Chapter: 12

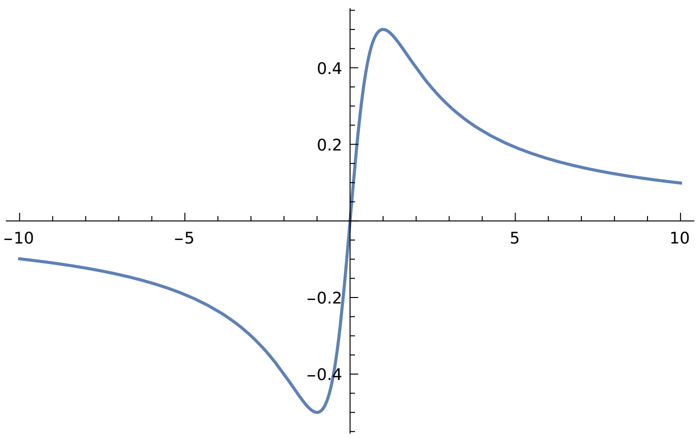
Ques 1. Graph each of the functions:

a. f(x)=x/(1+x^2)

In[36]:=
```
f[x_] := x / (1 + (x ^ 2))
```

In[37]:=
```
Plot[f[x], {x, -10, 10}]
```

Out[37]=



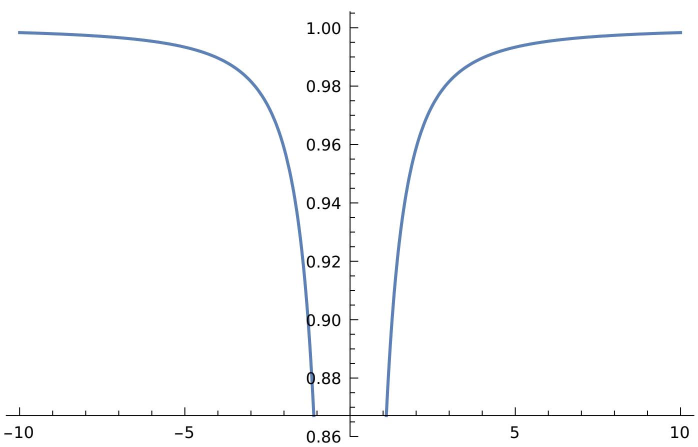
In[38]:=
```
ClearAll[f]
```

b. y=x Sin(1/x)

In[39]:=
```
y = x * Sin[1 / x];
```

In[40]:=
```
Plot[y, {x, -10, 10}]
```

Out[40]=



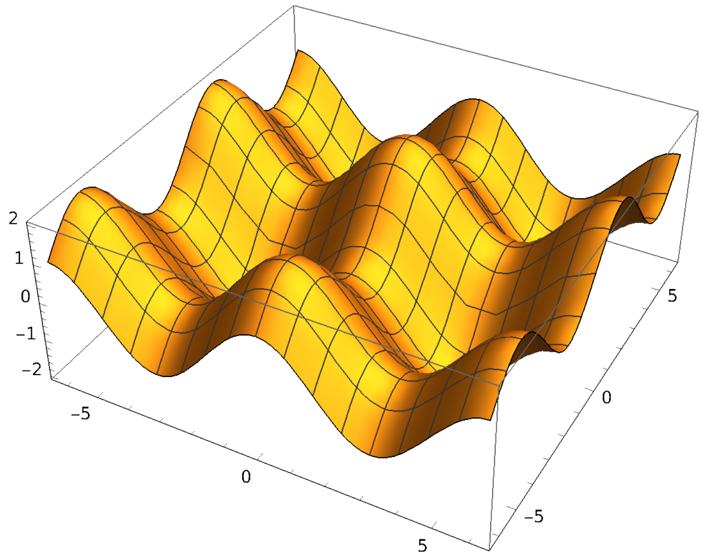
In[41]:=
```
ClearAll[y]
```

c. g(x,y) = Cos(x)+Sin(y)

