~CHETNA MEHRA (MAT/19/9)~

Chapter 12

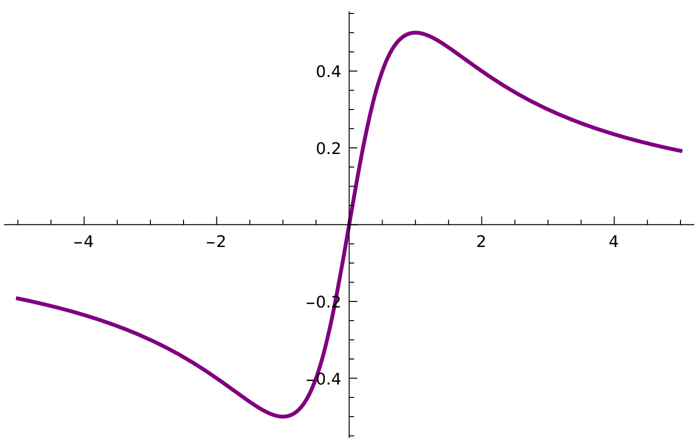
Q 1. Graph each of the functions. Experiment with different domains or viewpoints.

(a) f(x) = x/(1+x^2)

In[1]:=
```
f[x_] := x / (1 + x ^ 2)
```

In[2]:=
```
Plot[f[x], {x, -5, 5}, PlotStyle → {Purple , Thick}]
```

Out[2]=

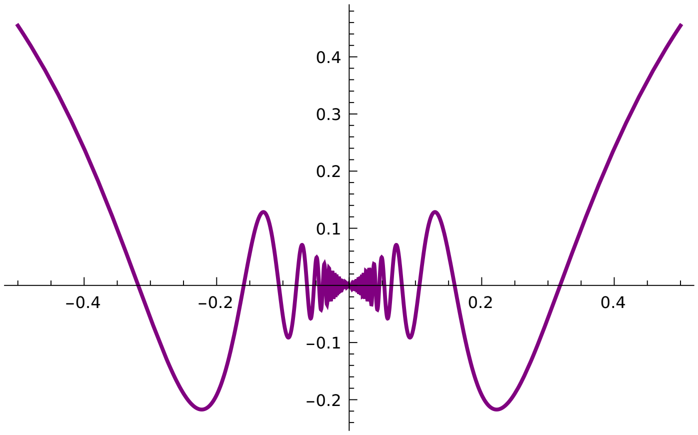


(b) f(x) = x Sin(1/x)

In[3]:=
```
Clear[f]
```

In[4]:=
```
f[x_] := x Sin[1 / x]
```

In[5]:=
```
Plot[f[x], {x, -0.5, 0.5}, PlotStyle → {Purple, Thick}]
```

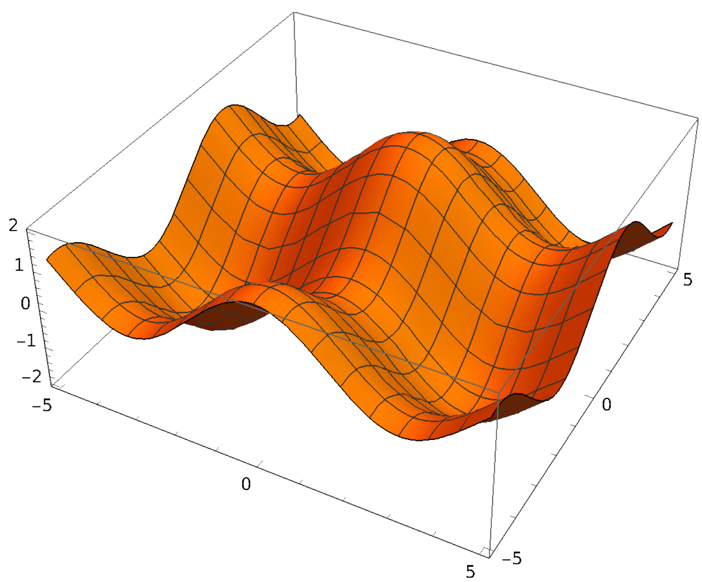
Out[5]= 

(c) f(x) = cos(x) + sin(y)

