MAT/19/109.
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Assignment
Chapter-12

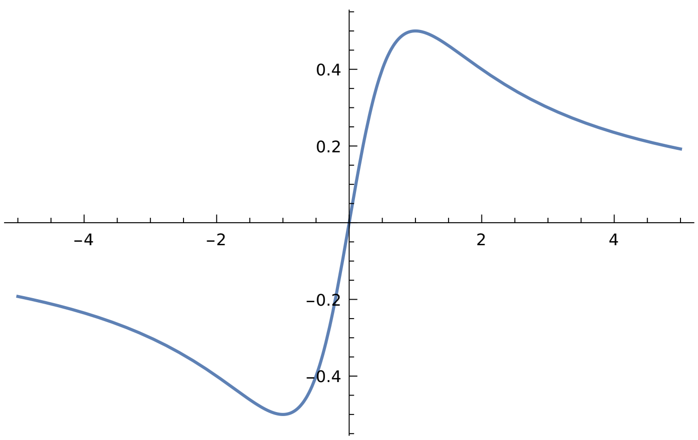
1. Graph each of the functions. Experiment with different domains or viewpoints to display the best image.

(a) f(x) = x/1+x^2

In[16]:=

```
Plot[x / (1 + x ^ 2), {x, -5, 5}]
```

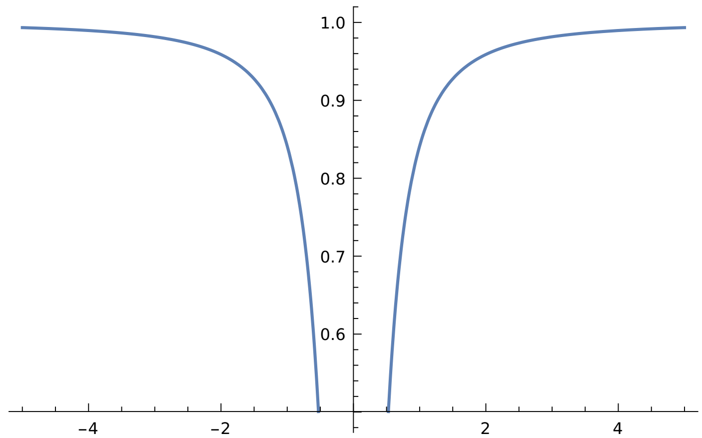
Out[16]=



(b) y = x Sin (1 / x)

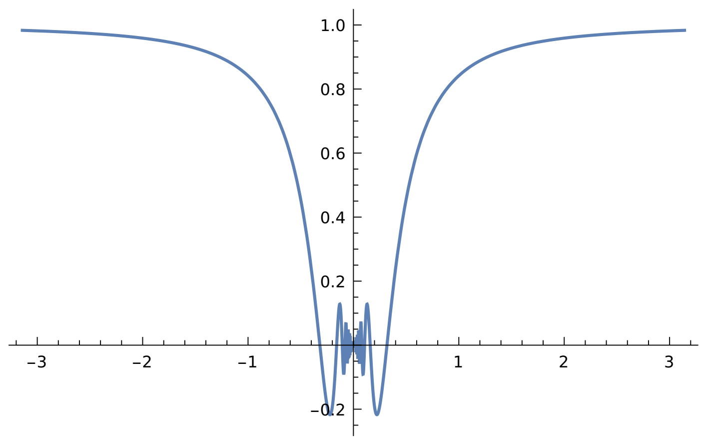
In[8]:= `Plot[x Sin[1 / x], {x, -5, 5}]`

Out[8]=



In[11]:= `Plot[x Sin[1 / x], {x, -Pi, Pi}]`

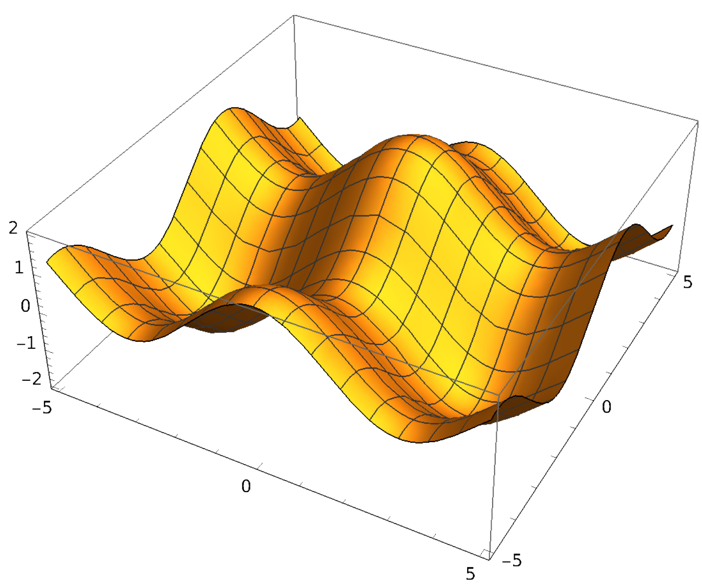
Out[11]=



(c) g(x,y)=cos(x)+sin(y)