In[31]:=
```
g[x_, y_] := Cos[x] + Sin[y]
```

In[33]:=
```
Plot3D[g[x, y], {x, -2 Pi, 2 Pi}, {y, -2 Pi, 2 Pi}]
```

Out[33]=



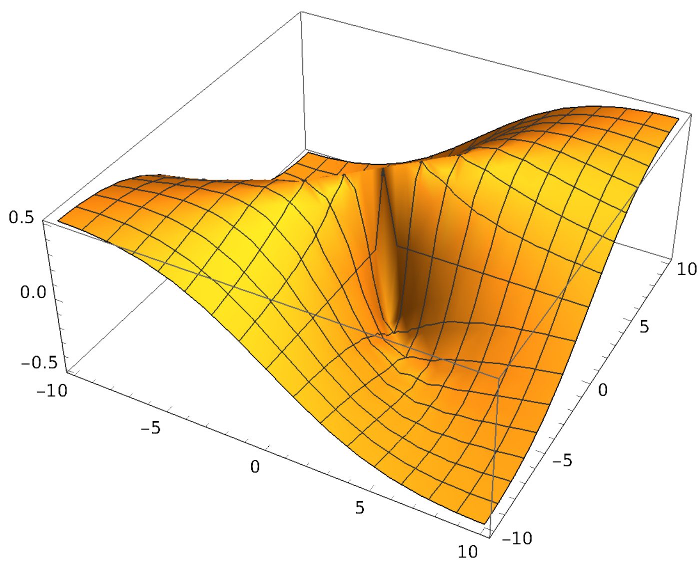
In[44]:=
```
ClearAll[g]
```

d. z = (xy)/(x^2+y^2)

In[45]:=
```
z := (x * y) / (x ^ 2 + y ^ 2)
```

In[46]:=
```
Plot3D[z, {x, -10, 10}, {y, -10, 10}]
```

Out[46]=



In[47]:=
```
ClearAll[z]
```

Ques 2. Let f[x]=x/(1+x^2)

a. Find f'(x) and f''(x)

In[48]:=
```
f[x_] := x / (1 + x ^ 2)
```

In[49]:=
```
D[f[x], x]
```
Out[49]=
$$-\frac{2\,x^2}{\left(1+x^2\right)^2}+\frac{1}{1+x^2}$$

In[50]:=
```
D[f'[x], x]
```
Out[50]=
$$\frac{8\,x^3}{\left(1+x^2\right)^3}-\frac{6\,x}{\left(1+x^2\right)^2}$$

b. Find f'(-1) and f'(0)

In[51]:=
```
f'[-1]
```
Out[51]= 0

In[52]:=
```
f'[0]
```
Out[52]= 1

c. Find f''(0) and f''(1)

In[53]:=
```
f''[0]
```
Out[53]= 0

In[54]:=
```
f''[1]
```
Out[54]=
$$-\frac{1}{2}$$

Ques 3. Find the prime factorization of each integer

a. 3,527,218,133,309,949,276,293

In[55]:=
```
FactorInteger[3 527 218 133 309 949 276 293 ]
```
Out[55]= {{15 013 , 2}, {25 013 , 3}}

b. 471,945,325,930,166,269

In[56]:=
```
FactorInteger[471 945 325 930 166 269 ]
```
Out[56]= {{4211 , 1}, {34 589 , 1}, {46 747 , 1}, {69 313 , 1}}

c. 471,945,325,930,166,281

In[57]:=
```
FactorInteger[471 945 325 930 166 281 ]
```
Out[57]= {{471 945 325 930 166 281 , 1}}

Ques 4. Compute each expression.

a. 3^6 mod 7

In[58]:=
```
Mod[3 ^ 6, 7]
```
Out[58]= 1

b. 6^10 mod 11

In[59]:=
```
Mod[6 ^ 10, 11]
```
Out[59]= 1

c. 7^20 mod 21

In[60]:=
```
Mod[7 ^ 20, 21]
```
Out[60]= 7

d. 7^22 mod 23

In[61]:=
```
Mod[7 ^ 22, 23]
```
Out[61]= 1

Ques 8. Let M={{1,1},{1,0}}

a. Find M^2, M^3, ..., M^10.

In[62]:=
```
M = {{1, 1}, {1, 0}}
```
Out[62]= {{1, 1}, {1, 0}}

In[63]:=
```
MatrixPower[M, 2]
```
Out[63]= {{2, 1}, {1, 1}}

In[64]:=
```
MatrixPower[M, 3]
```
Out[64]= {{3, 2}, {2, 1}}

In[65]:=
```
MatrixPower[M, 4]
```
Out[65]= {{5, 3}, {3, 2}}

In[66]:=
```
MatrixPower[M, 5]
```
Out[66]= {{8, 5}, {5, 3}}

In[67]:=
```
MatrixPower[M, 6]
```
Out[67]= {{13, 8}, {8, 5}}

In[68]:=
```
MatrixPower[M, 7]
```
Out[68]= {{21, 13}, {13, 8}}

In[69]:=
```
MatrixPower[M, 8]
```
Out[69]= {{34, 21}, {21, 13}}

In[70]:=
```
MatrixPower[M, 9]
```
Out[70]= {{55, 34}, {34, 21}}

In[71]:=
```
MatrixPower[M, 10]
```
Out[71]= {{89, 55}, {55, 34}}

b. Find the 100th Fibonacci number.

