

CHAPTER 12

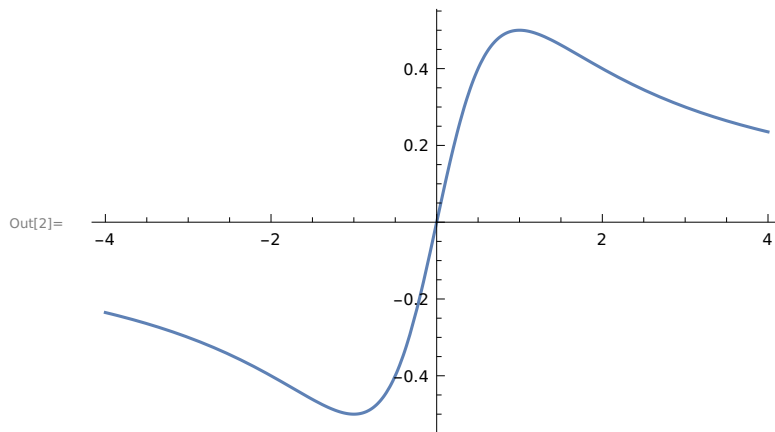
Getting Started with Mathematica.

1. Graph each of the functions. Experiment with different domains or viewpoints to display the best images.

a) $f(x) = x/(1+x^2)$

```
In[1]:= f[x_] := x / (1 + x ^ 2);
```

```
In[2]:= Plot[f[x], {x, -4, 4}]
```



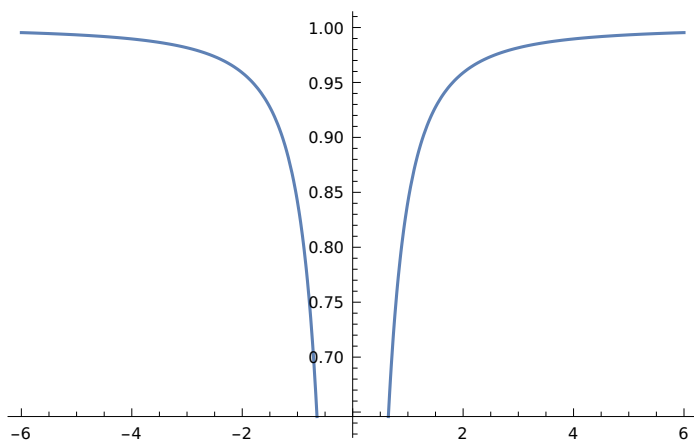
b) $y = x \sin [1/x]$

```
In[3]:= ClearAll[f, x];
```

```
In[4]:= f[x_] := x * Sin[1 / x];
```