In[12]:= `Plot3D[Cos[x] + Sin[y], {x, -5, 5}, {y, -5, 5}]`

Out[12]=

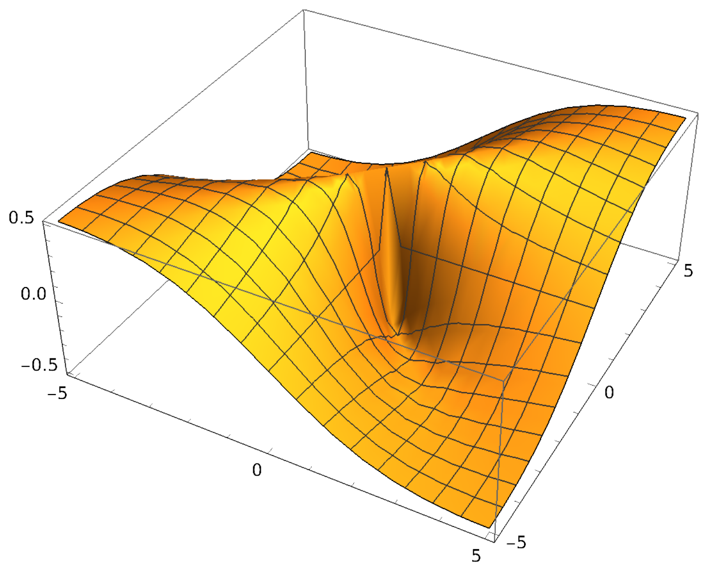


(d) z=xy/x^2+y^2

In[15]:=

```
Plot3D[x y / (x ^ 2 + y ^ 2), {x, -5, 5}, {y, -5, 5}]
```

Out[15]=



2.Let f(x)=x/(1+x^2).

(a) Find f'(x) and f''(x).

In[19]:=

```
f[x_] := x / (1 + x ^ 2)
```

In[20]:=

```
D[f[x], x]
```

Out[20]=

$$-\frac{2x^2}{(1+x^2)^2}+\frac{1}{1+x^2}$$

In[22]:=

```
f '[x]
```

Out[22]=

$$-\frac{2x^2}{(1+x^2)^2}+\frac{1}{1+x^2}$$

In[21]:=

```
f ''[x]
```

Out[21]=

$$\frac{8x^3}{(1+x^2)^3}-\frac{6x}{(1+x^2)^2}$$

(b) Find f '(-1) and f '(0)

In[23]:=

```
f '[-1]
```

Out[23]= 0

In[24]:=
```
f'[0]
```
Out[24]= 1

(c) Find f''(0) and f''(1)

In[26]:=
```
f''[0]
```
Out[26]= 0

In[25]:=
```
f''[1]
```
Out[25]= $-\frac{1}{2}$

3. Find the prime factorisation of each integer.

(a) 3,527,218,133,309,949,276,293

In[49]:=
```
FactorInteger[3 527 218 133 309 949 276 239]
```
Out[49]= {{2, 3}, {3, 3}, {7, 1}, {13, 1}, {17, 1}, {19, 1},
{23, 1}, {31, 1}, {73, 1}, {103, 1}, {109, 1}, {239, 1}}

(b) 471,945,325,930,166,269

In[50]:=
```
FactorInteger[471 945 325 930 166  269]
```
Out[50]= {{2, 2}, {3, 5}, {5, 4}, {7, 1}, {13, 1}, {31, 1}, {83, 1}, {157, 1}, {269, 1}}

(c) 471,945,325,930,166,281

In[51]:=
```
FactorInteger[471 945 325 930 166  281]
```
Out[51]= {{2, 2}, {3, 5}, {5, 4}, {7, 1}, {13, 1}, {31, 1}, {83, 1}, {157, 1}, {281, 1}}

4.Compute each expression. Do you notice a pattern?

(a) 3^6 mod 7

In[36]:=
```
Mod[3^6, 7]
```
Out[36]= 1

(b) 6^10 mod 11

In[37]:=
```
Mod[6^10, 11]
```
Out[37]= 1

(c) 7^20 mod 21

In[38]:=
```
Mod[7^20, 21]
```
Out[38]= 7

(d) 7^22 mod 23

In[39]:=
```
Mod[7^22, 23]
```
Out[39]=
```
1
```

8. (a) Find M^2,M^3<....<M^10.

