Multivariable Calculus (Vectors)

- MAT/19/99
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- DEFINITION

A vector is a measurement or a quantity that has both magnitude and direction.

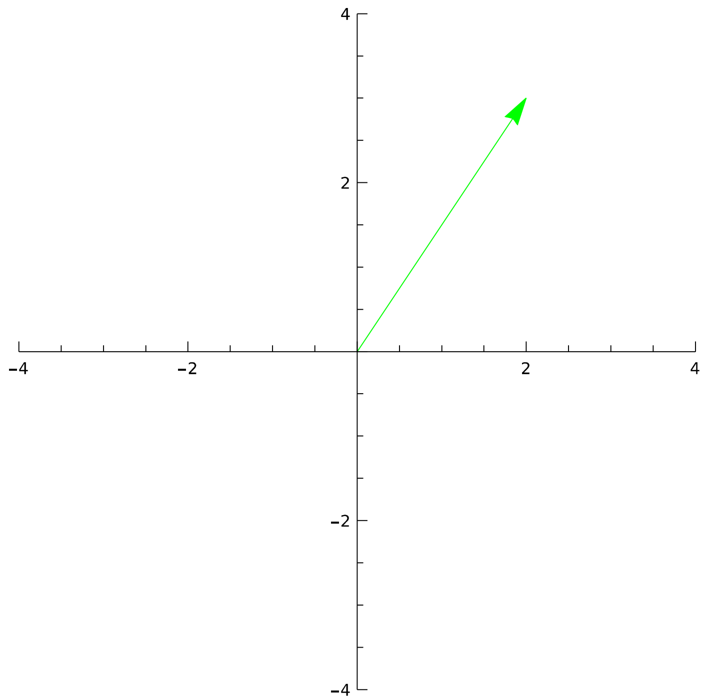
Standard notation for vectors in Mathematica is {a,b} , which represents that the vector has its tail at the origin and head at the point with x coordinate a and y coordinate b. Another standard notation for this vector is ai+bj, where i and j denote the unit vectors in the x and y directions respectively.

In[16]:=
```
a = {2, 3};
```

In[17]:=
```
b = {4, 2};
```

In[42]:=
```
a = Graphics[{Green, Arrow[{{0, 0}, {2, 3}}]}, PlotRange → 4, Axes → True]
```

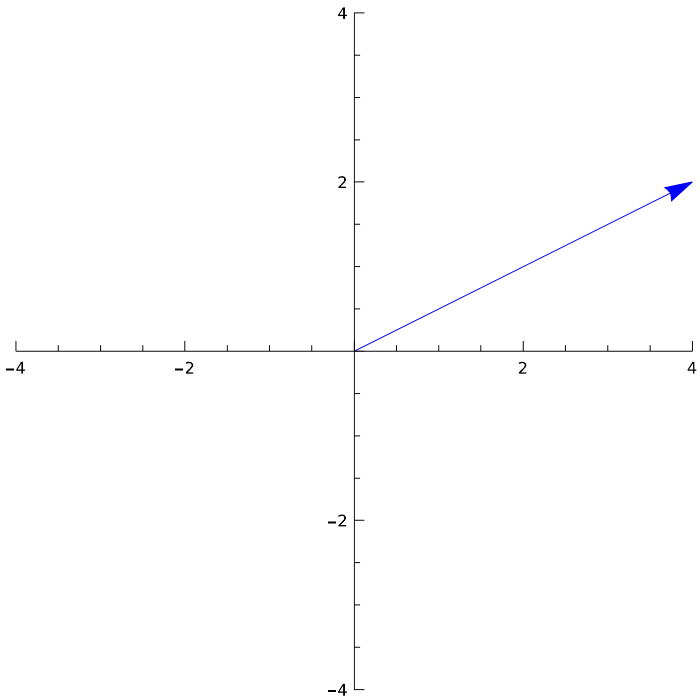
Out[42]=



In[44]:=

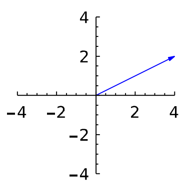
```
b = Graphics[{Blue, Arrow[{{0, 0}, {4, 2}}]}, PlotRange → 4, Axes → True]
```

