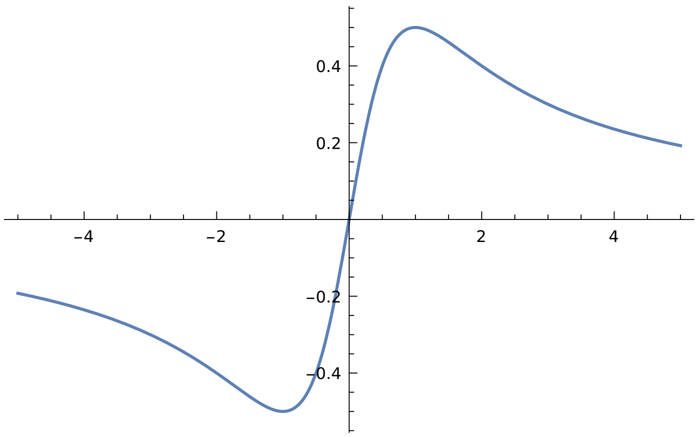
1. Graph each of the functions.

In[1]:=
```
f[x_] := x / (1 + x ^ 2)
```

In[2]:=
```
Plot[f[x], {x, -5, 5}]
```

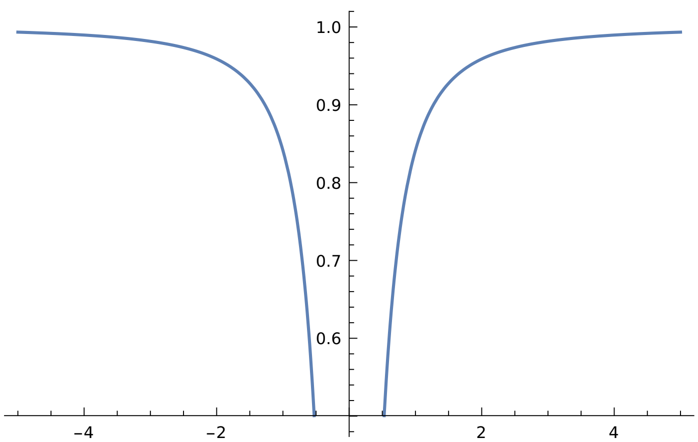
Out[2]=



In[3]:=
```
f[x_] := x Sin[1 / x]
```

In[4]:=
```
Plot[f[x], {x, -5, 5}]
```

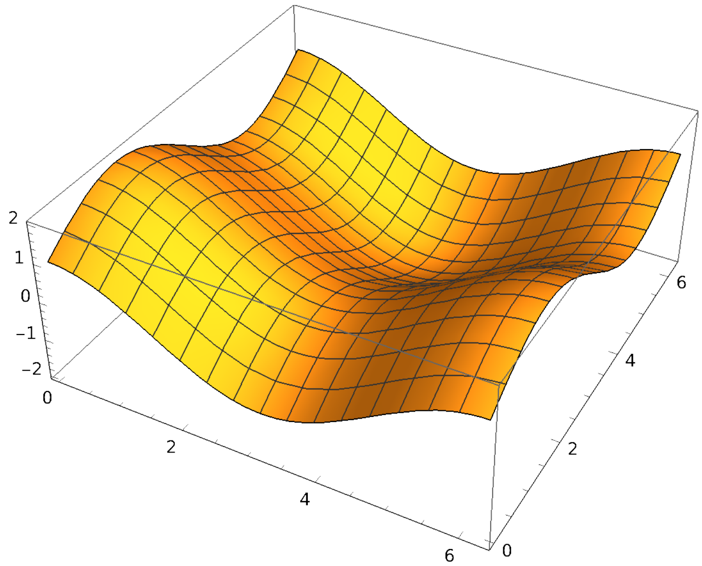
Out[4]=



In[5]:=
```
g[x_, y_] := Cos[x] + Sin[y]
```

In[9]:= `Plot3D[g[x, y], {x, 0, 2 Pi}, {y, 0, 2 Pi}]`

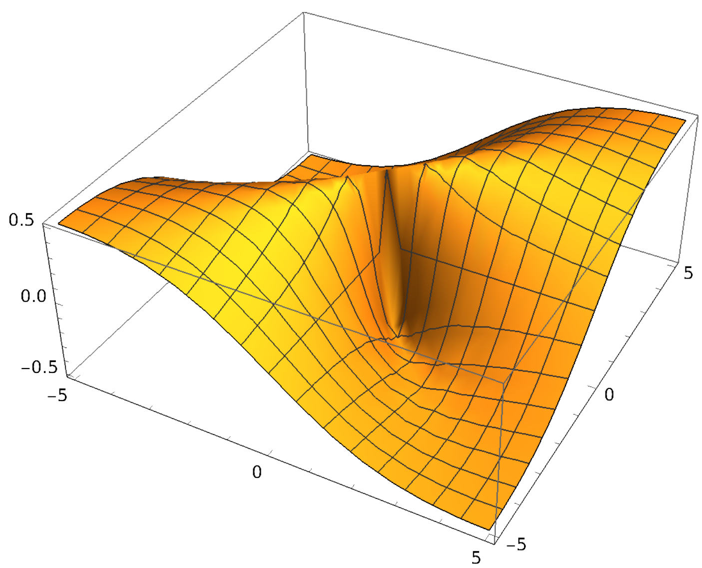
Out[9]=



In[13]:= `f[x_, y_] := (x y) / (x ^ 2 + y ^ 2)`

In[14]:= `Plot3D[f[x, y], {x, −5, 5}, {y, −5, 5}]`

Out[14]=



2. let f(x)= x/1+x^2

In[15]:= `f[x_] := x / (1 + x ^ 2)`

In[16]:= `f '[x]`

Out[16]= $-\frac{2 x^2}{(1 + x^2)^2} + \frac{1}{1 + x^2}$

In[17]:=
```
f''[x]
```

Out[17]=
$$\frac{8\,x^3}{\left(1+x^2\right)^3}-\frac{6\,x}{\left(1+x^2\right)^2}$$

In[18]:=
```
f'[-1]
```

Out[18]= 0

In[19]:=
```
f'[0]
```

Out[19]= 1

In[20]:=
```
f''[0]
```

Out[20]= 0

In[21]:=
```
f''[1]
```

Out[21]=
$$-\frac{1}{2}$$

3. Find the prime factorization of each integer

3, 527, 218, 133, 309, 949, 276, 293

In[22]:=
```
FactorInteger [3]
```

Out[22]= {{3, 1}}

In[23]:=
```
FactorInteger [527]