In[5]:= `Plot[f[x], {x, -6, 6}]`

Out[5]=

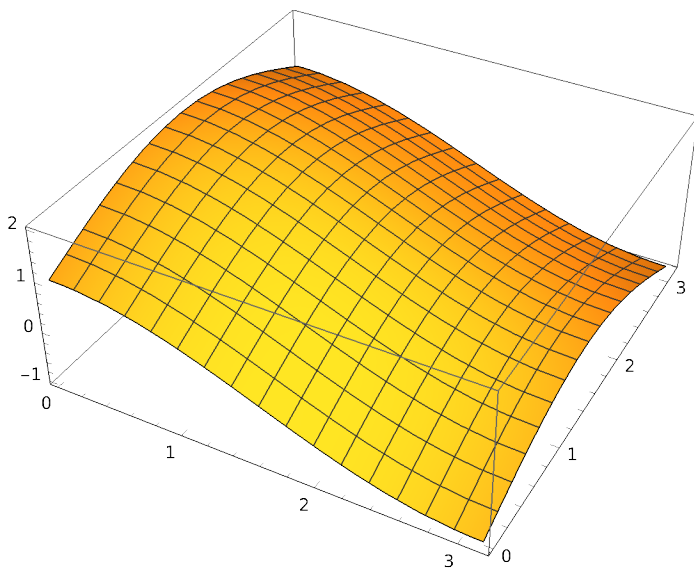


c) $g(x,y) = \cos[x] + \sin[y]$

In[6]:= `g[x_, y_] := Cos[x] + Sin[y];`

In[7]:= `Plot3D[g[x, y], {x, 0, Pi}, {y, 0, Pi}]`

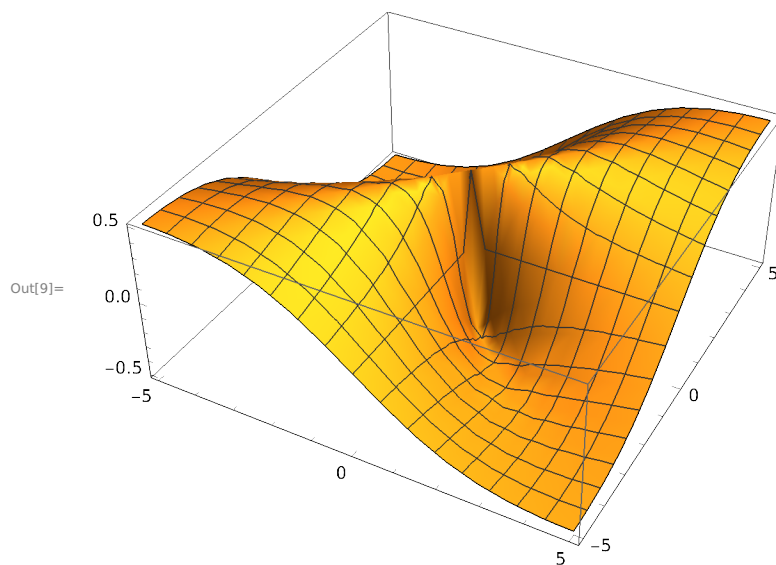
Out[7]=



d) $z = x y / (x^2 + y^2)$

In[8]:= `h[x_, y_] := x * y / (x^2 + y^2);`

In[9]:= `Plot3D[h[x, y], {x, -5, 5}, {y, -5, 5}]`



2. Let $f(x) = x/(1+x^2)$

In[10]:= `f[x_] := x / (1 + x ^ 2);`

a) Find $f'(x)$ and $f''(x)$.

In[11]:= `f'[x]`

Out[11]=
$$-\frac{2x^2}{(1+x^2)^2} + \frac{1}{1+x^2}$$

In[12]:= `f''[x]`

Out[12]=
$$\frac{8x^3}{(1+x^2)^3} - \frac{6x}{(1+x^2)^2}$$

b) Find $f'(-1)$ and $f'(0)$.

In[13]:= `f'[-1]`

Out[13]= 0

In[14]:= `f'[0]`

Out[14]= 1

c) Find $f''(0)$ and $f''(1)$.In[15]:= $f''[0]$

Out[15]= 0

In[16]:= $f''[1]$ Out[16]= $-\frac{1}{2}$ In[17]:= `ClearAll[f, x]`**3. Find the prime factorization of each integer.****a) 3, 527, 218, 133, 309, 949, 276, 293.**In[18]:= `FactorInteger[3 527 218 133 309 949 276 293]`

Out[18]= {{15 013, 2}, {25 013, 3}}

b) 471, 945, 325, 930, 166, 269.In[19]:= `FactorInteger[471 945 325 930 166 269]`

Out[19]= {{4211, 1}, {34 589, 1}, {46 747, 1}, {69 313, 1}}

c) 471, 945, 325, 930, 166, 281In[20]:= `FactorInteger[471 945 325 930 166 281]`

Out[20]= {{471 945 325 930 166 281, 1}}

4. Compute each expression. Do you notice a pattern.**a) $3^6 \text{ Mod } 7$** In[21]:= `Mod[3^6, 7]`

Out[21]= 1

b) $6^{10} \text{ Mod } 11$ In[22]:= `Mod[6^10, 11]`

Out[22]= 1

c) $7^{20} \bmod 21$

In[23]:= **Mod[7 ^ 20, 21]**

Out[23]= 7

d) $7^{22} \bmod 23$

In[24]:= **Mod[7 ^ 22, 23]**

Out[24]= 1

8. Let $M = \{\{1,1\}, \{1,0\}\}$

In[25]:= **M = {{1, 1}, {1, 0}}**

Out[25]= {{1, 1}, {1, 0}}

a) Find $M^2, M^3, \dots, M^{10}$.

In[26]:= **MatrixPower [M, 2]**

Out[26]= {{2, 1}, {1, 1}}

In[27]:= **MatrixPower [M, 3]**

Out[27]= {{3, 2}, {2, 1}}

In[28]:= **MatrixPower [M, 4]**

Out[28]= {{5, 3}, {3, 2}}

In[29]:= **MatrixPower [M, 5]**

Out[29]= {{8, 5}, {5, 3}}

In[30]:= **MatrixPower [M, 6]**

Out[30]= {{13, 8}, {8, 5}}

In[31]:= **MatrixPower [M, 7]**

Out[31]= {{21, 13}, {13, 8}}

In[32]:= **MatrixPower [M, 8]**

Out[32]= {{34, 21}, {21, 13}}

In[33]:= **MatrixPower [M, 9]**

Out[33]= {{55, 34}, {34, 21}}

```
In[34]:= MatrixPower [M, 10]
Out[34]= {{89, 55}, {55, 34}}
```

b) Do your answers suggest the way to complete Fibonacci numbers ? Find the 100th Fibonacci number.

```
In[35]:= f[0] = 1;
          f[1] = 1;
          f[n_] := f[n] = f[n - 2] + f[n - 1]

In[38]:= f[100]
Out[38]= 573 147 844 013 817 084 101
```