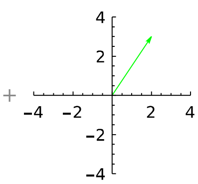
Out[44]=



In[45]:=

```
a + b = {2, 3} + {4, 2}
```

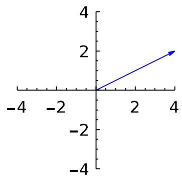
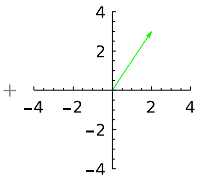
Set : Tag Plus in 

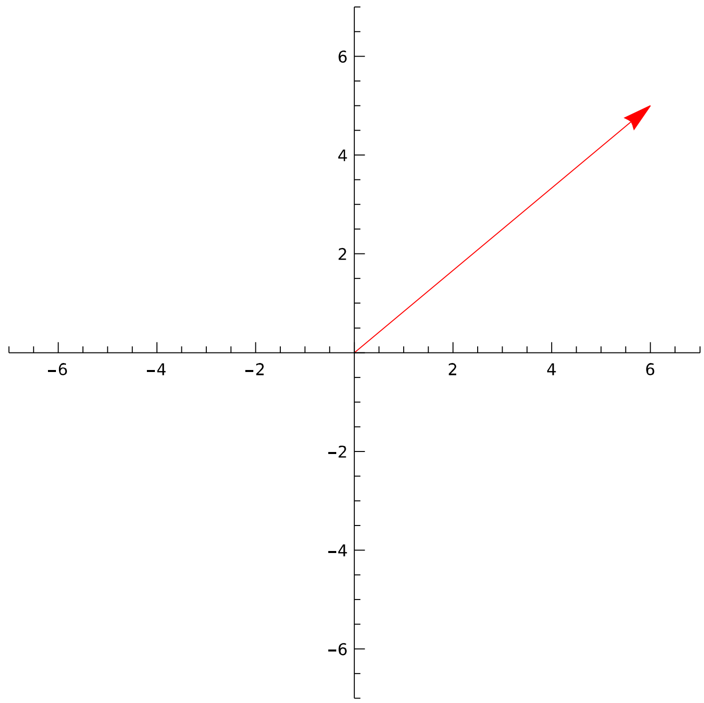
\+ 

is Protected .

Out[45]= {6, 5}

In[47]:=
```
a + b = Graphics[{Red, Arrow[{{0, 0}, {6, 5}}]}, PlotRange → 7, Axes → True]
```

Set : Tag Plus in  +  is Protected .

Out[47]=


In[20]:=
```
2 a + 3 b
```

Out[20]= {16, 12}

DOT PRODUCT

The dot product of the vectors u1, u2, …, un and v1, v2, …, vn is the scalar u1 v1 +u2v2+……….+un vn. in Mathematica it s computed by placing a dot between vectors

In[31]:=
```
a = {2, 3};
```

In[32]:=
```
b = {4, 2};
```

In[34]:=
```
a.b
```

Out[34]= 14

In[4]:=
```
{a1, a2}.{b1, b2}
```

Out[4]= a1 b1 + a2 b2

```
In[8]:= {2, 8}.{4, 6}
Out[8]= 56
```

NORM (magnitude)

We can compute magnitude of the vector by norm command.

```
In[9]:= Norm[{a1, a2}]
```

Out[9]= $\sqrt{\text{Abs[a1]}^2 + \text{Abs[a2]}^2}$

```
In[10]:= Norm[{3, 8}]
```

Out[10]= $\sqrt{73}$

For real vectors, this is equivalent to the square root of the dot product of the vector with itself.

```
In[11]:= Simplify[Norm[{a1, a2}], {a1, a2} ∈ Reals]
```

