# Assignment

Chapter-12

- MAT/19/99
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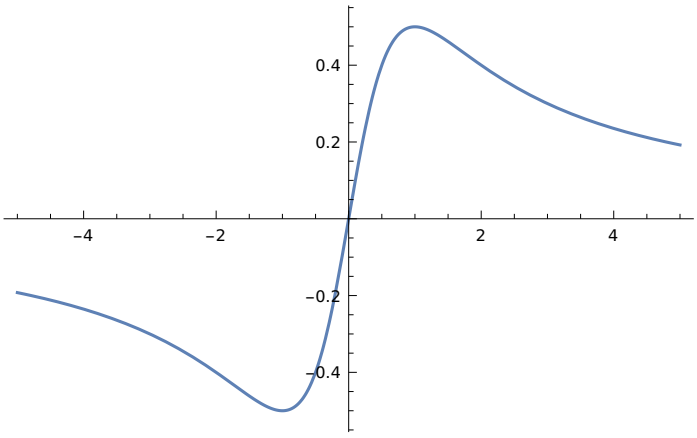
## QUES 1 : Graph Each Of The Functions:

### a) f(x)=x/(1+x^2)

In[1]:=
```
f[x_] := x / (1 + x ^ 2)
Plot[f[x], {x, -5, 5}]
```

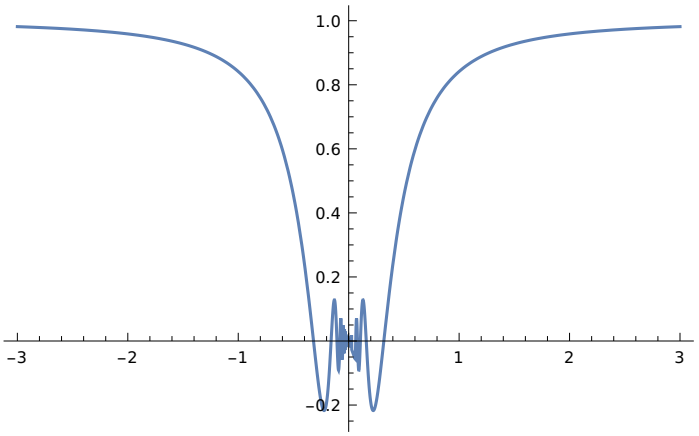
Out[2]=



## b) y=xsin(1/x)

In[5]:=
```
f[x_] := x Sin[1 / x]
Plot[f[x], {x, -3, 3}]
```

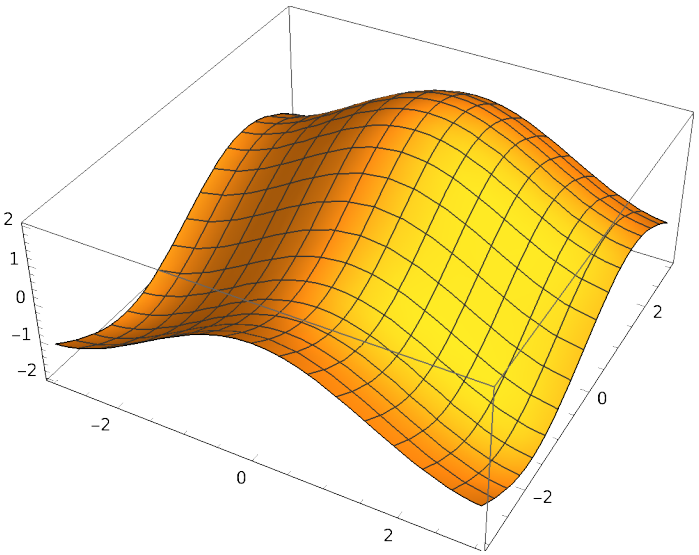
Out[6]=




## c) g(x,y)=cos(x)+sin(y)

In[9]:=
```
f[x_, y_] := Cos[x] + Sin[y]
Plot3D[f[x, y], {x, -3, 3}, {y, -3, 3}]
```

