ASSIGNMENT 2

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1) Let x=$(x_1, x_2, \ldots, x_n)$ where the x_i are non negative real numbers. Set

$$M_r(x) = \left(\frac{(x_1^r + x_2^r + \ldots + x_n^r)}{n}\right)^{\frac{1}{r}}, r \in R\backslash\{0\},$$

and

$$M_0(x) = (x_1 x_2 \ldots x_n)^{\frac{1}{n}}$$

We call $M_r(x)$ *the rth power mean of x.*
Claim:

$$\lim_{r \to 0} M_r(x) = M_0(x)$$

2) Define

$$V_n = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ x_1 & x_2 & x_3 & \dots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \dots & x_n^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \vdots & x_n^{n-1} \end{bmatrix}$$

We call V_n the *vandermonde matrix* of order n.

Claim: det

$$V_n = \prod_{1 \le i < j \le n} x_j - x_i$$

Q4 Make the following equations

- $$3^3 + 4^3 + 5^3 = 6^3$$
- $$\sqrt{100} + 10$$
- $$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
- $$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$
- $$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

Q4 make the following equations

- $$\cos\theta = \sin(90^\circ - \theta)$$
- $$e^{i\theta} = \cos\theta + i\sin\theta$$
- $$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$$
- $$\lim_{x\to\infty} \frac{\pi(x)}{\frac{x}{x\log x}} = 1$$
- $$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

Q5 Typeset the following sequences.

- Positive numbers a,b and c are the side lengths of a triangle if and only if $a + b > c, b + c > a$, and $c + a > b$
- The area of triangle with side lengths *a,b,c* is given by *Heron's formula :*

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where s is the semi perimeter $\frac{(a+b+c)}{2}$

- The volume of a regular tetrahedron of edge length 1 is $\frac{\sqrt{2}}{12}$
- The quadratic equation $ax^2 + bx + c = 0$ has roots

$$r_1.r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Q5 Typeset the following sequences.

- The *derivative* of a function *f*.denoted *f'* , is defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- A real valued function *f* is *convex* on an interval *I* if

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda)f(y)$$

  , for all $x,y \in$ I and $0 \leq \lambda \leq 1$.

- The general solution to the differential equation

$$y" - 3y' + 2y = 0$$

  is

$$y = C_1 e^x + C_2 e^{2x}$$

- The *Fermat number* F_n is defined as

$$F_n = 2^{2^n}, n \geq 0.$$

Q6 Make the following equations. Notice the large delimiters

- $$\frac{d}{dx}\left(\frac{x}{x+1}\right) = \frac{1}{(x+1)^2}$$

- $$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$

- $$\left|\begin{array}{cc} a & b \\ c & d \end{array}\right| = ad - bc$$

- $$R_\theta = \left[\begin{array}{cc} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{array}\right]$$

Q6 Make the following equations. Notice the large delimiters

- $$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

- $$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{12} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

- $$f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & 0 \le x \le 2 \\ 4, & x > 2 \end{cases}$$

Q7. Make the following multi line equations

$$\begin{aligned}
1+2 &= 3\\
4+5+6 &= 7+8\\
9+10+11+12 &= 13+14+15\\
16+17+18+19+20 &= 21+22+23+24\\
25+26+27+28+29+30 &= 31+32+33+34+35
\end{aligned}$$

Q7. Make the following multi line equations

$$
\begin{aligned}
(a+b)^2 &= (a+b)(a-b) \\
&= (a+b)a+(a+b)b \\
&= a(a+b)+b(a+b) \\
&= a^2+ab+ba+b^2 \\
&= a^2+ab+ab+b^2 \\
&= a^2+2ab+b^2
\end{aligned}
$$

Q7. Make the following multi line equations

$$
\begin{aligned}
\tan(\alpha+\beta+\gamma) &= \frac{\tan(\alpha+\beta)+\tan\gamma}{1-\tan(\alpha+\beta)\tan\gamma} \\
&= \frac{\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}+\tan\gamma}{1-(\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta})\tan\gamma} \\
&= \frac{\tan\alpha+\tan\beta+(1-\tan\alpha\tan\beta)\tan\gamma}{1-\tan\alpha\tan\beta-(\tan\alpha+\tan\beta)\tan\gamma} \\
&= \frac{\tan\alpha+\tan\beta+\tan\gamma-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\beta-\tan\alpha\tan\gamma-\tan\beta\tan\gamma}
\end{aligned}
$$

Q7. Make the following multi line equations

$$
\begin{aligned}
\prod_p \left(1-\frac{1}{p^2}\right) &= \prod_p \frac{1}{1+\frac{1}{p^2}+\frac{1}{p^4}+\cdots} \\
&= \left(\prod_p \left(1+\frac{1}{1+\frac{1}{p^2}+\frac{1}{p^4}+\cdots}\right)\right)^{-1} \\
&= \left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots\right)^{-1} \\
&= \frac{6}{\pi^2}
\end{aligned}
$$


Thank You