# CHAPTER -12 EXERCISE

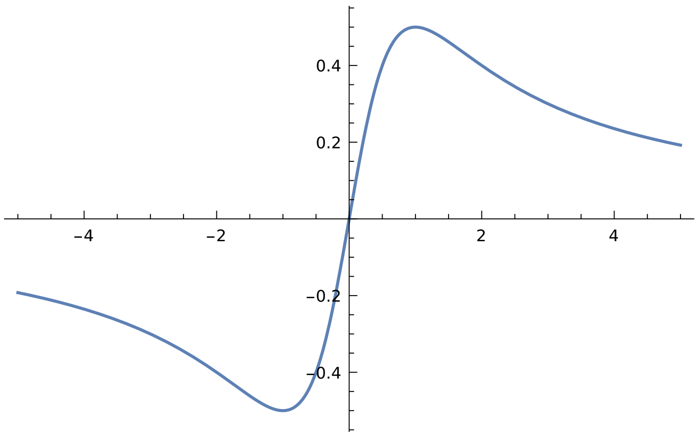
## Q1. Graph each of the following functions. Experiment with different domains or viewpoints to display the best images.

a) f(x)= x/(1+x^2)

In[1]:=
```
f[x_] := x / (1 + x ^ 2);
```

In[2]:=
```
Plot[f[x], {x, -5, 5}]
```

Out[2]=

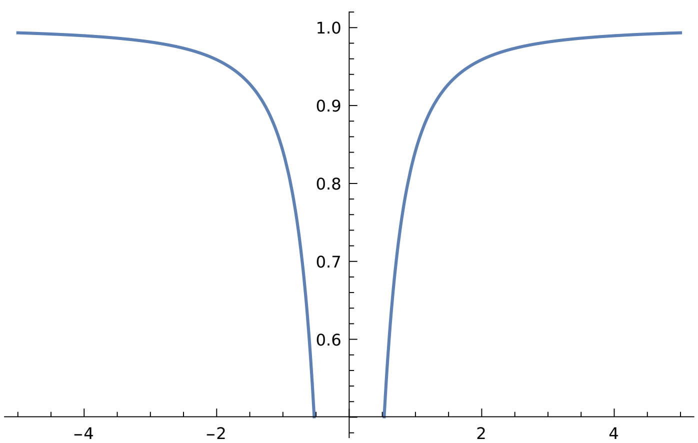



b) y= x Sin[1/x]

In[3]:=
```
f[x_] := x Sin[1 / x];
```

In[4]:=
```
Plot[f[x], {x, -5, 5}]
```

Out[4]=

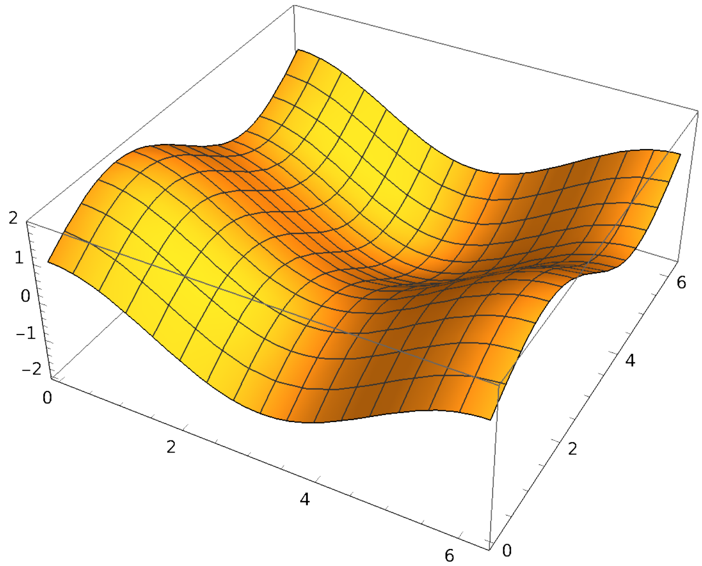



c) g(x,y)= Cos[x]+Sin[y]