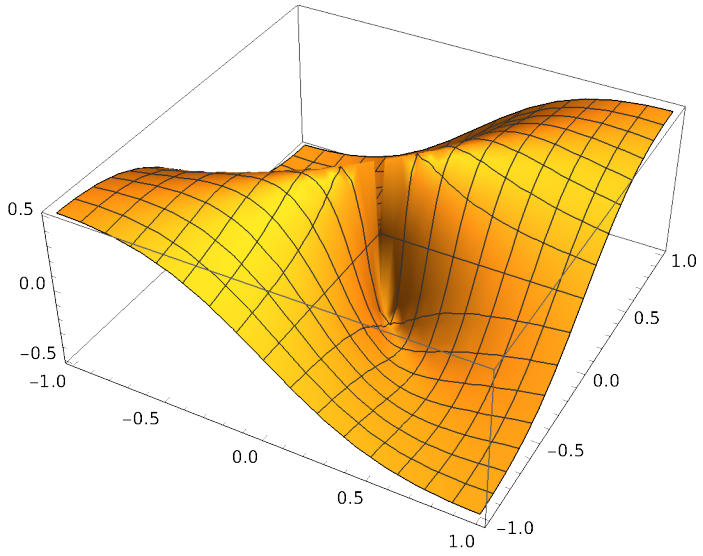
Out[10]=



## d) z=xy/(x^2+y^2)

In[11]:=
```
f[x_, y_] := x y / (x ^ 2 + y ^ 2)
Plot3D[f[x, y], {x, -1, 1}, {y, -1, 1}]
```

Out[12]=




## QUES 2 : let f(x)= x/(1+x^2)

## a) Find f'(x) and f''(x)

In[13]:=
```
f[x_] := x / (1 + x ^ 2)
f'[x]
```

Out[14]=
$$-\frac{2\,x^2}{(1+x^2)^2}+\frac{1}{1+x^2}$$

In[15]:=
```
f''[x]
```

Out[15]=
$$\frac{8\,x^3}{(1+x^2)^3}-\frac{6\,x}{(1+x^2)^2}$$

## b) Find f'(-1) and f'(0)

In[16]:=
```
f'[x] /. {x → -1}
```

Out[16]=
```
0
```

In[17]:=
```
f'[-1]
```

Out[17]=
```
0
```

In[18]:=
```
f'[0]
```

Out[18]=
```
1
```

## c) Find f''(0) and f''(1)

In[19]:=
```
f''[0]
```

Out[19]= 0

In[20]:=
```
f''[1]
```

Out[20]= $-\frac{1}{2}$

## QUES 3: Find the prime factorixation of each integer:

## a) 3,527,218,133,309,949,276,293

In[22]:=
```
FactorInteger [3 527 218 133 309 949 276 293 ]
```

Out[22]= {{15 013 , 2}, {25 013 , 3}}

## b) 471,945,325,930,166,269

In[23]:=
```
FactorInteger [471 945 325 930 166 269 ]
```

Out[23]= {{4211 , 1}, {34 589 , 1}, {46 747 , 1}, {69 313 , 1}}

## c) 471,945,325,930,166,281

In[24]:=
```
FactorInteger [471 945 325 930 166 281 ]
```

Out[24]= {{471 945 325 930 166 281 , 1}}

## QUES 4 : Compute each expression:

## a) 3^6 mod 7

In[25]:=
```
Mod[3 ^ 6, 7]
```

Out[25]= 1

## b) 6^10 mod 11

In[26]:=
```
Mod[6 ^ 10, 11]
```

Out[26]= 1

## c) 7^20 mod 21

In[28]:=
```
Mod[7 ^ 20, 21]
```

Out[28]= 7

## d) 7^22 mod 23

In[29]:=
```
Mod[7^22, 2]
```

Out[29]=
```
1
```

## QUES 8 :

In[30]:=
```
M = {{1, 1}, {1, 0}}
MatrixForm[M]
```

Out[30]=
```
{{1, 1}, {1, 0}}
```

Out[31]//MatrixForm=

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

## a) Find M 2, M 3,....,M 10

## Now at every step after defining function F, F[n] is nothing but M^n:

In[32]:=
```
F[n_] := MatrixPower[M, (n - 1)].M
List[F[2], F[3], F[4], F[5], F[6], F[7], F[8], F[9], F[10]]
```

Out[33]=
```
{{{2, 1}, {1, 1}}, {{3, 2}, {2, 1}}, {{5, 3}, {3, 2}}, {{8, 5}, {5, 3}}, {{13, 8}, {8, 5}},
 {{21, 13}, {13, 8}}, {{34, 21}, {21, 13}}, {{55, 34}, {34, 21}}, {{89, 55}, {55, 34}}}
```

In[34]:=
```
MatrixForm[F[2]]
```

Out[34]//MatrixForm=

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

In[35]:=
```
MatrixForm[F[3]]
```

Out[35]//MatrixForm=

$$\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$$

In[36]:=
```
MatrixForm[F[4]]
```

Out[36]//MatrixForm=

$$\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$$

In[37]:=
```
MatrixForm[F[5]]
```

Out[37]//MatrixForm=

$$\begin{pmatrix} 8 & 5 \\ 5 & 3 \end{pmatrix}$$

In[38]:=
```
MatrixForm[F[6]]
```
Out[38]//MatrixForm=
$$\begin{pmatrix} 13 & 8 \\ 8 & 5 \end{pmatrix}$$

