Assignment 2

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Starting of Questions

Example 9.5

My document

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Example 9.5

1. Let x $= (x_1, \ldots, x_n)$, where the x_i are non negative real numbers .
   Set $M_r(x){=}\left(\frac{x_1^r+x_2^r+\cdots+x_n^r}{n}\right)^{\frac{1}{r}}$, $r \in \mathrm{R} \setminus \{0\}$,
   and
   $$M_0(\mathrm{x}) = (x_1 x_2 \cdots x_n)^{1/n}$$
   we call $M_r(\mathrm{x})$ the *rth power mean* of x.
   Claim:
   $$\lim_{r \to 0} M_r(\mathrm{x}) = M_0(\mathrm{x})$$

IInd part of 9.5

1. Define

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ x_1 & x_2 & x_3 & \cdots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \cdots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \cdots & x_n^{n-1} \end{bmatrix}$$

We call V_n the *Vandermonde matrix* of order n.
Claim:

$$\det V_n = \prod_{1 \leq i < j \leq n} (x_j - x_i).$$

Question number 4

- $$3^3 + 4^3 + 5^3 = 6^3$$
- $$\sqrt{100} = 10$$
- $$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
- $$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$
- $$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots$$

IInd part of 4

- $$\cos\theta = \sin(90 - \theta)$$

- $$e^{i\theta} = \cos\theta + i\sin\theta$$

- $$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$$

- $$\lim_{x \to \infty} \frac{\pi(x)}{x/logx} = 1$$

- $$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

Question number 5

- Positive numbers a,b, and c are the side lengths of a triangle if and only of a+b $>$ c, b+c $>$ a, and c+a $>$ b.
- The area of triangle with side lengths a,b,c is given by Heron's formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where s is the semiperimeter (a+b+c)/2

- The volume of a regular tetrahedron of edge length 1 is

$$\sqrt{2}/12$$

IInd part of 5

- The quadratic equation $ax^2 + bx + c = 0$ has roots $r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- The derivatives of a function f, denoted f', is defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- A real-valued function f is *convex* on an interval I if

$$f(\gamma x + (1-\gamma)y) \leq \gamma f(x) + (1-\gamma) f(y)$$

for all $x, y \in I$ and

$$0 \leq \gamma \leq 1$$

IIIrd part of 5

- The general solution to the differential equation

$$y'' - 3y' + 2y = 0$$

is

$$y = C_1 e^x + C_2 e^{2x}$$

IVth part of 5

- The fermat number F_n is defined as

$$F_n = 2^{2n}, n \geq 0$$

Question number 6

- $$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$

- $$\frac{d}{dx}\left(\frac{x}{x+1}\right) = \frac{1}{(x+1)^2}$$

- $$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

IInd part of 6

- $$R_\theta = \begin{bmatrix} \cos & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

- $$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

IIIrd part of Question 6

- $$\begin{bmatrix} \mathrm{a}_{11} & \mathrm{a}_{12} \\ \mathrm{a}_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \mathrm{b}_{11} & b_{12} \\ \mathrm{b}_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

- $$\mathrm{f(x)} = \begin{cases} \text{-x}^2, & x < 0 \\ x^2, & 0 \leq x \leq 2 \\ 4, & x > 2 \end{cases}$$

Question number 7

- $$1 + 2 = 3$$
$$4 + 5 + 6 = 7 + 8$$
$$9 + 10 + 11 + 12 = 13 + 14 + 15$$
$$16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24$$
$$25 + 26 + 27 + 28 + 29 + 30 = 31 + 32 + 33 + 34 + 35$$

IInd part of 7

- $$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= (a+b)a+(a+b)b\\ &= a(a+b)+b(a+b)\\ &= a^2+ab+ba+b^2\\ &= a^2+ab+ab+b^2\\ &= a^2+2ab+b^2\end{aligned}$$

IIIrd part of 7

- $$\begin{aligned} tan(\alpha+\beta+\gamma) &= \frac{tan(\alpha+\beta)+tan\gamma}{1-tan(\alpha+\beta)tan\gamma} \\ &= \frac{\left(\frac{tan\alpha+tan\beta}{1-tan\alpha tan\beta}\right)+tan\gamma}{1-\left(\frac{tan\alpha+tan\beta}{1-tan\alpha tan\beta}\right)tan\gamma} \\ &= \frac{tan\alpha+tan\beta+(1-tan\alpha tan\beta)tan\gamma}{1-tan\alpha tan\beta-(tan\alpha+tan\beta)tan\gamma} \\ &= \frac{tan\alpha+tan\beta+tan\gamma-tan\alpha tan\beta tan\gamma}{1-tan\alpha tan\beta-tan\alpha tan\gamma-tan\beta tan\gamma} \end{aligned}$$

IVth part of 7

- $$\begin{aligned}\prod_p \left(1 - \frac{1}{p^2}\right) &= \prod_p \frac{1}{1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots} \\ &= \left(\prod_p \left(1 + \frac{1}{p^2} + \frac{1}{p^4} + \cdots\right)\right)^{-1} \\ &= \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\right)^{-1} \\ &= \frac{6}{\pi^2}\end{aligned}$$

Thank you