In[5]:=
```
f[x_, y_] := Cos[x] + Sin[y];
```

In[6]:= `Plot3D[f[x, y], {x, 0, 2 Pi}, {y, 0, 2 Pi}]`

Out[6]=

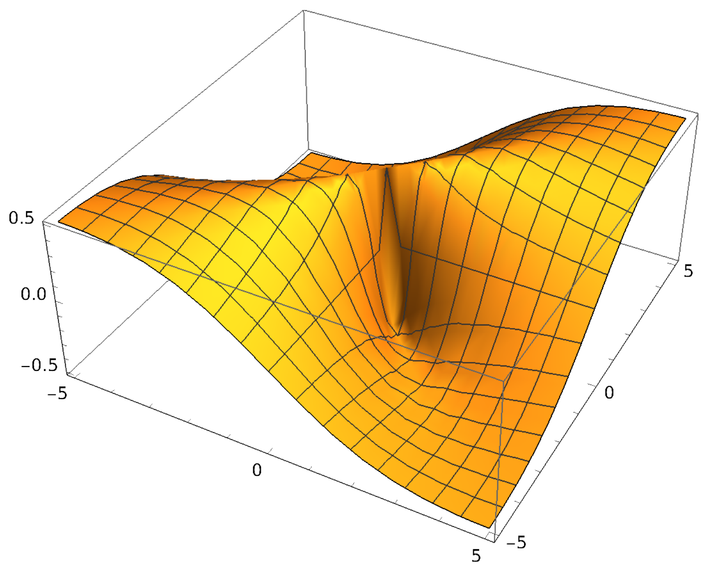



d) z= x y/(x^2+y^2)

In[7]:= `f[x_, y_] := (x y) / (x ^ 2 + y ^ 2);`

In[8]:= `Plot3D[f[x, y], {x, −5, 5}, {y, −5, 5}]`

Out[8]=




## Q 2. Let f(x)=x/(1+x^2)

In[9]:= `f[x_] := x / (1 + x ^ 2);`

a) Find f'(x) and f"(x).

In[10]:= `f'[x]`

Out[10]= $-\frac{2 x^2}{(1+x^2)^2}+\frac{1}{1+x^2}$

In[11]:= `f''[x]`

Out[11]= $\frac{8 x^3}{(1+x^2)^3}-\frac{6 x}{(1+x^2)^2}$

b) Find f'(-1) and f'(0)

In[12]:= `f'[-1]`

Out[12]= 0

In[13]:= `f'[0]`

Out[13]= 1

c) Find f''(0) and f''(1)

In[14]:= `f''[0]`

Out[14]= 0

In[15]:= `f''[1]`

Out[15]= $-\frac{1}{2}$

## Q3. Find the prime factorization of each integer.

a) 3, 527, 218, 133, 309, 949, 276, 293

In[16]:= `FactorInteger [3]`

Out[16]= {{3, 1}}

In[17]:= `FactorInteger [527]`

Out[17]= {{17, 1}, {31, 1}}

In[18]:= `FactorInteger [218]`

Out[18]= {{2, 1}, {109, 1}}

In[21]:= `FactorInteger [133]`

Out[21]= {{7, 1}, {19, 1}}

In[20]:= `FactorInteger [309]`

Out[20]= {{3, 1}, {103, 1}}

In[22]:= `FactorInteger [949]`

Out[22]= {{13, 1}, {73, 1}}

In[23]:= `FactorInteger [276]`

Out[23]= {{2, 2}, {3, 1}, {23, 1}}

In[24]:= `FactorInteger [293]`

Out[24]= {{293, 1}}

b) 471, 945, 325, 930, 166, 269

In[25]:= `FactorInteger [471]`

Out[25]= {{3, 1}, {157, 1}}

In[26]:= `FactorInteger [945]`

Out[26]= {{3, 3}, {5, 1}, {7, 1}}

In[27]:= `FactorInteger [325]`

Out[27]= {{5, 2}, {13, 1}}

In[28]:= `FactorInteger [930]`

Out[28]= {{2, 1}, {3, 1}, {5, 1}, {31, 1}}

In[30]:= `FactorInteger [166]`

Out[30]= {{2, 1}, {83, 1}}

In[31]:= `FactorInteger [269]`

Out[31]= {{269, 1}}

c) 471, 945, 325, 930, 166, 281

In[32]:= `FactorInteger [471]`

Out[32]= {{3, 1}, {157, 1}}

In[33]:= `FactorInteger [945]`

Out[33]= {{3, 3}, {5, 1}, {7, 1}}

In[34]:= `FactorInteger [325]`

Out[34]= {{5, 2}, {13, 1}}

In[35]:= `FactorInteger [930]`

Out[35]= {{2, 1}, {3, 1}, {5, 1}, {31, 1}}

In[36]:= `FactorInteger [166]`

Out[36]= {{2, 1}, {83, 1}}

In[37]:= `FactorInteger [281]`

Out[1]= {{281, 1}}

## Q4. Compute each expression. Do you notice a pattern.

a) 3^6 mod 7

b) 6^10 mod 11

c) 7^20 mod 21

d) 7^22 mod 23

```
In[2]:= Mod[3^6, 7]
Out[2]= 1

In[3]:= Mod[6^10, 11]
Out[3]= 1

In[4]:= Mod[7^20, 21]
Out[4]= 7

In[5]:= Mod[7^22, 23]
Out[5]= 1
```