```

Out[23]= {{17, 1}, {31, 1}}

In[24]:=
```
FactorInteger [218]
```

Out[24]= {{2, 1}, {109, 1}}

In[25]:=
```
FactorInteger [133]
```

Out[25]= {{7, 1}, {19, 1}}

In[26]:=
```
FactorInteger [309]
```

Out[26]= {{3, 1}, {103, 1}}

In[27]:=
```
FactorInteger [949]
```

Out[27]= {{13, 1}, {73, 1}}

In[28]:=
```
FactorInteger [276]
```

Out[28]= {{2, 2}, {3, 1}, {23, 1}}

In[29]:=
```
FactorInteger [293]
```

Out[29]= {{293, 1}}

4. Compute each expression.

3^6 mod 7

6^10 mod 11

7^20 mod 21

7^22 mod 23

```
In[30]:= Mod[3^6, 7]
Out[30]= 1

In[31]:= Mod[6^10, 11]
Out[31]= 1

In[32]:= Mod[7^20, 21]
Out[32]= 7

In[33]:= Mod[7^22, 23]
Out[33]= 1
```

8. Let M= [{1,1},{1,0}]

```
In[34]:= M = {{1, 1}, {1, 0}}
Out[34]= {{1, 1}, {1, 0}}

In[35]:= MatrixPower[M, 2]
Out[35]= {{2, 1}, {1, 1}}

In[36]:= MatrixPower[M, 3]
Out[36]= {{3, 2}, {2, 1}}

In[37]:= MatrixPower[M, 4]
Out[37]= {{5, 3}, {3, 2}}

In[38]:= MatrixPower[M, 5]
Out[38]= {{8, 5}, {5, 3}}

In[39]:= MatrixPower[M, 6]
Out[39]= {{13, 8}, {8, 5}}

In[40]:= MatrixPower[M, 7]
Out[40]= {{21, 13}, {13, 8}}

In[41]:= MatrixPower[M, 8]
Out[41]= {{34, 21}, {21, 13}}

In[42]:= MatrixPower[M, 9]
Out[42]= {{55, 34}, {34, 21}}
```

