

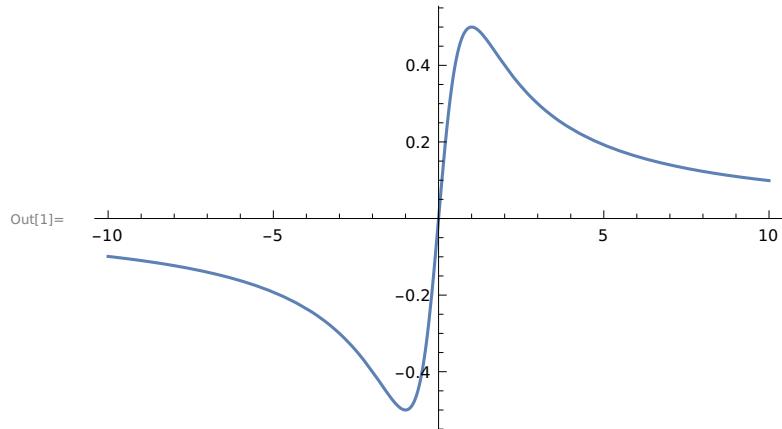
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MAT/19/4

CHAPTER 12

QUES 1 : Graph each of the functions . Experiment with different domains or viewpoints to display the best images .

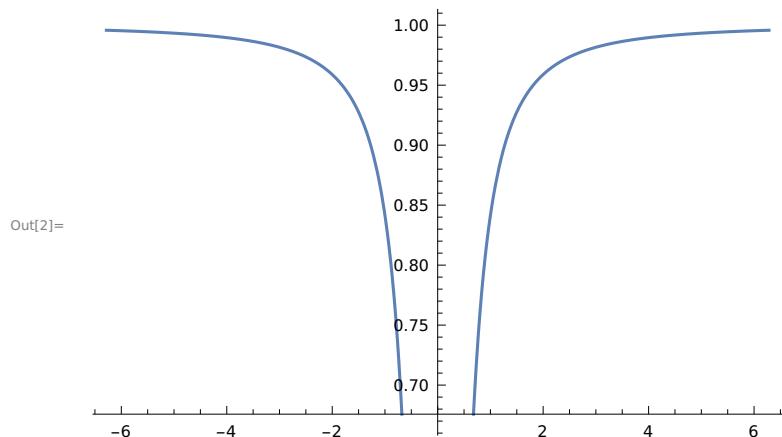
(a) $f(x) = x / (1 + x^2)$

In[1]:= Plot[x / (1 + x^2), {x, -10, 10}]



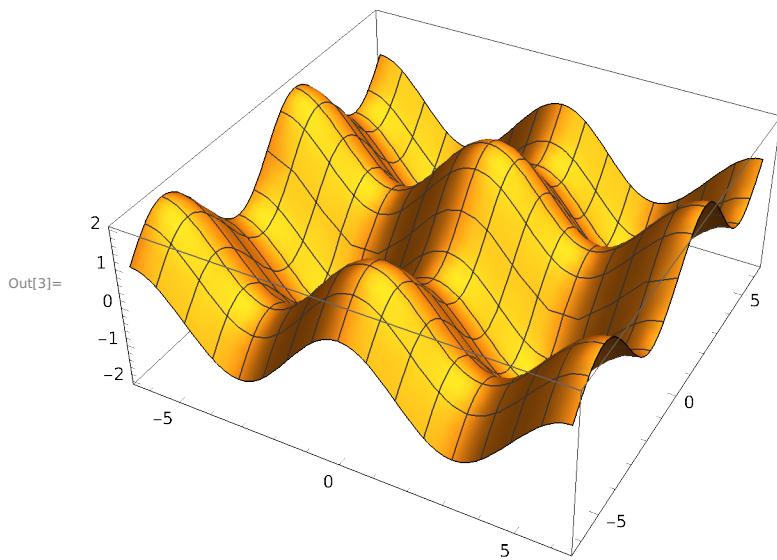
(b) $y = x \sin(1/x)$

In[2]:= Plot[x * Sin[1/x], {x, -2 Pi, 2 Pi}]



