**KINCHAT KAUR**
**MAT/19/57**

# CH-12 ASSIGNMENT

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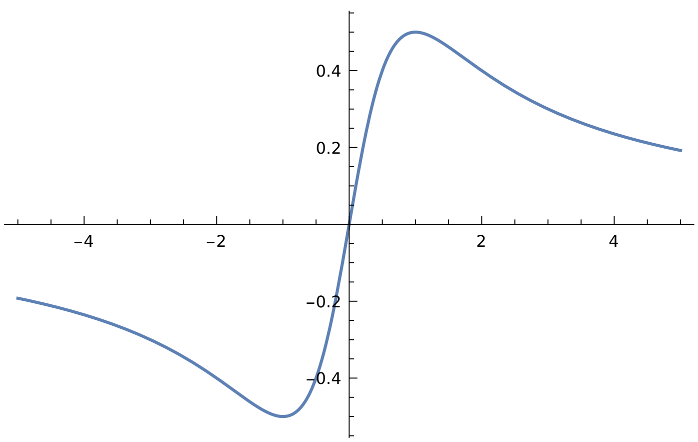
**Q.1 Graph each of the following functions. Experiment with different domains or viewpoints to display the best images.**

In[59]:=
```
f[x_] := x / (1 + x ^ 2)
```

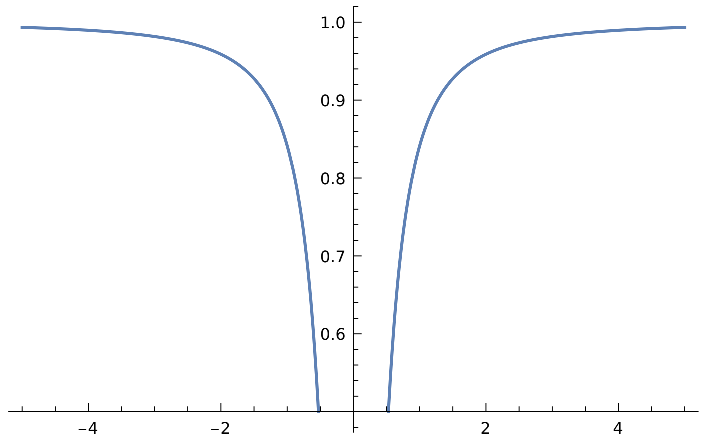
In[62]:=
```
Plot[f[x], {x, -5, 5}]
```

Out[62]=



In[11]:=
```
y[x_] := x * (Sin[1 / x])
```

In[12]:= `Plot[y[x], {x, -5, 5}]`

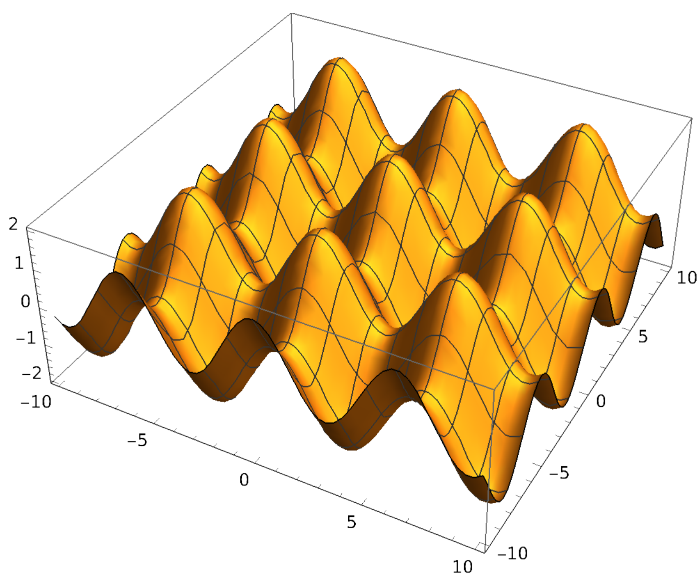
Out[12]=




In[5]:= `g[x_, y_] := Cos[x] + Sin[y]`

In[6]:= `Plot3D[g[x, y], {x, -10, 10}, {y, -10, 10}]`

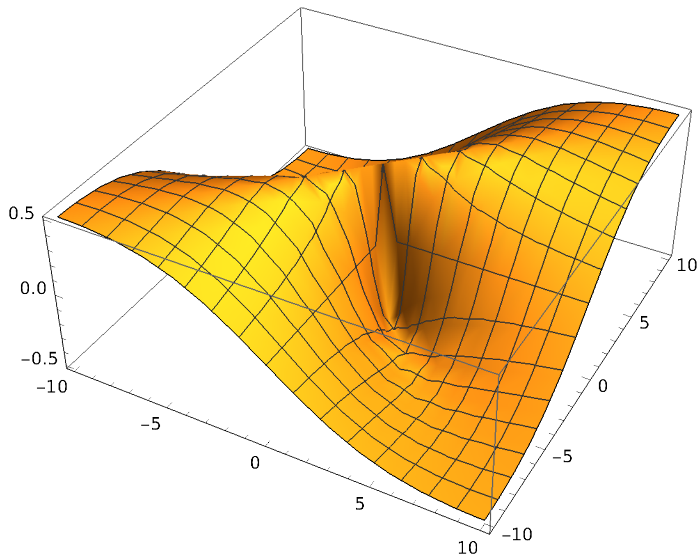
Out[6]=




In[16]:= `z[x_, y_] := (x y) / (x ^ 2 + y ^ 2)`

In[17]:= `Plot3D[z[x, y], {x, -10, 10}, {y, -10, 10}]`

Out[17]=




In[19]:= `ClearAll[f, x, y, g, z]`

## Q.2 Let f (x)=x

In[5]:= $f[x\_] := \frac{x}{1 + x^2}$

## (a) Find f'(x) and f"(x)

In[6]:= `f'[x]`

Out[6]= $-\frac{2 x^2}{(1 + x^2)^2} + \frac{1}{1 + x^2}$

In[7]:= `f''[x]`

Out[7]= $\frac{8 x^3}{(1 + x^2)^3} - \frac{6 x}{(1 + x^2)^2}$

## (b) Find f'(-1) and f'(0)

In[8]:= `f'[-1]`

Out[8]= 0

In[9]:= `f'[0]`

Out[9]= 1

## (c) Find f''(0) and f''(1)

In[10]:= `f''[0]`

Out[10]= 0

In[11]:=
```
f''[1]
```
Out[11]= $-\frac{1}{2}$

## Q.3 Find the prime factorization of each integer.

## (a) 3,527,218,133,309,949,276,293

In[12]:=
```
FactorInteger[3]
```
Out[12]= {{3, 1}}

In[13]:=
```
FactorInteger[527]
```
Out[13]= {{17, 1}, {31, 1}}

In[14]:=
```
FactorInteger[218]
```
Out[14]= {{2, 1}, {109, 1}}

In[15]:=
```
FactorInteger[133]
```
Out[15]= {{7, 1}, {19, 1}}

In[16]:=
```
FactorInteger[309]
```
Out[16]= {{3, 1}, {103, 1}}

In[17]:=
```
FactorInteger[949]
```
Out[17]= {{13, 1}, {73, 1}}

In[18]:=
```
FactorInteger[276]
```
Out[18]= {{2, 2}, {3, 1}, {23, 1}}

In[19]:=
```
FactorInteger[293]
```
Out[19]= {{293, 1}}

## (b) 471,945,325,930,166,269

In[20]:=
```
FactorInteger[471]
```
Out[20]= {{3, 1}, {157, 1}}

In[21]:=
```
FactorInteger[945]
```
Out[21]= {{3, 3}, {5, 1}, {7, 1}}

In[22]:=
```
FactorInteger[325]
```
Out[22]= {{5, 2}, {13, 1}}

In[23]:=
```
FactorInteger[930]
```
