ASSIGNMENT

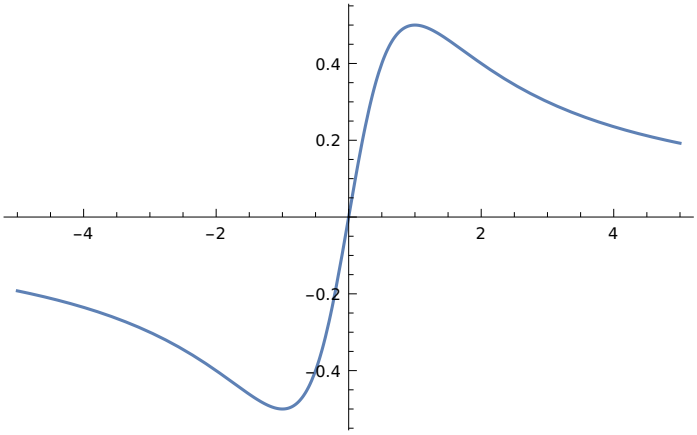
(CHAPTER-12)

QUESTION 1: Graph each of the functions:

a) f(x) = x/(1+x^2)

In[62]:=
```
f[x_] := x / (1 + x ^ 2)
Plot[f[x], {x, -5, 5}]
```

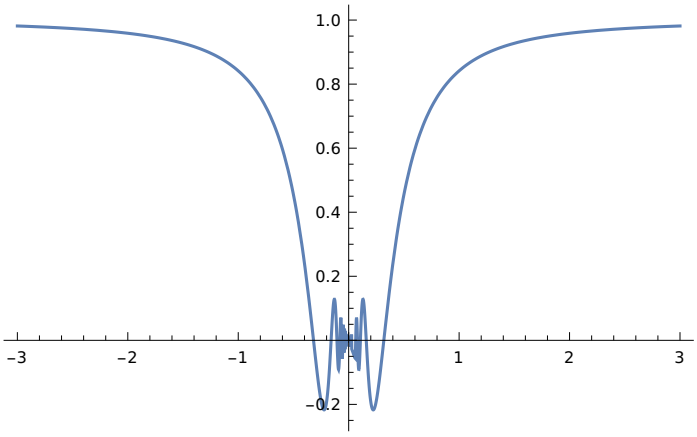
Out[63]=



b) y = x sin(1/x)

In[52]:=
```
f[x_] := x Sin[1 / x]
Plot[f[x], {x, -3, 3}]
```

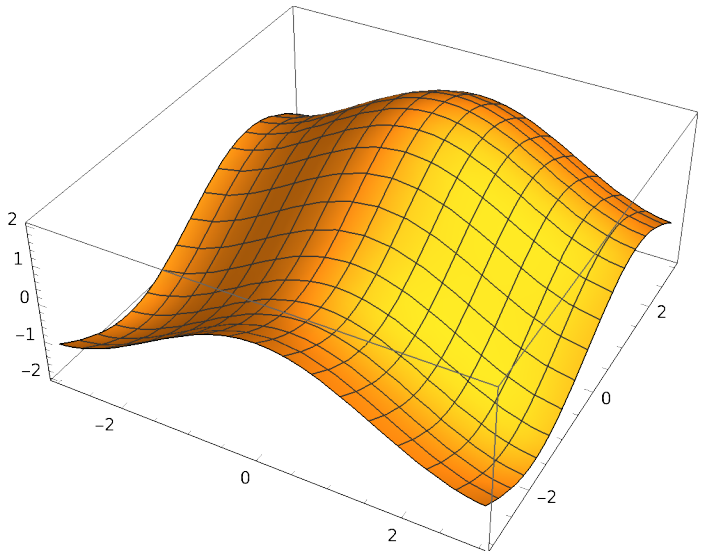
Out[53]=



c) g(x, y) = cos(x) + sin(y)

In[19]:=
```
f[x_, y_] := Cos[x] + Sin[y]
Plot3D[f[x, y], {x, -3, 3}, {y, -3, 3}]
```

