ASSIGNMENT-
Chapter 12

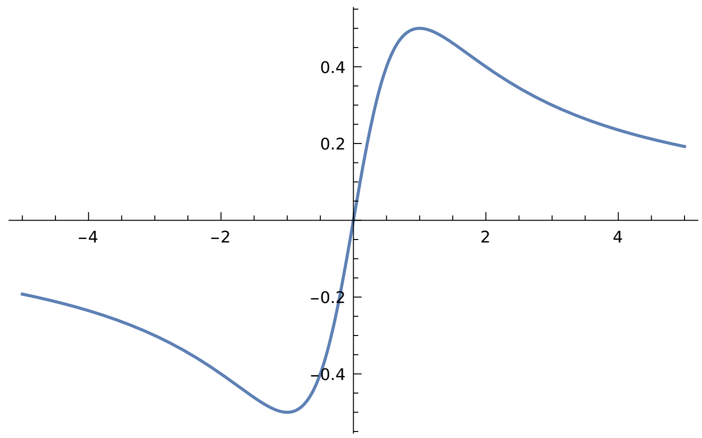
EXERCISES

1. Graph each of the following experiment with different domains or viewpoints to display the best image.

(a) f (x) = x / (1 + x ^ 2)

In[1]:=
```
Plot[x / (1 + x ^ 2), {x, -5, 5}]
```

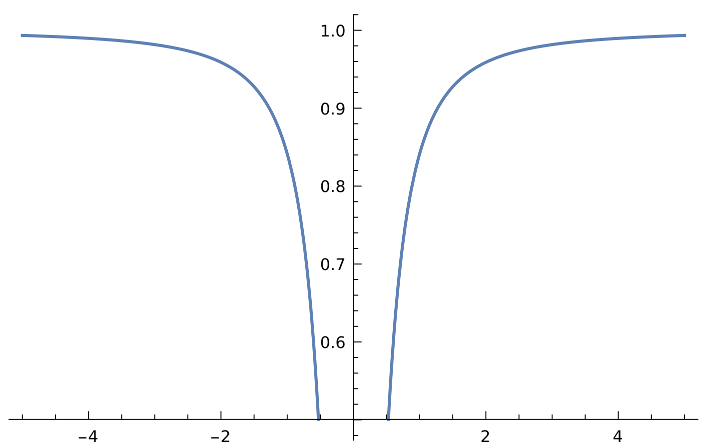
Out[1]=



(b) y=x sin(1/x)

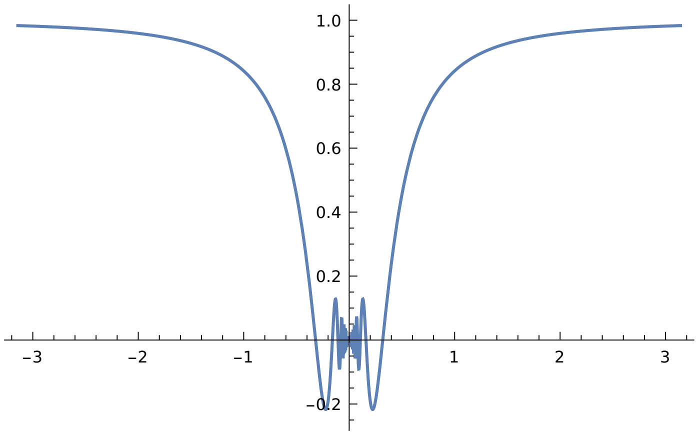
In[2]:=
```
Plot[x Sin[1 / x], {x, -5, 5}]
```

Out[2]=



In[3]:= **Plot[x Sin[1 / x], {x, -Pi, Pi}]**

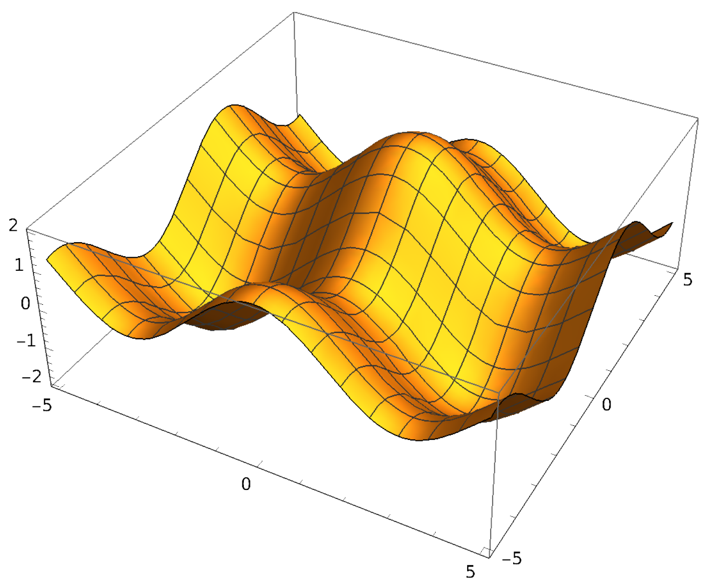
Out[3]=



(c) g(x,y) = cos(x) + sin(y)