In[40]:=
```
M = {{1, 1}, {1, 0}};
M // MatrixForm
```
Out[41]//MatrixForm=

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

In[42]:=
```
M.M;
M.M // MatrixForm
```
Out[43]//MatrixForm=

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

In[44]:=
```
M3 = {{1, 1}, {1, 0}}.{{1, 1}, {1, 0}}.{{1, 1}, {1, 0}};
M3 // MatrixForm
```
Out[45]//MatrixForm=

$$\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$$

In[46]:=
```
M4 = {{1, 1}, {1, 0}}.{{1, 1}, {1, 0}}.{{1, 1}, {1, 0}}.{{1, 1}, {1, 0}};
M4 // MatrixForm
```
Out[47]//MatrixForm=

$$\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$$

In[48]:=
```
M10 = MatrixPower[{{1, 1}, {1, 0}}, 10];
M10 // MatrixForm
```
Out[49]//MatrixForm=

$$\begin{pmatrix} 89 & 55 \\ 55 & 34 \end{pmatrix}$$

(b) Do your answers suggest a way to compute Fibonacci numbers? Find the 100th Fibonacci number.

In[50]:=
```
M100 = MatrixPower[{{1, 1}, {1, 0}}, 100];
M100 // MatrixForm
```
Out[51]//MatrixForm=

$$\begin{pmatrix} 573\,147\,844\,013\,817\,084\,101 & 354\,224\,848\,179\,261\,915\,075 \\ 354\,224\,848\,179\,261\,915\,075 & 218\,922\,995\,834\,555\,169\,026 \end{pmatrix}$$

Yes, by finding matrix power of M, we get more efficient way to compute fibonacci numbers. Thus, the first element of M100 matrix is 100th Fibonacci number, that is 573147844013817084101.

9. Find solutions to the following equations or system of equations.

(a) Find x if x^2+x=1

In[52]:=

```
? Solve
```

Out[52]=

Symbol

Solve[*expr*, *vars*] attempts to solve the system *expr* of equations or inequalities for the variables *vars*.

Solve[*expr*, *vars*, *dom*] solves over the domain *dom*. Common choices of *dom* are Reals, Integers, and Complexes.

In[53]:=

```
Solve[x^2 + x == 1, x]
```

Out[53]= $\left\{\left\{x \to \frac{1}{2}\left(-1 - \sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(-1 + \sqrt{5}\right)\right\}\right\}$

(b) Find x, if x^2+x=1.

In[54]:=

```
Solve[x^2 + x == -1, x]
```

Out[54]= $\{\{x \to -(-1)^{1/3}\}, \{x \to (-1)^{2/3}\}\}$

(c) Find x and y.

4x-3y = 5

6x+2y = 14

In[55]:=

```
Solve[{4 x - 3 y == 5, 6 x + 2 y == 14}, {x, y}]
```

Out[55]= $\{\{x \to 2, y \to 1\}\}$

(d) Find x, y, z and t.

-2x-2y+3z+t = 8

-3x+0y-6z+t = -19

6x-8y+6z+t = 47

x+3y-3z-t = -9

In[2]:=

```
Solve[{-2 x - 2 y + 3 z + t == 8, -3 x + 0 y - 6 z + t == -19,
  6 x - 8 y + 6 z + 5 t == 47, x + 3 y - 3 z - t == -9}, {x, y, z, t}]
```

Out[2]= $\{\{x \to 2, y \to 1, z \to 3, t \to 5\}\}$

10. Solve this equation for r: 250ⅇ^(1.0r)+300ⅇ^(0.75r)+350ⅇ^(0.5r)+400ⅇ^(0.25r)==1365

In[3]:=
```
? FindRoot
```

Out[3]=

Symbol

FindRoot [f, {x, x_0}] searches for a numerical root of f, starting from the point $x = x_0$.

FindRoot [$lhs == rhs$, {x, x_0}] searches for a numerical solution to the equation $lhs == rhs$.

FindRoot [{f_1, f_2, …}, {{x, x_0}, {y, y_0}, …}] searches for a simultaneous numerical root of all the f_i.

FindRoot [{eqn_1, eqn_2, …}, {{x, x_0}, {y, y_0}, …}]

searches for a numerical solution to the simultaneous equations eqn_i.

In[4]:=
```
FindRoot[{250 ⅇ^(1.0 r) + 300 ⅇ^(0.75 r) + 350 ⅇ^(0.5 r) + 400 ⅇ^(0.25 r) == 1365}, {r, 0}]
```

Out[4]=
```
{r → 0.084104}
```

11.

In[1]:=
```
mysqrt[n_] := Module[{i = 1, g = 1}, While [i ≤ 20, g = (g + n / g) / 2; i = i + 1]; g]
```

In[2]:=
```
N[mysqrt[2], 6]
```

Out[2]=
```
1.41421
```

In[3]:=
```
N[Sqrt[2], 6]
```

Out[3]=
```
1.41421
```

In[5]:=
```
N[mysqrt[3]]
```

Out[5]=
```
1.73205
```

12.

In[45]:=
```
Clear[collatz];
```

In[47]:=
```
collatz[n_] := Which[n == 1, collatz[n] = 0, EvenQ[n],
    collatz[n] = 1 + collatz[n / 2], OddQ[n], collatz[n] = 1 + collatz[3 * n + 1]];
```

In[48]:=
```
collatz[27]
```

Out[48]=
```
111
```