[27]= {x → 1.89088}

In[28]:= infpt = {x, g[x]} /. %
Out[28]= {1.89088, -0.177769}
```

In[29]:=
```
Plot[g[x], {x, 1, 3}, Epilog → {PointSize[0.02], Red, Point[infpt]}]
```

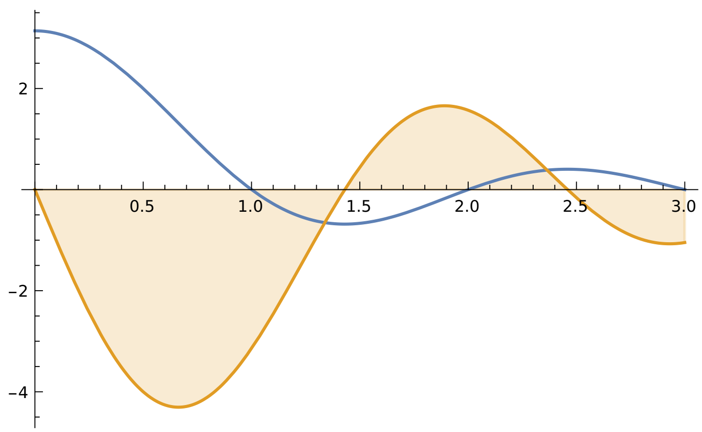
Out[29]=



This plot confirms that f has an inflection point at approximately x=1.89088.

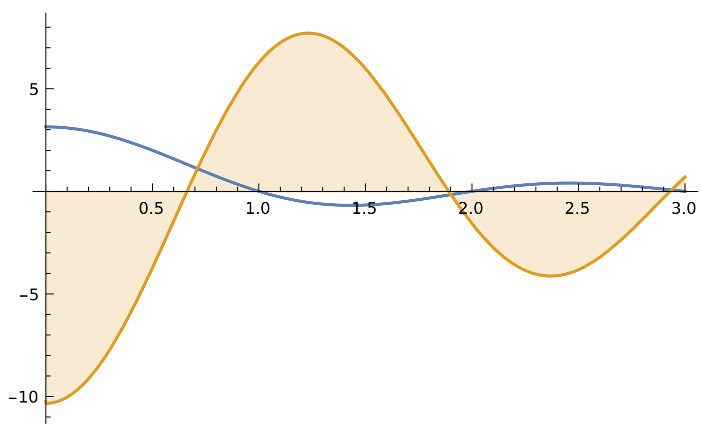
In[30]:=
```
Plot[{g[x], g'[x]}, {x, 0, 3}, Filling → {2 → Axis}]
```

Out[30]=



In[31]:=
```
Plot[{g[x], g''[x]}, {x, 0, 3}, Filling → {2 → Axis}]
```

Out[31]=



The zeroes of g'' correspond to the inflection points of g.