In[4]:= **Plot3D[Cos[x] + Sin[y], {x, -5, 5}, {y, -5, 5}]**

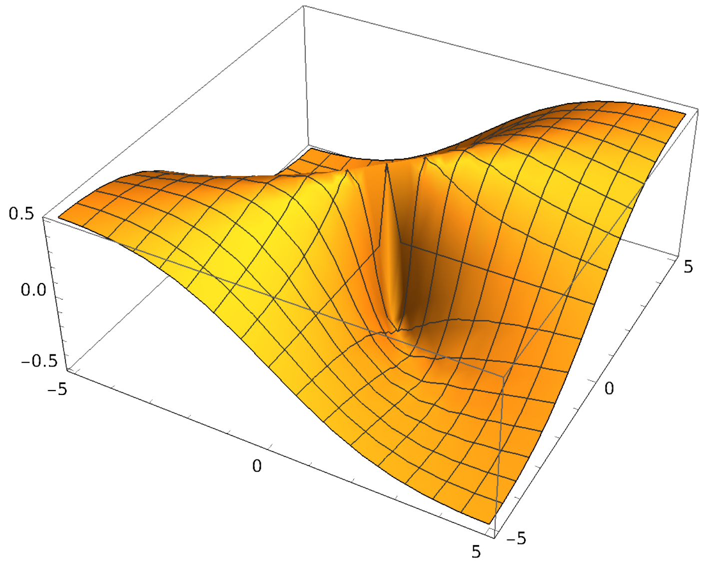
Out[4]=



(d) z = x y/x^2 + y^2

In[5]:=
```
Plot3D[x y / (x ^ 2 + y ^ 2), {x, -5, 5}, {y, -5, 5}]
```

Out[5]=



2. Let f(x) = x/(1+x^2)

(a) Find f'(x) and f''(x)

In[7]:=
```
f[x_] := x / (1 + x ^ 2)
D[f[x], x]
```

Out[8]= $-\frac{2x^2}{(1+x^2)^2}+\frac{1}{1+x^2}$

In[9]:=
```
f '[x]
```

Out[9]= $-\frac{2x^2}{(1+x^2)^2}+\frac{1}{1+x^2}$

In[10]:=
```
f ''[x]
```

Out[10]= $\frac{8x^3}{(1+x^2)^3}-\frac{6x}{(1+x^2)^2}$

(b) Find f'(-1) and f'(0)

In[11]:=
```
f '[-1]
```

Out[11]= 0

In[12]:=
```
f '[0]
```

Out[12]= 1

(c) Find f''(0) and f''(1)

In[13]:=
```
f ''[0]
```

Out[13]= 0

In[14]:=
```
f''[1]
```
Out[14]=
$$-\frac{1}{2}$$

3. Find the prime factorisation of each integer.

(a) 3,527,218,133,309,949,276,293

In[15]:=
```
FactorInteger[3 527 218 133 309 949 276 293 ]
```
Out[15]=
```
{{15 013 , 2}, {25 013 , 3}}
```

(b) 471,945,325,930,166,269

In[17]:=
```
FactorInteger[471 945 325 930 166 269 ]
```
Out[17]=
```
{{4211 , 1}, {34 589 , 1}, {46 747 , 1}, {69 313 , 1}}
```

(c) 471,945,325,930,166,281

In[18]:=
```
FactorInteger[471 945 325 930 166 281 ]
```
Out[18]=
```
{{471 945 325 930 166 281 , 1}}
```

4. Compute each expression. Do you notice a pattern?

(a) 3^6 mod 7

In[19]:=
```
Mod[3 ^ 6, 7]
```
Out[19]=
```
1
```

(b) 6^10 mod 11

In[20]:=
```
Mod[6 ^ 10, 11]
```
Out[20]=
```
1
```

(c) 7^20 mod 21

In[21]:=
```
Mod[7 ^ 20, 21]
```
Out[21]=
```
7
```

(d) 7^22 mod 23

In[22]:=
```
Mod[7 ^ 22, 23]
```
Out[22]=
```
1
```

8.

(a) Find M^2 , M^3 , ... , M^10

In[23]:=
```
M = {{1, 1}, {1, 0}};
M // MatrixForm
```
Out[24]//MatrixForm=
$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

In[25]:=
```
M.M;
M.M // MatrixForm
```
Out[26]//MatrixForm=

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

In[27]:=
```
M3 = {{1, 1}, {1, 0}}.{{1, 1}, {1, 0}}.{{1, 1}, {1, 0}};
M3 // MatrixForm
```
Out[28]//MatrixForm=

$$\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$$

In[34]:=
```
M4 = {{1, 1}, {1, 0}}.{{1, 1}, {1, 0}}.{{1, 1}, {1, 0}}.{{1, 1}, {1, 0}}
M4 // MatrixForm
```
Out[35]//MatrixForm=

$$\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$$

In[36]:=
```
M10 = MatrixPower[{{1, 1}, {1, 0}}, 10];
M10 // MatrixForm
```
Out[37]//MatrixForm=

$$\begin{pmatrix} 89 & 55 \\ 55 & 34 \end{pmatrix}$$

(b) Do your answer suggest a way to complete Fibonacci numbers? Find the 100th Fibonacci number.

