

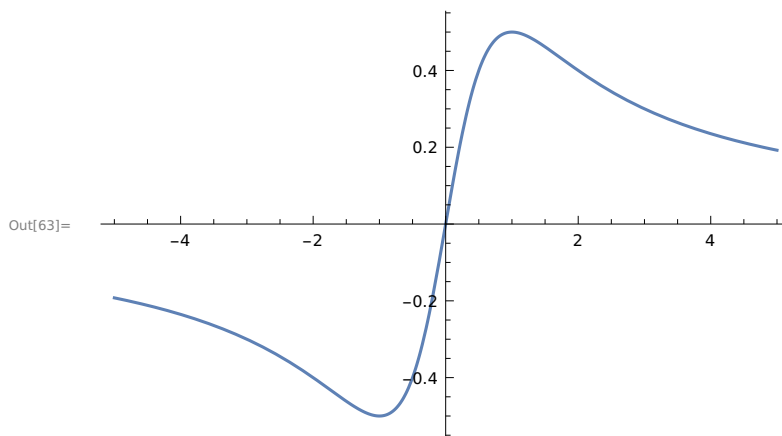
ASSIGNMENT

(CHAPTER-12)

QUESTION 1: Graph each of the functions:

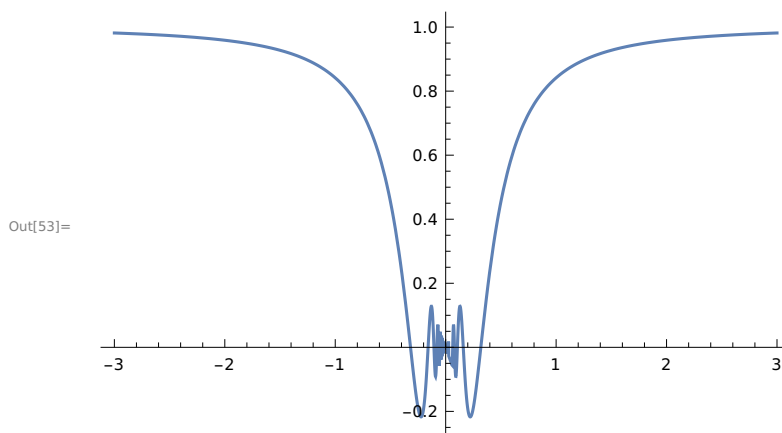
a) $f(x) = x/(1+x^2)$

```
In[62]:= f[x_] := x / (1 + x ^ 2)  
Plot[f[x], {x, -5, 5}]
```



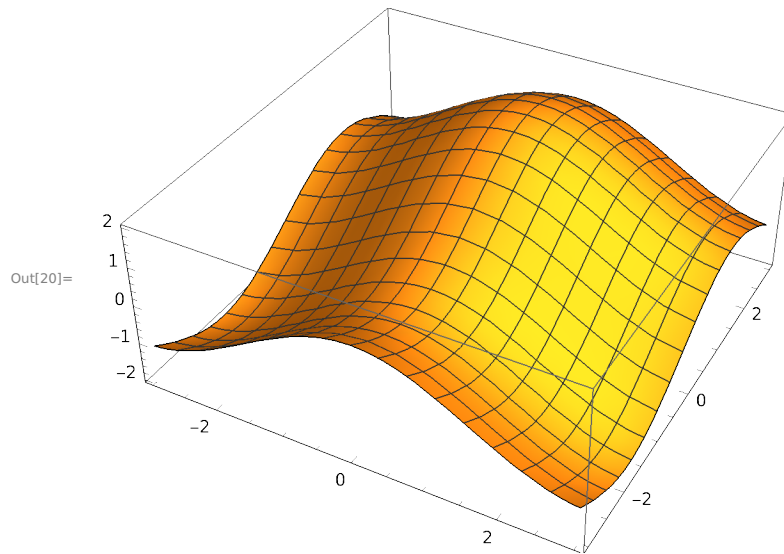
b) $y = x \sin(1/x)$

```
In[52]:= f[x_] := x Sin[1 / x]  
Plot[f[x], {x, -3, 3}]
```