## Q8. Let M=[{1,1},{1,0}].

```
In[7]:= M = {{1, 1}, {1, 0}}
Out[7]= {{1, 1}, {1, 0}}
```

a) Find M^2, M^3, ...., M^10.

```
In[8]:= MatrixPower[M, 2]
Out[8]= {{2, 1}, {1, 1}}

In[9]:= MatrixPower[M, 3]
Out[9]= {{3, 2}, {2, 1}}

In[10]:= MatrixPower[M, 4]
Out[10]= {{5, 3}, {3, 2}}

In[11]:= MatrixPower[M, 5]
Out[11]= {{8, 5}, {5, 3}}

In[12]:= MatrixPower[M, 6]
Out[12]= {{13, 8}, {8, 5}}

In[13]:= MatrixPower[M, 7]
Out[13]= {{21, 13}, {13, 8}}
```

In[14]:=
```
MatrixPower[M, 8]
```
Out[14]= {{34, 21}, {21, 13}}

In[15]:=
```
MatrixPower[M, 9]
```
Out[15]= {{55, 34}, {34, 21}}

In[16]:=
```
MatrixPower[M, 10]
```
Out[16]= {{89, 55}, {55, 34}}

b) Do your answers suggest the way to compute Fibonacci numbers ? Find the 100th Fibonacci number .

In[18]:=
```
f[0] = 1;
```
In[19]:=
```
f[1] = 1;
```
In[20]:=
```
f[n_] := f[n] = f[n - 2] + f[n - 1]
```
In[21]:=
```
f[100]
```
Out[21]= 573 147 844 013 817 084 101

## Q 9. Find solutions to the following equations or system of equations.

a) Find x, if x^2+x=1.

b) Find x, if x^2+x=-1.

c) Find x and y

4x-3y=5

6x+2y=14

d) Find x, y, z and t.

-2x-2y+3z+t=8

-3x+0y-6z+t=-19

6x-8y+6z+5t=47

x+3y-3z-t=-9

In[22]:=
```
Solve[x ^ 2 + x == 1, x]
```
Out[22]= $\left\{\left\{x \to \frac{1}{2}\left(-1-\sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(-1+\sqrt{5}\right)\right\}\right\}$

In[23]:=
```
Solve[x ^ 2 + x == -1, x]
```
Out[23]= $\{\{x \to -(-1)^{1/3}\}, \{x \to (-1)^{2/3}\}\}$

In[24]:=
```
Solve[4 x - 3 y == 5 && 6 x + 2 y == 14, {x, y}]
```
Out[24]= {{x → 2, y → 1}}

```
In[25]:= Solve[-2 x - 2 y + 3 z + t == 8 && -3 x + 0 y - 6 z + t == -19 &&
           6 x - 8 y + 6 z + 5 t == 47 && x + 3 y - 3 z - t == -9, {x, y, z, t}]
Out[25]= {{x → 2, y → 1, z → 3, t → 5}}
```

Q 10. Some equations are difficult or impossible to solve explicitly, even with software. In such situations , we often resort to numerical methods. Mathematica uses FindRoot, Maple uses fsolve(), and Maxima uses find_root() to find numerical solutions to equations. Here is an example where a numerical approach works well.
Assume that I invest $250 at the beginning of the year $300 at the beginning of the second quarter, $350 at the beginning of the third quarter, and $400 at the beginning of the fourth quarter. At the end of the year, I have $1365 ( because my investments grow) . To find my (continious) rate of return , solve this equation for r:

250Exp[1.0r]+300Exp[0.75r]+350Exp[0.5r]+400Exp[0.25r]=1365

```
In[26]:= FindRoot[250 Exp[1.0 r] + 300 Exp[0.75 r] + 350 Exp[0.5 r] + 400 Exp[0.25 r] == 1365, {r, 0}]
Out[26]= {r → 0.084104}
```

Q 11.

```
In[1]:= mysqrt[n_] := Module[{i = 1, g = 1}, While[i ≤ 20, g = (g + n / g) 2; i = i + 1]; g]

In[2]:= N[mysqrt[2], 6]
Out[2]= 3.37550 × 10^6

In[3]:= N[Sqrt[2], 6]
Out[3]= 1.41421

In[4]:= N[mysqrt[3]]
Out[4]= 4.45334 × 10^6
```

Q 12.

```
In[5]:= Clear[collatz];

In[6]:= collatz[n_] := Which[n == 1, collatz[n] = 0, EvenQ[n],
            collatz[n] = 1 + collatz[n / 2], OddQ[n], collatz[n] = 1 + collatz[3 * n + 1]];

In[7]:= collatz[27]
Out[7]= 111
```