Out[11]= $\sqrt{a1^2 + a2^2}$

```
In[13]:= Sqrt[{a1, a2}.{a1, a2}]
```

Out[13]= $\sqrt{a1^2 + a2^2}$

ANGLE BETWEEN VECTORS

The dot product can also be employed to find the angle between a pair of vectors. we can find the angle (in radians) between vectors like this:

```
In[14]:= c = {5, 4}; d = {5, 2};
In[51]:= ArcCos[(c.d) / (Norm[c] × Norm[d])]
```

Out[51]= $\text{ArcCos}\left[\frac{c.d}{\text{Norm[c]} \times \text{Norm[d]}}\right]$

for numeric value

```
In[16]:= ArcCos[(c.d) / (Norm[c] × Norm[d])] // N
Out[16]= 0.294235
```

for degree value of angle between vectors

In[17]:=
```
% Degree
```
Out[17]=
```
0.00513536
```

CROSS PRODUCT

The cross product of u and v is a normal vector to the plane determined by u and v, whose magnitude is equal to the area of the parallelogram determined by u and v.
in Mathematica we use cross command to take cross product of vectors.

In[22]:=
```
Cross[{u1, u2, u3}, {v1, v2, v3}]
```
Out[22]=
```
{-u3 v2 + u2 v3, u3 v1 - u1 v3, -u2 v1 + u1 v2}
```

In[63]:=
```
g = {2, 3, 4};
```
In[64]:=
```
h = {4, 2, 5};
```
In[65]:=
```
Cross[{2, 3, 4}, {4, 2, 5}]
```
Out[65]=
```
{7, 6, -8}
```

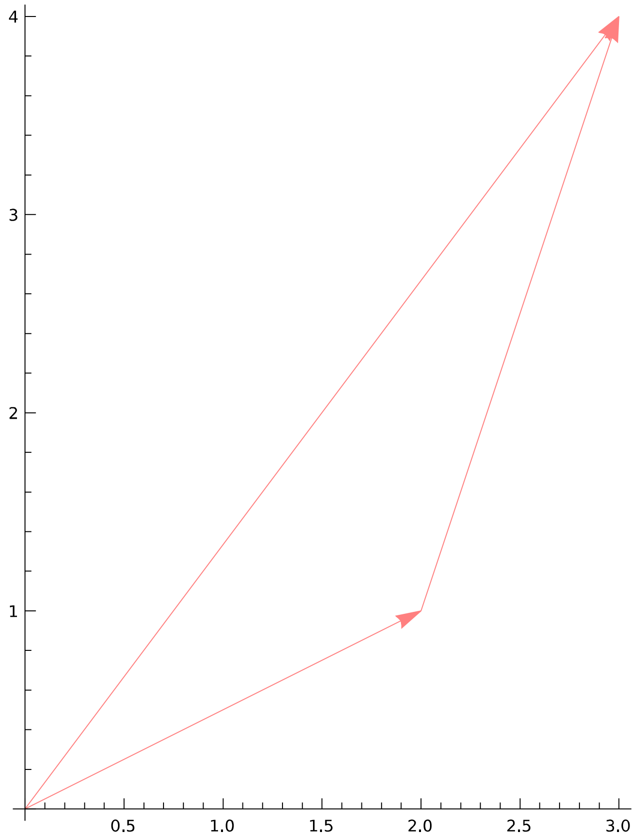
VECTOR ADDITION

we have two rules to find sum of vectors that are triangle rule and parallelogram rule.

1. Triangle rule

In[69]:=
```
Graphics[
 {Pink, Arrow[{{0, 0}, {2, 1}}], Arrow[{{0, 0}, {3, 4}}], Arrow[{{2, 1}, {3, 4}}]}, Axes → True]
```