In[43]:=
```
MatrixPower[M, 10]
```
Out[43]=
```
{{89, 55}, {55, 34}}
```

Do your answers suggest the way to compute Fibonacci numbers ? Find the 100th Fibonacci number

In[44]:=
```
f[0] = 1;
```
In[45]:=
```
f[1] = 1;
```
In[46]:=
```
f[n_] := f[n] = f[n - 2] + f[n - 1]
```
In[47]:=
```
f[100]
```
Out[47]=
```
573 147 844 013 817 084 101
```

9. Find solutions to the following equations.

```
Find x, if x^2 + x = 1
Find x, if x^2 + x = -1
Find x and y
4 x - 3 y = 5
6 x + 2 y = 14
 Find x, y, z and t.
 -2 x - 2 y + 3 z + t = 8
 -3 x + 0 y - 6 z + t = -19
 6 x - 8 y + 6 z + 5 t = 47
 x + 3 y - 3 z - t = -9
```

In[48]:=
```
Solve[x^2 + x == 1, x]
```
Out[48]=

$$\left\{\left\{x \to \frac{1}{2}\left(-1-\sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(-1+\sqrt{5}\right)\right\}\right\}$$

In[49]:=
```
Solve[x^2 + x == -1, x]
```
Out[49]=

$$\left\{\left\{x \to -(-1)^{1/3}\right\}, \left\{x \to (-1)^{2/3}\right\}\right\}$$

In[50]:=
```
Solve[4 x - 3 y == 5 && 6 x + 2 y == 14, {x, y}]
```
Out[50]=
```
{{x → 2, y → 1}}
```

In[51]:=
```
Solve[-2 x - 2 y + 3 z + t == 8 && -3 x + 0 y - 6 z + t == -19 &&
   6 x - 8 y + 6 z + 5 t == 47 && x + 3 y - 3 z - t == -9, {x, y, z, t}]
```
Out[51]=
```
{{x → 2, y → 1, z → 3, t → 5}}
```

10. . Some equations are difficult or impossible to solve explicitly, even with software. In such situations , we often resort to numerical methods. Mathematica uses FindRoot, Maple uses fsolve(), and Maxima uses find_root() to find numerical solutions to equations. Here is an example where a numerical approach works well.
Assume that I invest $250 at the beginning of the year $300 at the beginning of the second quarter, $350 at the beginning of the third quarter, and $400 at the beginning of the fourth quarter. At the end of the

year, I have $1365 (because my investments grow) . To find my (continious) rate of return , solve this equation for r:

250Exp[1.0r]+300Exp[0.75r]+350Exp[0.5r]+400Exp[0.25r]=1365

```
In[52]:= FindRoot[250 Exp[1.0 r] + 300 Exp[0.75 r] + 350 Exp[0.5 r] + 400 Exp[0.25 r] == 1365, {r, 0}]
Out[52]= {r → 0.084104}
```

11.

```
In[53]:= mysqrt[n_] := Module[{i = 1, g = 1}, While[i ≤ 20, g = (g + n / g) 2; i = i + 1]; g]
In[54]:= N[mysqrt[2], 6]
Out[54]= 3.37550 × 10^6
In[56]:= N[Sqrt[2], 6]
Out[56]= 1.41421
In[57]:= N[mysqrt[3]]
Out[57]= 4.45334 × 10^6
```

12.

```
In[58]:= Clear[collatz];
In[61]:= collatz[n_] := Which[n == 1, collatz[n] = 0, EvenQ[n],
           collatz[n] = 1 + collatz[n / 2], OddQ[n], collatz[n] = 1 + collatz[3 * n + 1]];
In[62]:= collatz[27]
Out[62]= 111
```