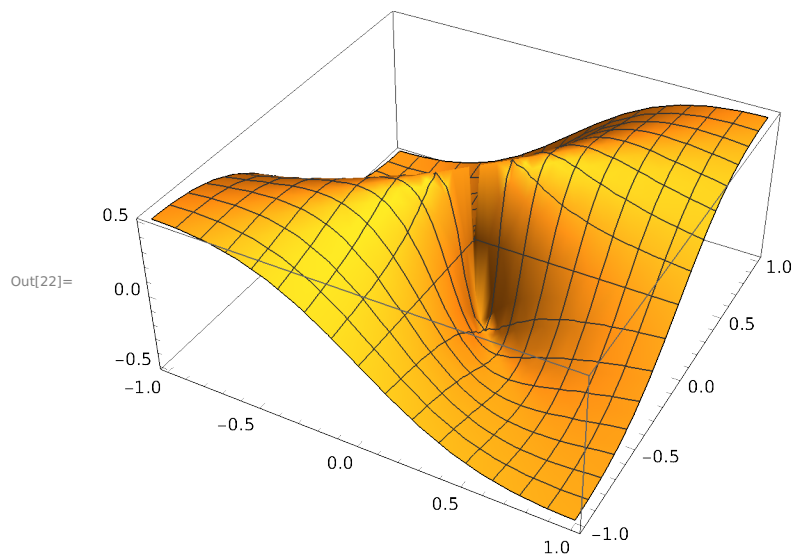
c) $g(x, y) = \cos(x) + \sin(y)$

```
In[19]:= f[x_, y_] := Cos[x] + Sin[y]
Plot3D[f[x, y], {x, -3, 3}, {y, -3, 3}]
```



d) $z = x y / (x^2 + y^2)$

```
In[21]:= f[x_, y_] := x y / (x^2 + y^2)
Plot3D[f[x, y], {x, -1, 1}, {y, -1, 1}]
```



QUESTION 2: Let $f(x) = x/(1+x^2)$

a) Find $f'(x)$ and $f''(x)$

```
In[23]:= f[x_] := x / (1 + x ^ 2)
         f '[x]
```

$$\text{Out[24]= } -\frac{2x^2}{(1+x^2)^2} + \frac{1}{1+x^2}$$

```
In[19]:= f ''[x]
```

$$\text{Out[19]= } \frac{8x^3}{(1+x^2)^3} - \frac{6x}{(1+x^2)^2}$$

b) Find $f'(-1)$ and $f'(0)$

```
In[25]:= f '[x] /. {x -> -1}
```

```
Out[25]= 0
```

```
In[26]:= f '[-1]
```

```
Out[26]= 0
```

```
In[27]:= f '[0]
```

```
Out[27]= 1
```

c) Find $f''(0)$ and $f''(1)$

```
In[22]:= f ''[0]
```

```
Out[22]= 0
```

```
In[23]:= f ''[1]
```

$$\text{Out[23]= } -\frac{1}{2}$$

QUESTION 3: Find the prime factorization of each integer:

a) 3,527,218,133,309,949,276,293

```
In[1]:= FactorInteger [3 527 218 133 309 949 276 293 ]
```

```
Out[1]= {{15 013 , 2}, {25 013 , 3}}
```

b) 471,945,325,930,166,269

```
In[39]:= FactorInteger [471 945 325 930 166 269 ]
Out[39]= {{4211 , 1}, {34 589 , 1}, {46 747 , 1}, {69 313 , 1}}
```

c) 471,945,325,930,166,281

```
In[3]:= FactorInteger [471 945 325 930 166 281 ]
Out[3]= {{471 945 325 930 166 281 , 1}}
```

QUESTION 4: Compute each expression:

a) $3^6 \bmod 7$

```
In[40]:= Mod[3 ^ 6, 7]
Out[40]= 1
```

b) $6^{10} \bmod 11$

```
In[41]:= Mod[6 ^ 10, 11]
Out[41]= 1
```

c) $7^{20} \bmod 21$

```
In[42]:= Mod[7 ^ 20, 21]
Out[42]= 7
```

d) $7^{22} \bmod 23$

```
In[43]:= Mod[7 ^ 22, 23]
Out[43]= 1
```

QUESTION 8:

```
In[31]:= M = {{1, 1}, {1, 0}}
          MatrixForm[M]
Out[31]= {{1, 1}, {1, 0}}
Out[32]//MatrixForm=
          
$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

```

(a) Find $M_2, M_3, \dots, M_{10}$

Now at every step after defining function F , $F[n]$ is nothing but M^n :

```
In[33]:= F[n_] := MatrixPower[M, (n - 1)].M
```

```
In[34]:= List[F[2], F[3], F[4], F[5], F[6], F[7], F[8], F[9], F[10]]
```

```
Out[34]:= {{{2, 1}, {1, 1}}, {{3, 2}, {2, 1}}, {{5, 3}, {3, 2}}, {{8, 5}, {5, 3}}, {{13, 8}, {8, 5}},  
          {{21, 13}, {13, 8}}, {{34, 21}, {21, 13}}, {{55, 34}, {34, 21}}, {{89, 55}, {55, 34}}}
```

```
In[35]:= MatrixForm[F[2]]
```

```
Out[35]//MatrixForm=
```