Out[23]= {{2, 1}, {3, 1}, {5, 1}, {31, 1}}

In[24]:= `FactorInteger[166]`

Out[24]= {{2, 1}, {83, 1}}

In[25]:= `FactorInteger[269]`

Out[25]= {{269, 1}}

## (c) 471,945,325,930,166,281

In[26]:= `FactorInteger[471]`

Out[26]= {{3, 1}, {157, 1}}

In[27]:= `FactorInteger[945]`

Out[27]= {{3, 3}, {5, 1}, {7, 1}}

In[28]:= `FactorInteger[325]`

Out[28]= {{5, 2}, {13, 1}}

In[29]:= `FactorInteger[930]`

Out[29]= {{2, 1}, {3, 1}, {5, 1}, {31, 1}}

In[30]:= `FactorInteger[166]`

Out[30]= {{2, 1}, {83, 1}}

In[31]:= `FactorInteger[281]`

Out[31]= {{281, 1}}

## Q.4 Compute each expression. Do you notice a pattern?

## (a) 3 mod 7

In[32]:= `Mod[3^6, 7]`

Out[32]= 1

## (b) 6 mod 11

In[33]:= `Mod[6^10, 11]`

Out[33]= 1

## (c) 7 mod 21

In[38]:= `Mod[7^20, 21]`

Out[38]= 7

## (d) 7 mod 23

In[35]:= `Mod[7^22, 23]`

Out[35]= 1

## Q.8 Let M=[{1,1},{1,0}]

In[33]:=
```
M = {{1, 1}, {1, 0}}
```
Out[33]= {{1, 1}, {1, 0}}

**(a) Find M^2,M^3,...,M^10**

In[34]:=
```
MatrixPower[M, 2]
```
Out[34]= {{2, 1}, {1, 1}}

In[35]:=
```
MatrixPower[M, 3]
```
Out[35]= {{3, 2}, {2, 1}}

In[36]:=
```
MatrixPower[M, 4]
```
Out[36]= {{5, 3}, {3, 2}}

In[37]:=
```
MatrixPower[M, 5]
```
Out[37]= {{8, 5}, {5, 3}}

In[38]:=
```
MatrixPower[M, 6]
```
Out[38]= {{13, 8}, {8, 5}}

In[40]:=
```
MatrixPower[M, 7]
```
Out[40]= {{21, 13}, {13, 8}}

In[41]:=
```
MatrixPower[M, 8]
```
Out[41]= {{34, 21}, {21, 13}}

In[42]:=
```
MatrixPower[M, 9]
```
Out[42]= {{55, 34}, {34, 21}}

In[43]:=
```
MatrixPower[M, 10]
```
Out[43]= {{89, 55}, {55, 34}}

**(b) Do your answers suggest the way to compute Fibonacci numbers? Find the 100th Fibonacci number.**

In[44]:=
```
f[0] = 1;
f[1] = 1;
f[n_] := f[n] = f[n - 2] + f[n - 1]
```

In[47]:=
```
f[100]
```
Out[47]= 573 147 844 013 817 084 101

**Q.9 Find solutions to the following equations or system of equations.**
**(a) Find x, if x^2+x=1**
**(b) Find x, if x^2+x=-1**
**(c) Find x and y**

**4x-3y=5**

**6x+2y=14**

**(d) Find x,y,z and t**

**-2x-2y+3z+t=8**

**-3x+0y-6z+t=-19**

**6x-8y+6z+5t=47**

**x+3y-3z-t=-9**

In[49]:=
```
Solve[x^2 + x == 1, x]
```

Out[49]= $\left\{\left\{x \to \frac{1}{2}\left(-1-\sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(-1+\sqrt{5}\right)\right\}\right\}$