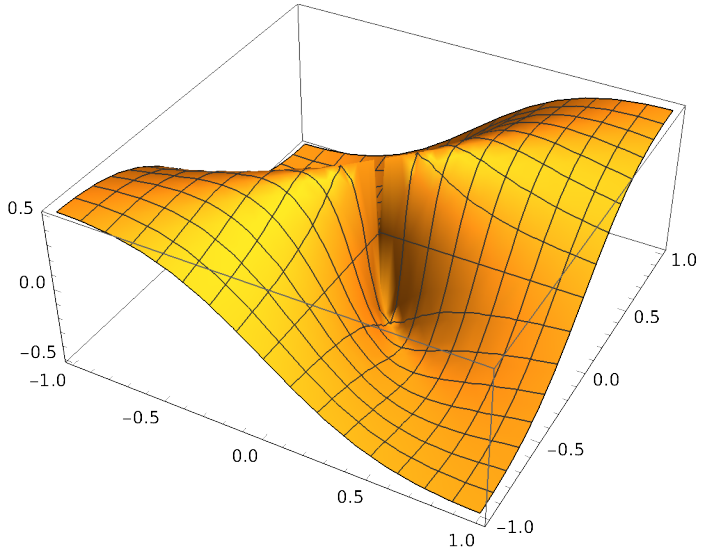
Out[20]=



d) z = x y/(x^2 +y^2)

In[21]:=
```
f[x_, y_] := x y / (x ^ 2 + y ^ 2)
Plot3D[f[x, y], {x, -1, 1}, {y, -1, 1}]
```

Out[22]=



QUESTION 2: Let f(x) = x/(1+x^2)

a)Find f'(x) and f''(x)

In[23]:=
```
f[x_] := x / (1 + x ^ 2)
f'[x]
```

Out[24]=
$$-\frac{2\,x^2}{(1+x^2)^2}+\frac{1}{1+x^2}$$

In[19]:=
```
f''[x]
```

Out[19]=
$$\frac{8\,x^3}{(1+x^2)^3}-\frac{6\,x}{(1+x^2)^2}$$

b) Find f'(-1) and f'(0)

In[25]:=
```
f'[x] /. {x → -1}
```

Out[25]=
```
0
```

In[26]:=
```
f'[-1]
```

Out[26]=
```
0
```

In[27]:=
```
f'[0]
```

Out[27]=
```
1
```

c) Find f''(0) and f''(1)

In[22]:=
```
f''[0]
```

Out[22]=
```
0
```

In[23]:=
```
f''[1]
```

Out[23]=
$$-\frac{1}{2}$$

QUESTION 3: Find the prime factorization of each integer:

a)3,527,218,133,309,949,276,293

In[1]:=
```
FactorInteger [3 527 218 133 309 949 276 293 ]
```

Out[1]=
```
{{15 013 , 2}, {25 013 , 3}}
```

b) 471,945,325,930,166,269

In[39]:=
```
FactorInteger[471 945 325 930 166 269]
```

Out[39]=
```
{{4211, 1}, {34 589, 1}, {46 747, 1}, {69 313, 1}}
```

c)471,945,325,930,166,281

In[3]:=
```
FactorInteger[471 945 325 930 166 281]
```

Out[3]=
```
{{471 945 325 930 166 281, 1}}
```

QUESTION 4: Compute each expression:

a) 3^6 mod 7

In[40]:=
```
Mod[3^6, 7]
```

Out[40]=
```
1
```

b)6^10 mod 11

In[41]:=
```
Mod[6^10, 11]
```

Out[41]=
```
1
```

c)7^20 mod 21

In[42]:=
```
Mod[7^20, 21]
```

Out[42]=
```
7
```

d)7^22 mod 23

In[43]:=
```
Mod[7^22, 23]
```

Out[43]=
```
1
```

QUESTION 8:

In[31]:=
```
M = {{1, 1}, {1, 0}}
MatrixForm[M]
```

Out[31]=
```
{{1, 1}, {1, 0}}
```

Out[32]//MatrixForm=

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

(a) Find M2, M3, . . . , M10

Now at evey step after defining function F , F[n]is nothing but M^n :

In[33]:=
```
F[n_] := MatrixPower[M, (n - 1)].M
```

In[34]:=
```
List[F[2], F[3], F[4], F[5], F[6], F[7], F[8], F[9], F[10]]
```

Out[34]=
```
{{{2, 1}, {1, 1}}, {{3, 2}, {2, 1}}, {{5, 3}, {3, 2}}, {{8, 5}, {5, 3}}, {{13, 8}, {8, 5}},
 {{21, 13}, {13, 8}}, {{34, 21}, {21, 13}}, {{55, 34}, {34, 21}}, {{89, 55}, {55, 34}}}
```