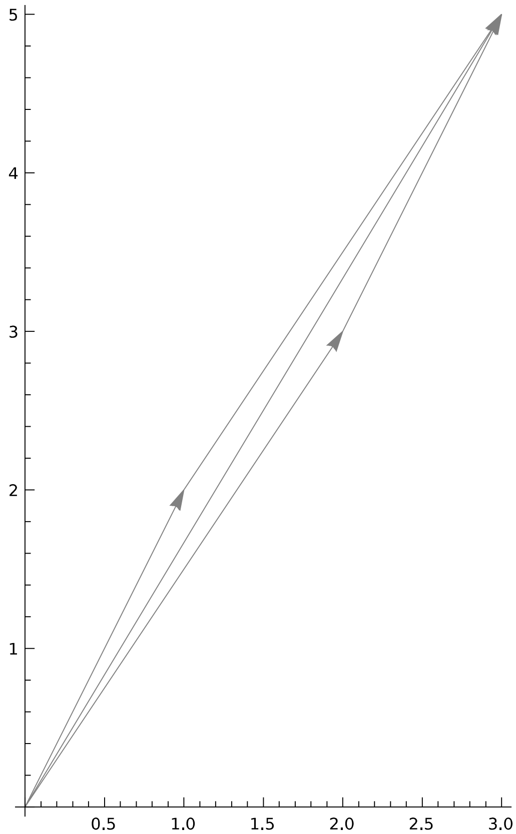
Out[69]=



2. Parallelogram rule

In[3]:=
```
Graphics [{Gray, Arrow[{{0, 0}, {1, 2}}], Arrow[{{0, 0}, {2, 3}}],
Arrow[{{0, 0}, {3, 5}}], Arrow[{{1, 2}, {3, 5}}], Arrow[{{2, 3}, {3, 5}}]}, Axes → True]
```

Out[3]=



SOME MORE COMMANDS

find the projection of one vector on another.

In[2]:=
```
Projection [{5, 6}, {1, 0}]
```

Out[2]= {5, 0}

Number of elements in a vector.

In[4]:=
```
Length[{2, 3, 4}]
```

Out[4]= 3

To find whether an expression is a vector or not.

In[9]:=
```
VectorQ[{2, 3}]
```

Out[9]= True

```
In[10]:= VectorQ[{2 i + 3 j}]
Out[10]= True

In[13]:= VectorQ[x ^ 2]
Out[13]= False

In[14]:= VectorQ[x ^ 3 + 2 x ^ 2]
Out[14]= False
```

Total of elements in a vector.

```
In[15]:= Total[{2, 3}]
Out[15]= 5
```

Normalize a vector to unit length.

```
In[16]:= Normalize[{2, 3}]
```

Out[16]= $\left\{\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}\right\}$

To find angle between vectors.

```
In[24]:= VectorAngle[{2, 3}, {3, 2}]
```

Out[24]= $\text{ArcCos}\left[\frac{12}{13}\right]$

```
In[25]:= VectorAngle[{2, 3}, {3, 2}] // N
Out[25]= 0.394791
```

UnitVector — unit vector along a coordinate direction

```
In[27]:= UnitVector[2, 2]
Out[27]= {0, 1}

In[30]:= UnitVector[3, 3]
Out[30]= {0, 0, 1}
```

Constant array command generates a list of n copies of the element c.

```
In[1]:= a = ConstantArray[3, 5]
Out[1]= {3, 3, 3, 3, 3}
```

```
In[2]:= b = ConstantArray [4, {3, 5}]
Out[2]= {{4, 4, 4, 4, 4}, {4, 4, 4, 4, 4}, {4, 4, 4, 4, 4}}
```

Angle vector gives the list representing the 2D unit vector at angle θ relative to the axis.

```
In[3]:= AngleVector [Pi / 3]
```

Out[3]= $\left\{\frac{1}{2}, \frac{\sqrt{3}}{2}\right\}$

```
In[4]:= AngleVector [Theta]
Out[4]= {Cos[Theta], Sin[Theta]}
```

Display in row form

```
In[9]:= Row[{1, 2, 5}]
Out[9]= 125
```

Display in column form

```
In[10]:= Column[{6, 8, 5}]
         6
Out[10]= 8
         5
```

Extract an element of a vector

```
In[11]:= {a, b, c, d, e, f}[[3]]
Out[11]= c
```

```
In[12]:= {a, b, c, d, e, f}[[-2]]
Out[12]= e
```

Part works with expressions of any kind, not just lists

```
In[13]:= (1 + 2 x ^ 2 + y ^ 2)[[2]]
```

Out[13]= $2\,x^2$