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Donald book eg 9.5

Part1

Let $\mathrm{x} = (x_1, \dots, x_n)$, where the x_i are nonnegative real numbers. Set

$$M_r(\mathrm{x}) = \left(\frac{x_1^r + x_2^r + \cdot + x_n^r}{n}\right)^{1/r}, \quad r \in \mathrm{R} \setminus \{0\},$$

and

$$M_0(\mathrm{x}) = (x_1 x_2 \dots x_n)^{1/n}.$$

We call $M_r(\mathrm{x})$ the *rth power mean* of x.
Claim:

$$\lim_{r \to 0} M_r(\mathrm{x}) = M_0(\mathrm{x}),$$

Donald book eg 9.5

Part2

Define

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 0 & \dots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_n^{n-1} \end{bmatrix}.$$

We call V_n the *Vandermonde matrix* of order n. Claim:

$$\det V_n = \prod_{\leq i < j \leq n} (x_{j\,i}).$$

Question 4

- $$3^3 + 4^3 + 5^3 = 6^3$$
- $$\sqrt{100} = 10$$
- $$(a+b)^3 = a^3 + 3a^2 + 3ab^2 + b^3$$
- $$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

Question 4 [part 2]

- $$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots$$

- $$\cos\theta = \sin(90 - \theta)$$

- $$e^{i0} = \cos\theta + i\sin\theta$$

Question 4 [part 3]

- $$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$$
- $$\lim_{x \to \infty} \frac{\pi(x)}{x/\log x}$$
- $$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

Question 5[Part 1]

- Positive numbers a,b,c are the side lengths of a triangle if and only if $a + b > c$, $b + c > a$ and $c + a > b$.
- The area of triangle with side length a,b,c is given by HERON's FORMULA:
  A=$\sqrt{s(s-a)(s-b)(s-c)}$
  where s is semiperimeter$(a+b+c)/2$. .

Question 5[Part 2]

- The volume of regular tetrahedon of edge length 1 is$\sqrt{2}/12$.
- The quadritic equation $ax^2 + bx + c = 0$ has root

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The derivative of a function f , denoted f',is defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Question 5[Part 3]

- A real valued function f is convex on an interval I if

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda)f(y),$$

for all $x, y \in I$ and $=0 \leq \lambda \leq 1$.

- The general solution to the differential equation

y''-3y'+2y=0

is

$$y = c_1 e^x + c_2 e^{2x}$$

- The Fermat number F_n is defined lowing equations as $F_n = 2^{2^n}, n \geq 0$.

Question 6[Part1]

- $\frac{d}{dx}(\frac{x}{x+1}) = \frac{1}{x+1^2}$
- $\lim_{n\to\infty}(1 + \frac{1}{n}^n)$
- $\begin{vmatrix} a & d \\ c & d \end{vmatrix}$ = ad-bc
- $R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Question 6[Part2]

- $$\begin{bmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$$

$$= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \boldsymbol{i}\text{-} \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \boldsymbol{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \boldsymbol{k}$$

- $$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{120} + a_{22}b_{22} \end{bmatrix}$$

- $$f(x) = \begin{cases} -x^2, & x0 \\ x^2, & 0 \leq \\ 1, & x > 2 \end{cases}$$

Question-7 [part 1]

$$\begin{aligned}
1+2 &= 3 \\
4+5+6 &= 7+8 \\
9+10+11+12 &= 13+14+15 \\
16+17+18+19+19+20 &= 13+14+15 \\
25+26+27+28+29+30 &= 31+32+33+34+35
\end{aligned}$$

question 7[Part2]

$$
\begin{aligned}
(a+b)^2 &= (a+b)(a+b) \\
&= (a+b)a+(a+b)b \\
&= a(a+b)b(a+b) \\
&= a^2+ab+ba+b^2 \\
&= a^2+ab+ab+b^2 \\
&= a^2+2ab+b^2
\end{aligned}
$$

$$\tan(\alpha+\beta+\gamma) = \frac{\tan(\alpha+\beta)+\tan\gamma}{1-\tan\alpha+\beta)+\tan\gamma}$$

$$= \frac{\frac{tan\alpha+tan\beta}{1-tan\alpha tan\beta}+\tan\gamma}{1-(\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta})\tan\gamma}$$

$$= \frac{\tan\alpha+\tan\beta+(1-\tan\alpha\tan\beta)\tan\gamma}{1-\tan\alpha\tan\beta-(\tan\alpha+\tan\beta)\tan\gamma}$$

$$= \frac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\beta-\tan\alpha\tan\gamma-\tan\beta\tan\gamma}$$

Question 7

$$\prod_p \left(1 - \frac{1}{p^2}\right) = \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots}$$

$$= \left(\prod_p (1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots)\right)^{-1}$$

$$= \left(1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots\right)^{-1}$$

$$= \frac{6}{\pi^2}$$


Thank
You