Assignment - 2

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- Let x $= (x_1, \ldots, x_n)$,where the x_i are non negative real numbers.Set

$$M_r(\mathrm{x}) = \left(\frac{x_1^r + x_2^r + \cdots + x_n^r}{n} \right)^{1/r}, \quad r \in \mathrm{R} \setminus \{0\},$$

and

$$M_0(\mathrm{x}) = (x_1 x_2 \ldots x_n)^{1/n}.$$

We call $M_r(\mathrm{x})$ the *rth power mean* of x.
Claim:

$$\lim_{r \to 0} M_r(\mathrm{x}) = M_0(\mathrm{x}).$$

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- Define

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ x_1 & x_2 & x_3 & \cdots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \cdots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \cdots & x_n^{n-1} \end{bmatrix}$$

We call V_n the *Vandermonde matrix* of order n.
Claim:

$$\det V_n = \prod_{1 \le i < j \le n} (x_j - x_i).$$

Question No.4

1. $$3^3 + 4^3 + 5^3 = 6^3$$

2. $$\sqrt{100} = 10$$

3. $$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

4. $$\sum_{k=1}^{n} k = \frac{n(n+2)}{2}$$

5. $$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots$$

Remaining parts of Question No. 4

6

$$\cos\theta = \sin(90^\circ - \theta)$$

7

$$e^{i\theta} = \cos\theta + i\sin\theta$$

8

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$

9

$$\lim_{x\to\infty}\frac{\pi(x)}{\frac{x}{logx}} = 1$$

10

$$\int_{-\infty}^{\infty} e^{-x^2}dx = \sqrt{\pi}$$

Question No.5

- Positive numbers a , b and c are the side lengths of a triangle if and only if $a + b > c$, $b + c > a$, and $c + a > b$.
- The area of a triangle with side lengths a ,b and c is given by *Heron's formula*:

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

  where s is the semiperimeter $(a + b + c)/2$.
- The volume of a regular tetrahedron of edge length 1 is $\sqrt{2}/12$.

Remaining parts of Question No. 5

- The quadratic equation $ax^2 + bx + c = 0$ has roots

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

- The *derivative* of a function f , denoted f' , is defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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Remaining parts of Question No.5

- A real - valued function f is *convex* on an interval I if

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda) f(y),$$

for all $x, y \in I$ and $0 \leq \lambda \leq 1$

- The general solution to the differential eqaution

$$y" - 3y' + 2y = 0$$

is

$$y = C_1 e^x + C_2 e^{2x}$$

- The *Fermat number* F_n is defined as

$$F_n = 2^{2^n}, n \leq 0.$$

Question No.6

- $$\frac{d}{dx}\left(\frac{x}{x+1}\right) = \frac{1}{(x+1)^2}$$

- $$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$

- $$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

- $$\mathcal{R}_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Remaining parts of Question No.6

- $$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

- $$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Remaining parts of Question No.6

- $$f(x) = \begin{cases} -x^2 & , x > 0 \\ x^2 & , 0 \leq x \leq 2 \\ 4 & , x > 2 \end{cases}$$

Multi-Line Equations

(i) part of Question No.7

$$\begin{aligned} 1+2 &= 3 \\ 4+5+6 &= 7+8 \\ 9+10+11+12 &= 13+14+15 \\ 16+17+18+19+20 &= 21+22+23+24 \\ 25+26+27+28+29+30 &= 31+32+33+34+35 \end{aligned}$$

Multi-Line Equations

(ii) part of Question No.7

$$
\begin{aligned}
(a+b)^2 &= (a+b)(a+b) \\
&= (a+b)a + (a+b)b \\
&= a(a+b) + b(a+b) \\
&= a^2 + ab + ba + b^2 \\
&= a^2 + ab + ab + b^2 \\
&= a^2 + 2ab + b^2
\end{aligned}
$$

Multi-Line Equations

(iii) part of Question No.7

$$\begin{aligned}
\tan(\alpha+\beta+\gamma) &= \frac{\tan(\alpha+\beta)+\tan\gamma}{1-\tan(\alpha+\beta)\tan\gamma} \\
&= \frac{\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}+\tan\gamma}{1-\left(\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}\right)\tan\gamma} \\
&= \frac{\tan\alpha+\tan\beta+(1-\tan\alpha\tan\beta)\tan\gamma}{1-\tan\alpha\tan\beta-(\tan\alpha+\tan\beta)\tan\gamma} \\
&= \frac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\beta-\tan\alpha\tan\gamma-\tan\beta\tan\gamma}
\end{aligned}$$

Multi-Line Equations

(iv) part of Question No.7

$$\begin{aligned}
\prod_p (1 - \frac{1}{p^2}) &= \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots} \\
&= \left(\prod_p \left(1 + \frac{1}{p^2} + \frac{1}{p^4} + \dots \right) \right)^{-1} \\
&= \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots \right)^{-1} \\
&= \frac{6}{\pi^2}
\end{aligned}$$


Thank you