In[72]:=
```
f[0] = 1;
```

In[73]:=
```
f[1] = 1;
```

In[74]:=
```
f[n_] := f[n] = f[n - 2] + f[n - 1]
```

In[75]:=
```
f[100]
```

Out[75]= 573 147 844 013 817 084 101

Ques 9. Find Solutions to the following system of equations:

a. Find x, if x^2+x=1

In[76]:=
```
Solve[x ^ 2 + x == 1, x]
```

Out[76]= $\left\{\left\{x \to \frac{1}{2}\left(-1-\sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(-1+\sqrt{5}\right)\right\}\right\}$

b. Find x, if x^2+x=-1

In[77]:=
```
Solve[x ^ 2 + x == -1, x]
```

Out[77]= $\{\{x \to -(-1)^{1/3}\}, \{x \to (-1)^{2/3}\}\}$

c. Find x and y: 4x-3y=5, 6x+2y=14

In[78]:=
```
Solve[{4 x - 3 y == 5, 6 x + 2 y == 14}, {x, y}]
```

Out[78]= $\{\{x \to 2, y \to 1\}\}$

d. Find x,y,z and t

In[79]:=
```
Solve[{-2 x - 2 y + 3 z + t == 8, -3 x + 0 y - 6 z + t == -19,
  6 x - 8 y + 6 z + 5 t == 47, x + 3 y - 3 z - t == -9}, {x, y, z, t}]
```

Out[79]= $\{\{x \to 2, y \to 1, z \to 3, t \to 5\}\}$

Ques 10. To find countinuos rate of return, solve the equation for r: 250Exp[1.0r]+300Exp[0.75r]+350Exp[0.5r]+400Exp[0.25r]=1365.

In[80]:=
```
Solve[250 Exp[1.0 * r] + 300 Exp[0.75 * r] + 350 Exp[0.5 * r] + 400 Exp[0.25 * r] == 1365, r]
```

Out[80]= $\{\{r \to 4.\,((0.598729 + 3.14159\, i) + (0. + 6.28319\, i)\, c_1) \text{ if } c_1 \in \mathbb{Z}\},$
$\{r \to 4.\,(0.021026 + (0. + 6.28319\, i)\, c_1) \text{ if } c_1 \in \mathbb{Z}\},$
$\{r \to 4.\,((0.538847 - 1.68817\, i) + (0. + 6.28319\, i)\, c_1) \text{ if } c_1 \in \mathbb{Z}\},$
$\{r \to 4.\,((0.538847 + 1.68817\, i) + (0. + 6.28319\, i)\, c_1) \text{ if } c_1 \in \mathbb{Z}\}\}$

In[81]:=
```
FindRoot[250 Exp[1.0 * r] + 300 Exp[0.75 * r] + 350 Exp[0.5 * r] + 400 Exp[0.25 * r] == 1365, {r, 0}]
```

Out[81]= $\{r \to 0.084104\}$

Ques 11. If n is a positive number, and g > 0 is any “guess” for the square root of n, then a better estimate of $\sqrt{n}$ is the average of g and n/g, i.e., (g + n/g)/2. Write a function called mysqrt that accepts one argument, begins with an initial guess of 1.0, finds 20 new guesses, and returns the answer.

```
In[8]:= mysqrt[n_] := Module[{i = 1, g = 1}, While[i ≤ 20, g = (g + (n / g)) / 2; i = i + 1]; g]

In[9]:= N[mysqrt[2], 6]
Out[9]= 1.41421

In[10]:= N[Sqrt[2], 6]
Out[10]= 1.41421

In[11]:= N[mysqrt[3]]
Out[11]= 1.73205

In[12]:= N[mysqrt[5], 5]
Out[12]= 2.2361

In[11]:= N[mysqrt[8], 4]
Out[11]= 2.828
```

Ques 12. a. Write a (recursive) function called collatz that accepts a single argument, n.

```
In[12]:= Clear[collatz];

In[1]:= collatz[n_] := Which[n == 1, collatz[n] = 0, EvenQ[n],
    collatz[n] = 1 + collatz[n / 2], OddQ[n], collatz[n] = 1 + collatz[3 * n + 1]];
```

b. Verify the values:

```
In[3]:= collatz[1]
Out[3]= 0

In[4]:= collatz[2]
Out[4]= 1

In[5]:= collatz[6]
Out[5]= 8

In[2]:= collatz[27]
Out[2]= 111
```