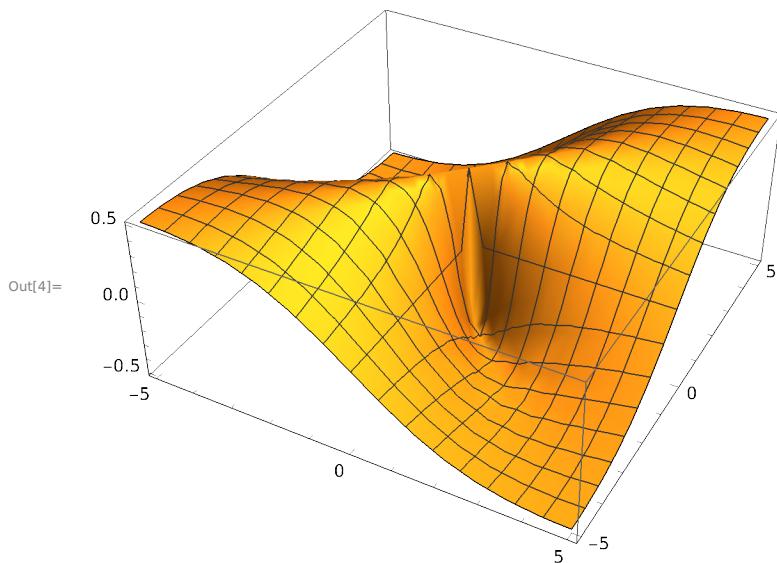
(c) $g(x, y) = \cos(x) + \sin(y)$

In[3]:= Plot3D[Cos[x] + Sin[y], {x, -2 Pi, 2 Pi}, {y, -2 Pi, 2 Pi}]



(d) $z = xy / (x^2 + y^2)$

In[4]:= Plot3D[x * y / (x^2 + y^2), {x, -5, 5}, {y, -5, 5}]



QUES 2 : Let $f(x) = x / (1 + x^2)$

In[5]:= f[x_] = x / (1 + x^2)

Out[5]=

$$\frac{x}{1+x^2}$$

(a) Find $f'(x)$ and $f''(x)$.

In[6]:= $f'[x]$
Out[6]= $-\frac{2x^2}{(1+x^2)^2} + \frac{1}{1+x^2}$

In[7]:= $f''[x]$
Out[7]= $\frac{8x^3}{(1+x^2)^3} - \frac{6x}{(1+x^2)^2}$

(b) Find $f'(-1)$ and $f'(0)$.

In[8]:= $f'[-1]$

Out[8]= 0

In[9]:= $f'[0]$

Out[9]= 1

(c) Find $f''(0)$ and $f''(1)$.

In[10]:= $f''[0]$

Out[10]= 0

In[11]:= $f''[1]$

Out[11]= $-\frac{1}{2}$

In[12]:= ClearAll[f]

QUES 3: Find the prime factorization of each integer .

(a) 3, 527, 218, 133, 309, 949, 276, 293

In[13]:= FactorInteger [3 527 218 133 309 949 276 293]

Out[13]= {{15 013 , 2}, {25 013 , 3}}

(b) 471, 945, 325, 930, 166, 269

In[14]:= FactorInteger [471 945 325 930 166 269]

Out[14]= {{4211 , 1}, {34 589 , 1}, {46 747 , 1}, {69 313 , 1}}

(c) 471, 945, 325, 930, 166, 281

```
In[15]:= FactorInteger [471 945 325 930 166 281 ]
Out[15]= {{471 945 325 930 166 281 , 1}}
```

QUES 4 : Compute each expression .