In[39]:=
```
MatrixForm[F[7]]
```
Out[39]//MatrixForm=
$$\begin{pmatrix} 21 & 13 \\ 13 & 8 \end{pmatrix}$$

In[40]:=
```
MatrixForm[F[8]]
```
Out[40]//MatrixForm=
$$\begin{pmatrix} 34 & 21 \\ 21 & 13 \end{pmatrix}$$

In[41]:=
```
MatrixForm[F[9]]
```
Out[41]//MatrixForm=
$$\begin{pmatrix} 55 & 34 \\ 34 & 21 \end{pmatrix}$$

In[42]:=
```
MatrixForm[F[10]]
```
Out[42]//MatrixForm=
$$\begin{pmatrix} 89 & 55 \\ 55 & 34 \end{pmatrix}$$

In[67]:=
```
ClearAll
```
Out[67]=
```
ClearAll
```

## b) Find the 100th Fibonacci number.

In[47]:=
```
f[0] = 1;
f[1] = 1;
f[n_] := f[n] = f[n – 2] + f[n – 1]
f[100]
```
Out[50]=
```
573 147 844 013 817 084 101
```

## QUES 9 : Find solutions to the following equations or system of equations:

## a) Find x, if x^2+x=1

In[51]:=
```
Solve[x^2 + x == 1, x]
```
Out[51]=
$$\left\{\left\{x \to \frac{1}{2}\left(-1-\sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(-1+\sqrt{5}\right)\right\}\right\}$$

## b) Find x, if x^2+x=-1

In[52]:=
```
Solve[x^2 + x == -1, x]
```
Out[52]=
$$\{\{x \to -(-1)^{1/3}\}, \{x \to (-1)^{2/3}\}\}$$

## c) Find x and y:
## 4x-3y=5
## 6x+2y=14

In[53]:=
```
Solve[4 x - 3 y == 5 && 6 x + 2 y == 14]
```
Out[53]=
$$\{\{x \to 2, y \to 1\}\}$$

## d) Find x,y,z and t
## -2x-2y+3z+t=8
## -3x+0y-6z+t=-19
## 6x-8y+6z+5t=47
## x+3y-3z-t=-9

In[54]:=
```
Solve[-2 x - 2 y + 3 z + t == 8 &&
 -3 x + 0 y - 6 z + t == -19 && 6 x - 8 y + 6 z + 5 t == 47 && x + 3 y - 3 z - t == -9]
```
Out[54]=
$$\{\{t \to 5, x \to 2, y \to 1, z \to 3\}\}$$

```
ClearAll[x, y, z, t]
```

## QUES 10 : Assume that I invest $250 at the beginning of the year, $300 at the beginning of the second quarter, $350 at the beginning of the third quarter and $400 at the beginning of the fourth quarter. At the end of the year, I have $1365 (because my investments grow). To find my rate of return, solve this equation for r:
## 250e^1.0r + 300e^0.75r + 350e^0.5r + 400e^0.25r=1365

In[55]:=
```
eqn1 = 250 * Exp[1.0 r] + 300 * Exp[0.75 r] + 350 * Exp[0.5 r] + 400 * Exp[0.25 r] == 1365
```
Out[55]=
$$400\, e^{0.25\, r} + 350\, e^{0.5\, r} + 300\, e^{0.75\, r} + 250\, e^{1.\, r} == 1365$$

In[56]:=
```
FindRoot[eqn1, {r, 1}]
```
Out[56]=
$$\{r \to 0.084104\}$$

## QUES 11 :

In[57]:=
```
mysqrt[n_] := Module[{i = 1, g = 1}, While[i ≤ 20, g = (g + n / g) / 2; i = i + 1]; g]
N[mysqrt[2]]
```

Out[58]=
```
1.41421
```

In[59]:=
```
N[mysqrt[10]]
```

Out[59]=
```
3.16228
```

## QUES 12 : a)

In[60]:=
```
collatz[n_] := Which[n == 1, collatz[n] = 0, EvenQ[n],
collatz[n] = 1 + collatz[n / 2], OddQ[n], collatz[n] = 1 + collatz[3 * n + 1]];
collatz[15]
```

Out[61]=
```
17
```

## b)

In[62]:=
```
collatz[1]
```

Out[62]=
```
0
```

In[63]:=
```
collatz[2]
```

Out[63]=
```
1
```

In[64]:=
```
collatz[6]
```

Out[64]=
```
8
```

In[65]:=
```
collatz[27]
```

Out[65]=
```
111
```