In[38]:=
```
M100 = MatrixPower[{{1, 1}, {1, 0}}, 100];
M100 // MatrixForm
```
Out[39]//MatrixForm=

$$\begin{pmatrix} 573\,147\,844\,013\,817\,084\,101 & 354\,224\,848\,179\,261\,915\,075 \\ 354\,224\,848\,179\,261\,915\,075 & 218\,922\,995\,834\,555\,169\,026 \end{pmatrix}$$

Yes, by finding matrix power of M , we get more efficient way to compute fibonacci numbers. Thus the first element of M100 matrix is 100th Fibonacci number, that is 573147844013817084101 .

9. Find solutions to the following equations or system of equations .

(a) Find x, if x^2 + x=1.

In[40]:=
```
? Solve
```
Out[40]=

Symbol

Solve [*expr* , *vars*] attempts to solve the system *expr* of equations or inequalities for the variables *vars* .
Solve [*expr* , *vars* , *dom*] solves over the domain
dom . Common choices of *dom* are Reals , Integers , and Complexes .

In[41]:=
```
Solve[x^2 + x == 1, x]
```

Out[41]= $\left\{\left\{x \to \frac{1}{2}\left(-1-\sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(-1+\sqrt{5}\right)\right\}\right\}$

(b) Find x, if x^2 + x = -1

In[42]:=
```
Solve[x^2 + x == -1, x]
```

Out[42]= $\{\{x \to -(-1)^{1/3}\}, \{x \to (-1)^{2/3}\}\}$

(c) Find x and y .

4x - 3y = 5

6x + 2y = 14

In[43]:=
```
Solve[{4 x - 3 y == 5, 6 x + 2 y == 14}, {x, y}]
```

Out[43]= {{x → 2, y → 1}}

(d) Find x , y , z and t .

-2x - 2y + 3z + t = 8

-3x + 0y - 6z + t = - 19

6x - 8y +6z + 5t =47

x + 3y - 3z - t = - 9

In[44]:=
```
Solve[{-2 x - 2 y + 3 z + t == 8, -3 x + 0 y - 6 z + t == -19,
  6 x - 8 y + 6 z + 5 t == 47, x + 3 y - 3 z - t == -9}, {x, y, z, t}]
```

Out[44]= {{x → 2, y → 1, z → 3, t → 5}}

10. Solve this equation for r :

250 e^(1.0 r) + 300 e^(0.75 r) + 350 e^(0.5 r) + 400 e^(0.25 r)==1365 .

In[45]:=
```
? FindRoot
```

Out[45]=

Symbol

FindRoot $[f, \{x, x_0\}]$ searches for a numerical root of f, starting from the point $x = x_0$.

FindRoot $[lhs == rhs, \{x, x_0\}]$ searches for a numerical solution to the equation $lhs == rhs$.

FindRoot $[\{f_1, f_2, \ldots\}, \{\{x, x_0\}, \{y, y_0\}, \ldots\}]$ searches for a simultaneous numerical root of all the f_i.

FindRoot $[\{eqn_1, eqn_2, \ldots\}, \{\{x, x_0\}, \{y, y_0\}, \ldots\}]$

searches for a numerical solution to the simultaneous equations eqn_i.

In[49]:=
```
FindRoot[{250*e^(1.*r)  + 300*e^(0.75*r)  + 350*e^(0.5*r)  + 400*e^(0.25*r)  ==
    1365},  {r,  0}]
```

FindRoot : The function value $\{-1365 + 400\, e^{3.72529\times10^{-9}} + 350\, e^{7.45058\times10^{-9}} + 300\, e^{1.11759\times10^{-8}} + 250\, e^{1.49012\times10^{-8}}\}$ is not a list of numbers with dimensions {1} at {r} = $\{1.49012 \times 10^{-8}\}$.

Out[49]= {r → 0.}

11.

In[56]:=
```
mysqrt[n_] := Module[{i = 1, g = 1}, While[i ≤ 20, g = 1/2 (g + n/g); i = i + 1]; g]
N[mysqrt[2], 6]
```

Out[57]= 1.41421

In[58]:=
```
N[Sqrt[2], 6]
```

Out[58]= 1.41421

In[59]:=
```
N[mysqrt[3]]
```

Out[59]= 1.73205

12.

In[1]:=
```
Clear[collatz];
collatz[n_] := Which[n == 1, collatz[n_] 0, EvenQ[n],
   collatz[n] = 1 + collatz[n / 2], OddQ[n], collatz[n] = 1 + collatz[3 n + 1]];
collatz[
 27]
```

Out[3]= 111