9. Find solutions to the following equations or systems of equations. Hint: If you use Mathematica, the command you are looking for is Solve. Maple and Maxima both use solve() to find algebraic solutions. Check the help for examples and syntax.

a) Find x, if $x^2 + x = 1$.

```
In[39]:= Solve[x^2 + x == 1, x]
Out[39]= {{x -> -1/2 - Sqrt[5]/2}, {x -> -1/2 + Sqrt[5]/2}}
```

b) Find x, if $x^2 + x = -1$.

```
In[40]:= Solve[x^2 + x == -1, x]
Out[40]= {{x -> -(-1)^(1/3)}, {x -> (-1)^(2/3)}}
```

c) Find x and y.

$$4x - 3y = 5$$

$$6x + 2y = 14$$

```
In[41]:= Solve[4 x - 3 y == 5 && 6 x + 2 y == 14, {x, y}]
Out[41]= {{x -> 2, y -> 1}}
```

d) Find x, y, z and t.

$$-2x - 2y + 3z + t = 8$$

$$-3x + 0y - 6z + t = -19$$

$$6x - 8y + 6z + 5t = 47$$

$$x + 3y - 3z - t = -9$$

```
In[42]:= Solve[-2 x - 2 y + 3 z + t == 8 && -3 x + 0 y - 6 z + t == -19 &&
        6 x - 8 y + 6 z + 5 t == 47 && x + 3 y - 3 z - t == -9, {x, y, z, t}]
```

```
Out[42]= {{x -> 2, y -> 1, z -> 3, t -> 5}}
```

10. Some equations are difficult or impossible to solve explicitly, even with software. In such situations, we often resort to numerical methods. Mathematica uses FindRoot, Maple uses fsolve(), and Maxima uses find_root() to find numerical solutions to equations. Here is an example where a numerical approach works well. Assume that I invest \$250 at the beginning of the year, \$300 at the beginning of the second quarter, \$350 at the beginning of the third quarter, and \$400 at the beginning of the fourth quarter. At the end of the year, I have \$1365 (because my investments grow). To find my (continuous) rate of return, solve this equation for r .

$$250e^{(1.0)r} + 300e^{(0.75)r} + 350e^{(0.5)r} + 400e^{(0.25)r} = 1365$$

```
In[43]:= FindRoot[250 Exp[1.0 r] + 300 Exp[0.75 r] + 350 Exp[0.5 r] + 400 Exp[0.25 r] == 1365, {r, 0}]
```

```
Out[43]= {r -> 0.084104}
```

11. If n is a positive number, and $g > 0$ is any "guess" for the square root of n , then a better estimate of $\text{Sqrt}[n]$ is the average of g and n/g , i.e. $(g + n/g)/2$. Write a function called mysqrt that accepts one argument, begins with an initial guess of 1.0, finds 20 new guesses, and returns the answer.

```
In[44]:= mysqrt[n_] := Module[{i = 1, g = 1}, While[i <= 20, g = (g + n/g)/2; i = i + 1]; g]
```

```
In[45]:= N[mysqrt[2], 6]
```

```
Out[45]= 1.41421
```

```
In[46]:= N[Sqrt[2], 6]
```

```
Out[46]= 1.41421
```

```
In[47]:= N[mysqrt[3]]
```

```
Out[47]= 1.73205
```

12. The Collatz conjecture states that if we start from any natural number $a_0 = n$ and form a sequence by the rule :
 $a_{i+1} = \{a_i/2 \text{ if } a_i \text{ is even \& } 3a_i+1 \text{ if } a_i \text{ is odd.}$
then the sequence eventually contains the value 1. For example, starting from $a_0 = 6$, we get the sequence 6,3, 10, 5,16,8,4,2,1 (we reached 1 after eight steps).

a. Write a (recursive) function called collatz that accepts a single argument, n , and returns:

- 0 if n is equal to 1
- $1 + \text{collatz}(n/2)$ if n is even
- $1 + \text{collatz}(3*n+1)$ if n is odd

Thus, $\text{collatz}(n)$ is the number of steps needed to go from n to 1.

```
In[48]:= Clear[collatz];
```

```
In[49]:= collatz[n_] := Which[n == 1, collatz[n] = 0, EvenQ[n],  
collatz[n] = 1 + collatz[n/2], OddQ[n], collatz[n] = 1 + collatz[3 * n + 1]];
```

b. Verify the values:

$n \quad \text{collatz}(n)$

1 : 0

2 : 1

6 : 8

27 : 111

```
In[50]:= collatz[1]
```

```
Out[50]= 0
```

```
In[51]:= collatz[2]
```

```
Out[51]= 1
```

```
In[52]:= collatz[6]
```

```
Out[52]= 8
```

```
In[53]:= collatz[27]
```

```
Out[53]= 111
```