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

```
In[37]:= MatrixForm[F[3]]
```

```
Out[37]//MatrixForm=
```

$$\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$$

```
In[38]:= MatrixForm[F[4]]
```

```
Out[38]//MatrixForm=
```

$$\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$$

```
In[39]:= MatrixForm[F[5]]
```

```
Out[39]//MatrixForm=
```

$$\begin{pmatrix} 8 & 5 \\ 5 & 3 \end{pmatrix}$$

```
In[40]:= MatrixForm[F[6]]
```

```
Out[40]//MatrixForm=
```

$$\begin{pmatrix} 13 & 8 \\ 8 & 5 \end{pmatrix}$$

```
In[42]:= MatrixForm[F[7]]
```

```
Out[42]//MatrixForm=
```

$$\begin{pmatrix} 21 & 13 \\ 13 & 8 \end{pmatrix}$$

```
In[43]:= MatrixForm[F[8]]
```

```
Out[43]//MatrixForm=
```

$$\begin{pmatrix} 34 & 21 \\ 21 & 13 \end{pmatrix}$$

```
In[44]:= MatrixForm[F[9]]
```

```
Out[44]//MatrixForm=
```

$$\begin{pmatrix} 55 & 34 \\ 34 & 21 \end{pmatrix}$$

```
In[46]:= MatrixForm[F[10]]
```

```
Out[46]//MatrixForm=
```

$$\begin{pmatrix} 89 & 55 \\ 55 & 34 \end{pmatrix}$$

```
In[1]:= ClearAll
```

```
Out[1]= ClearAll
```

b) Find the 100th Fibonacci number.

```
In[23]:= f[0] = 1;
```

```
f[1] = 1;
```

```
f[n_] := f[n] = f[n - 2] + f[n - 1]
```

```
f[100]
```

```
Out[26]= 573 147 844 013 817 084 101
```

QUESTION 9: Find solutions to the following equations or systems of equations:

a) Find x , if $x^2 + x = 1$

```
In[55]:= Solve[x^2 + x == 1, x]
```

```
Out[55]= {{x -> 1/2 (-1 - Sqrt[5])}, {x -> 1/2 (-1 + Sqrt[5])}}
```

b) Find x , if $x^2 + x = -1$

```
In[56]:= Solve[x^2 + x == -1, x]
```

```
Out[56]= {{x -> -(-1)^(1/3)}, {x -> (-1)^(2/3)}}
```

c) Find x and y :

$$4x - 3y = 5$$

$$6x + 2y = 14$$

```
In[57]:= Solve[4 x - 3 y == 5 && 6 x + 2 y == 14]
```

```
Out[57]= {{x -> 2, y -> 1}}
```

d) Find x, y, z and t

$$-2x - 2y + 3z + t = 8$$

$$-3x + 0y - 6z + t = -19$$

$$6x - 8y + 6z + 5t = 47$$

$$x + 3y - 3z - t = -9$$

```
In[58]:= Solve[-2 x - 2 y + 3 z + t == 8 &&
  -3 x + 0 y - 6 z + t == -19 && 6 x - 8 y + 6 z + 5 t == 47 && x + 3 y - 3 z - t == -9]
Out[58]:= {{t -> 5, x -> 2, y -> 1, z -> 3}}

In[59]:= ClearAll[x, y, z, t]
```

QUESTION 10: Assume that I invest \$250 at the beginning of the year, \$300 at the beginning of the second quarter, \$350 at the beginning of the third quarter, and \$400 at the beginning of the fourth quarter. At the end of the year, I have \$1365 (because my investments grow). To find my (continuous) rate of return, solve this equation for r :

$$250e^{1.0r} + 300e^{0.75r} + 350e^{0.5r} + 400e^{0.25r} = 1365$$

```
In[60]:= eqn1 = 250 * Exp[1.0 r] + 300 * Exp[0.75 r] + 350 * Exp[0.5 r] + 400 * Exp[0.25 r] == 1365
Out[60]:= 400 e^{0.25 r} + 350 e^{0.5 r} + 300 e^{0.75 r} + 250 e^{1. r} == 1365

In[61]:= FindRoot[eqn1, {r, 1}]
Out[61]:= {r -> 0.084104}
```

QUESTION 11:

```
In[27]:= mysqrt[n_] := Module[{i = 1, g = 1}, While[i ≤ 20, g = (g + n/g)/2; i = i + 1]; g]
In[28]:= N[mysqrt[2]]
Out[28]:= 1.41421

In[29]:= N[mysqrt[10]]
Out[29]:= 3.16228
```

QUESTION 12:

a)

```
In[32]:= collatz[n_] := Which[n == 1, collatz[n] = 0, EvenQ[n],
  collatz[n] = 1 + collatz[n/2], OddQ[n], collatz[n] = 1 + collatz[3 * n + 1]];
In[34]:= collatz[15]
Out[34]:= 17
```

b)

```
In[35]:= collatz[1]
```

```
Out[35]= 0
```

```
In[36]:= collatz[2]
```

```
Out[36]= 1
```

```
In[37]:= collatz[6]
```

```
Out[37]= 8
```

```
In[38]:= collatz[27]
```

```
Out[38]= 111
```