In[6]:=
```
Clear[f]
```

In[7]:=
```
f[x_, y_] := Cos[x] + Sin[y]
```

In[10]:=
```
Plot3D[f[x, y], {x, -5, 5}, {y, -5, 5}, PlotStyle → {Orange}]
```

Out[10]= 

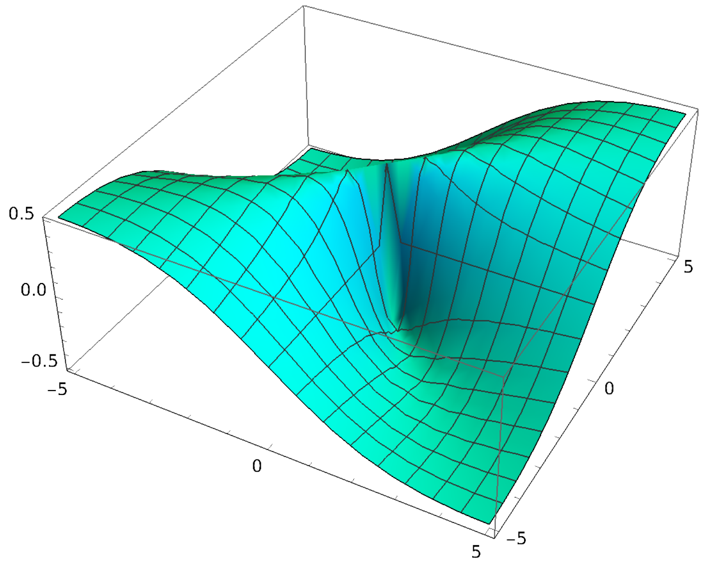
(d) f(x) = x y/(x^2 + y^2)

In[14]:=
```
Clear[f]
```

In[15]:=
```
f[x_, y_] := x y / (x ^ 2 + y ^ 2)
```

In[16]:= `Plot3D[f[x, y], {x, -5, 5}, {y, -5, 5}, PlotStyle → {Cyan}]`

Out[16]=



Q 2. Let f(x) = x/(1+x^2) then

In[18]:= `Clear[f]`

In[19]:= `f[x_] := x / (1 + x ^ 2)`

(a) Find f'(x) and f''(x).

In[20]:= `m = D[f[x], x]`

Out[20]= $-\frac{2 x^2}{(1 + x^2)^2} + \frac{1}{1 + x^2}$

In[21]:= `n = D[%, x]`

Out[21]= $\frac{8 x^3}{(1 + x^2)^3} - \frac{6 x}{(1 + x^2)^2}$

(b) Find f'(-1) and f'(0).

In[22]:= `m /. x → (-1)`

Out[22]= 0

In[24]:= `m /. x → (0)`

Out[24]= 1

(c) Find f''(0) and f''(1).

In[25]:=
```
n /. x → 0
```
Out[25]= 0

In[26]:=
```
n /. x → 1
```
Out[26]= $-\frac{1}{2}$

Q 3. Find the prime factorisation of each integer.

(a)3527218133309949276293

In[45]:=
```
FactorInteger [3 527 218 133 309 949 276 293 ]
```
Out[45]= {{15 013 , 2}, {25 013 , 3}}

(b) 471945325930166269

In[46]:=
```
FactorInteger [471 945 325 930 166 269 ]
```
Out[46]= {{4211 , 1}, {34 589 , 1}, {46 747 , 1}, {69 313 , 1}}

(c) 471945325930166281

In[47]:=
```
FactorInteger [471 945 325 930 166 281 ]
```
Out[47]= {{471 945 325 930 166 281 , 1}}

Q 4. Compute each expression

(a) 3 ^ 6 modulo 7

In[50]:=
```
Mod[3 ^ 6, 7]
```
Out[50]= 1

(b)6^10 modulo 11

In[51]:=
```
Mod[6 ^ 10, 11]
```
Out[51]= 1

(c)7^20 modulo 21

In[52]:=
```
Mod[7^20, 21]
```
Out[52]= 7

(d)7^22 modulo 23

In[53]:=
```
Mod[7^22, 23]
```
Out[53]= 1

Q 8. Let M = [{1,1},{1,0}]

In[54]:=
```
M = {{1, 1}, {1, 0}}
```
Out[54]= {{1, 1}, {1, 0}}

(a) Find M^2, M^3, ...,M^10.