In[35]:=
```
MatrixForm[F[2]]
```

Out[35]//MatrixForm=

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

In[37]:=
```
MatrixForm[F[3]]
```

Out[37]//MatrixForm=

$$\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$$

In[38]:=
```
MatrixForm[F[4]]
```

Out[38]//MatrixForm=

$$\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$$

In[39]:=
```
MatrixForm[F[5]]
```

Out[39]//MatrixForm=

$$\begin{pmatrix} 8 & 5 \\ 5 & 3 \end{pmatrix}$$

In[40]:=
```
MatrixForm[F[6]]
```

Out[40]//MatrixForm=

$$\begin{pmatrix} 13 & 8 \\ 8 & 5 \end{pmatrix}$$

In[42]:=
```
MatrixForm[F[7]]
```

Out[42]//MatrixForm=

$$\begin{pmatrix} 21 & 13 \\ 13 & 8 \end{pmatrix}$$

In[43]:=
```
MatrixForm[F[8]]
```

Out[43]//MatrixForm=

$$\begin{pmatrix} 34 & 21 \\ 21 & 13 \end{pmatrix}$$

In[44]:=
```
MatrixForm[F[9]]
```

Out[44]//MatrixForm=

$$\begin{pmatrix} 55 & 34 \\ 34 & 21 \end{pmatrix}$$

In[46]:=
```
MatrixForm[F[10]]
```

Out[46]//MatrixForm=

$$\begin{pmatrix} 89 & 55 \\ 55 & 34 \end{pmatrix}$$

In[22]:=
```
clear[all]
```

Out[22]=
```
clear[all]
```

b) Find the 100th Fibonacci number.

In[23]:=
```
f[0] = 1;
f[1] = 1;
f[n_] := f[n] = f[n - 2] + f[n - 1]
f[100]
```

Out[26]=
```
573 147 844 013 817 084 101
```

QUESTION 9: Find solutions to the following equations or systems of equations:

a)Find x,if x^2 + x = 1

In[55]:=
```
Solve[x^2 + x == 1, x]
```

Out[55]=

$$\left\{\left\{x \to \frac{1}{2}\left(-1 - \sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(-1 + \sqrt{5}\right)\right\}\right\}$$

b)Find x,if x^2 + x = –1

In[56]:=
```
Solve[x^2 + x == -1, x]
```

Out[56]=

$$\{\{x \to -(-1)^{1/3}\}, \{x \to (-1)^{2/3}\}\}$$

c)Find x and y:

4x – 3y = 5

6x + 2y = 14

In[57]:=
```
Solve[4 x - 3 y == 5 && 6 x + 2 y == 14]
```

Out[57]=

$$\{\{x \to 2, y \to 1\}\}$$

d)Find x,y,z and t

–2x – 2y + 3z + t = 8

–3x + 0y – 6z + t = –19

6x – 8y + 6z + 5t = 47

x + 3y – 3z – t = –9

In[58]:=
```
Solve[-2 x - 2 y + 3 z + t == 8 &&
  -3 x + 0 y - 6 z + t == -19 && 6 x - 8 y + 6 z + 5 t == 47 && x + 3 y - 3 z - t == -9]
```

Out[58]=
```
{{t → 5, x → 2, y → 1, z → 3}}
```

In[59]:=
```
ClearAll[x, y, z, t]
```

QUESTION 10:Assume that I invest $250 at the beginning of the year, $300 at the beginning of the second quarter, $350 at the beginning of the third quarter, and $400 at the beginning of the fourth quarter. At the end of the year, I have $1365 (because my investments grow). To find my (continuous) rate of return, solve this equation for r:
250e^1.0r + 300e^0.75r + 350e^0.5r + 400e^0.25r = 1365

In[60]:=
```
eqn1 = 250 * Exp[1.0 r] + 300 * Exp[0.75 r] + 350 * Exp[0.5 r] + 400 * Exp[0.25 r] == 1365
```

Out[60]=
$$400\, e^{0.25\, r} + 350\, e^{0.5\, r} + 300\, e^{0.75\, r} + 250\, e^{1.\, r} == 1365$$

In[61]:=
```
FindRoot[eqn1, {r, 1}]
```

Out[61]=
```
{r → 0.084104}
```

QUESTION 11:

In[27]:=
```
mysqrt[n_] := Module[{i = 1, g = 1}, While[i ≤ 20, g = (g + n / g) / 2; i = i + 1]; g]
```

In[28]:=
```
N[mysqrt[2]]
```

Out[28]=
```
1.41421
```

In[29]:=
```
N[mysqrt[10]]
```

Out[29]=
```
3.16228
```

QUESTION 12:

a)

In[32]:=
```
collatz[n_] := Which[n == 1, collatz[n] = 0, EvenQ[n],
    collatz[n] = 1 + collatz[n / 2], OddQ[n], collatz[n] = 1 + collatz[3 * n + 1]];
```

In[34]:=
```
collatz[15]
```

Out[34]=
```
17
```

b)

In[35]:=
```
collatz[1]
```

Out[35]=
```
0
```

In[36]:=
```
collatz[2]
```

Out[36]=
```
1
```

In[37]:=
```
collatz[6]
```

Out[37]=
```
8
```

In[38]:=
```
collatz[27]
```

Out[38]=
```
111
```