(a) $3^6 \bmod 7$

```
In[16]:= Mod[3 ^ 6, 7]
Out[16]= 1
```

(b) $6^{10} \bmod 11$

```
In[17]:= Mod[6 ^ 10, 11]
Out[17]= 1
```

(c) $7^{20} \bmod 21$

```
In[18]:= Mod[7 ^ 20, 21]
Out[18]= 7
```

(d) $7^{22} \bmod 23$

```
In[19]:= Mod[7 ^ 22, 23]
Out[19]= 1
```

QUES 8

(a)

```
In[20]:= M = {{1, 1}, {1, 0}}
Out[20]= {{1, 1}, {1, 0}}
```

```
In[21]:= M.M
Out[21]= {{2, 1}, {1, 1}}
```

```
In[22]:= M.M.M
Out[22]= {{3, 2}, {2, 1}}
```

```
In[23]:= M.M.M.M
Out[23]= {{5, 3}, {3, 2}}
```

```
In[24]:= M.M.M.M.M
Out[24]= {{8, 5}, {5, 3}}
```

```
In[25]:= M.M.M.M.M.M
Out[25]= {{13, 8}, {8, 5}}
```

```
In[26]:= M.M.M.M.M.M.M
Out[26]= {{21, 13}, {13, 8}}
```

```
In[27]:= M.M.M.M.M.M.M.M
Out[27]= {{34, 21}, {21, 13}}
```

```
In[28]:= M.M.M.M.M.M.M.M.M
Out[28]= {{55, 34}, {34, 21}}
```

```
In[29]:= M.M.M.M.M.M.M.M.M.M
Out[29]= {{89, 55}, {55, 34}}
```

(b)

```
In[30]:= f[0] = {{1, 1}, {1, 0}};
In[31]:= f[1] = {{2, 1}, {1, 1}};
In[32]:= f[n_] := f[n] = f[n - 2] + f[n - 1]
In[33]:= f[100]
Out[33]= {{927 372 692 193 078 999 176 , 573 147 844 013 817 084 101 },
{573 147 844 013 817 084 101 , 354 224 848 179 261 915 075 }}
```

QUES 9

(a)

```
In[34]:= Solve[{x^2 + x == 1}, {x}]
Out[34]= {{x →  $\frac{1}{2}(-1 - \sqrt{5})$ }, {x →  $\frac{1}{2}(-1 + \sqrt{5})$ }}
```

(b)

```
In[35]:= Solve[{x^2 + x == -1}, {x}]
Out[35]= {{x →  $-(-1)^{1/3}$ }, {x →  $(-1)^{2/3}$ }}
```

(c)

```
In[36]:= Solve[{4*x - 3*y == 5, 6*x + 2*y == 14}, {x, y}]
Out[36]= {{x → 2, y → 1}}
```

(d)

```
In[37]:= Solve[{-2*x - 2*y + 3*z + t == 8, -3*x + 0*y - 6*z + t == -19,
6*x - 8*y + 6*z + 5*t == 47, x + 3*y - 3*z - t == -9}, {x, y, z, t}]
Out[37]= {{x → 2, y → 1, z → 3, t → 5}}
```

QUES 10

```
In[38]:= FindRoot[250* $e^{1.0*r}$  + 300* $e^{0.75*r}$  + 350* $e^{0.5*r}$  + 400* $e^{0.25*r}$  == 1365, {r, 1}]
Out[38]= {r → 0.084104}
```

QUES 11

```
In[40]:= mysqrt[n_] := Module[{i = 1, g = 1}, While[i ≤ 20, g = (g + n/g)/2; i = i + 1]; g]
In[41]:= N[mysqrt[2], 6]
Out[41]= 1.41421
In[43]:= N[Sqrt[2], 6]
Out[43]= 1.41421
In[44]:= N[mysqrt[3]]
Out[44]= 1.73205
```

QUES 12

(a)

```
In[45]:= collatz[n_] := Which[n == 1, collatz[n] = 0, EvenQ[n],
collatz[n] = 1 + collatz[n/2], OddQ[n], collatz[n] = 1 + collatz[3*n + 1]]
```

(b)

```
In[47]:= collatz[1]
Out[47]= 0
In[48]:= collatz[2]
Out[48]= 1
```

```
In[49]:= collatz[6]
```

```
Out[49]= 8
```

```
In[50]:= collatz[27]
```

```
Out[50]= 111
```