In[55]:=
```
MatrixPower[M, 2]
```
Out[55]= {{2, 1}, {1, 1}}

In[56]:=
```
MatrixPower[M, 3]
```
Out[56]= {{3, 2}, {2, 1}}

In[57]:=
```
MatrixPower[M, 4]
```
Out[57]= {{5, 3}, {3, 2}}

In[58]:=
```
MatrixPower[M, 5]
```
Out[58]= {{8, 5}, {5, 3}}

In[59]:=
```
MatrixPower[M, 6]
```
Out[59]= {{13, 8}, {8, 5}}

In[60]:=
```
MatrixPower[M, 7]
```
Out[60]= {{21, 13}, {13, 8}}

In[61]:=
```
MatrixPower[M, 8]
```
Out[61]= {{34, 21}, {21, 13}}

In[62]:=
```
MatrixPower[M, 9]
```
Out[62]= {{55, 34}, {34, 21}}

In[63]:=
```
MatrixPower[M, 10]
```
Out[63]= {{89, 55}, {55, 34}}

(b) Find out the value of 100th Fibonacci number.

In[69]:=
```
Clear[f]
```
In[70]:=
```
f[0] = 1;
```
In[71]:=
```
f[1] = 1;
```
In[72]:=
```
f[n_] := f[n] = f[n - 1] + f[n - 2]
```
In[73]:=
```
f[100]
```
Out[73]= 573 147 844 013 817 084 101

Q 9. Find the algebraic solutions in the following questions.

(a) Find x if x^2 + x = 1

In[75]:=
```
Solve[x^2 + x == 1, x]
```
Out[75]= $\left\{\left\{x \to \frac{1}{2}\left(-1-\sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(-1+\sqrt{5}\right)\right\}\right\}$

(b) Find x if x^2 + x = -1

In[76]:=
```
Solve[x^2 + x == -1, x]
```
Out[76]= $\{\{x \to -(-1)^{1/3}\}, \{x \to (-1)^{2/3}\}\}$

(c) Find x and y if 4x - 3y=5 and 6x + 2y =14

In[77]:=
```
Solve[{4 x - 3 y == 5, 6 x + 2 y == 14}, {x, y}]
```
Out[77]= $\{\{x \to 2, y \to 1\}\}$

(d) Find x, y, z and t if -2x - 2y + 3z + t = 8, -3x + 0y - 6z + t = -19, 6x - 8y + 6z + 5t = 47 and x + 3y - 3z - t = -9

In[78]:=
```
Solve[{-2 x - 2 y + 3 z + t == 8, -3 x + 0 y - 6 z + t == -19,
  6 x - 8 y + 6 z + 5 t == 47, x + 3 y - 3 z - t == -9}, {x, y, z, t}]
```
Out[78]=
```
{{x → 2, y → 1, z → 3, t → 5}}
```

Q 10. Solve the equation 250e^(1.0r) + 300e^(0.75r) + 350e^(0.5r) + 400e^(0.25r) = 1365 for r.

In[1]:=
```
FindRoot[250 Exp[1.0 r] + 300 Exp[0.75 r] + 350 Exp[0.5 r] + 400 Exp[0.25 r] == 1365, {r, 0}]
```
Out[1]=
```
{r → 0.084104}
```

Q 11. If n is a positive number and g >0 is any "guess" for the square root of n, then a better estimate of $\sqrt{n}$ is the average of g and n/g, i.e., (g+n/g)/2. Write a function called mysqrt that accepts one argument, begines with an initial guess of 1.0, finds 20 new guesses and returns the answer.

In[1]:=
```
mysqrt[n_] := Module[{i = 1, g = 1}, While[i ≤ 20, g = (g + n / g) / 2; i = i + 1]; g]
```
In[2]:=
```
N[mysqrt[2], 6]
```
Out[2]=
```
1.41421
```
In[3]:=
```
N[mysqrt[3], 6]
```
Out[3]=
```
1.73205
```

Q 12. Use collatz function

(a) Write a (recursive) function called collatz that accepts a single argument, n and returns:

- 0 if n is equal to 1.
- 1 + Collatz(n/2) if n is even
- 1 + Collatz(3*n+1) if n is odd

In[12]:=
```
collatz[n_] := Which[n == 1, collatz[n] = 0, EvenQ[n],
   collatz[n] = 1 + collatz[n / 2], OddQ[n], collatz[n] = 1 + collatz[3 * n + 1]];
```

(b) Verify the values:

n : collatz(n)

1 : 0

2 : 1

6 : 8

27 : 111

In[13]:=
```
collatz[1]
```
Out[13]= 0

In[14]:=
```
collatz[2]
```
Out[14]= 1

In[15]:=
```
collatz[6]
```
Out[15]= 8

In[16]:=
```
collatz[27]
```
Out[16]= 111