In[51]:=
```
Solve[x^2 + x == -1, x]
```

Out[51]= $\{\{x \to -(-1)^{1/3}\}, \{x \to (-1)^{2/3}\}\}$

In[52]:=
```
Solve[4 x - 3 y == 5 && 6 x + 2 y == 14, {x, y}]
```

Out[52]= $\{\{x \to 2, y \to 1\}\}$

In[54]:=
```
Solve[-2 x - 2 y + 3 z + t == 8 && -3 x + 0 y - 6 z + t == -19 &&
  6 x - 8 y + 6 z + 5 t == 47 && x + 3 y - 3 z - t == -9, {x, y, z, t}]
```

Out[54]= $\{\{x \to 2, y \to 1, z \to 3, t \to 5\}\}$

**Q.10 Some equations are difficult or impossible to solve explicitly,even with software. In such situations, we often resort to numerical methods. Mathematica uses FindRoot, Maple uses fsolve(), and Maxima uses find_root() to find numerical solutions to equations. Here is an example where a numerical approach works well. Assume that I invest $250 at the beginning of the second quarter, $350 at the beginning of the third quarter and $400 at the beginning of the fourth quarter. At the end of the year, I have $1365 (because my investments grow). To find my (continuous) rate of return, solve this equation for r:**

**250Exp[1.0r]+ 300Exp[0.75r]+350Exp[0.5r]+400Exp[0.25r]=1365**

In[55]:=
```
FindRoot[250 Exp[1.0 r] + 300 Exp[0.75 r] + 350 Exp[0.5 r] + 400 Exp[0.25 r] == 1365, {r, 0}]
```

Out[55]= $\{r \to 0.084104\}$

**Q.11 If n is a positive number, g>0 is any "guess" for the square root of n, then a better estimate of $\sqrt{n}$ is the average of g and n/g,i.e.,(g+n/g)/2. Write a function called mysqrt that accepts one argument, begins with an initial guess of 1.0, finds 20 new guesses, and returns the answer.**

```
In[1]:= mysqrt[n_] := Module[{i = 1, g = 1}, While[i ≤ 20, g = (g + n / g) / 2; i = i + 1]; g]

In[2]:= N[mysqrt[2], 6]

Out[2]= 1.41421

In[3]:= N[Sqrt[2], 6]

Out[3]= 1.41421

In[4]:= N[mysqrt[3]]

Out[4]= 1.73205
```

**Q .12**

```
In[16]:= Clear[collatz];

In[17]:= collatz[n_] := Which[n == 1, collatz[n] = 0, EvenQ[n],
           collatz[n] = 1 + collatz[n / 2], OddQ[n], collatz[n] = 1 + collatz[(3 * n) + 1]];

In[18]:= collatz[1]

Out[18]= 0

In[19]:= collatz[2]

Out[19]= 1

In[20]:= collatz[6]

Out[20]= 8

In[21]:= collatz[27]

Out[21]= 111
```