CHAPTER - 12 EXERCISE

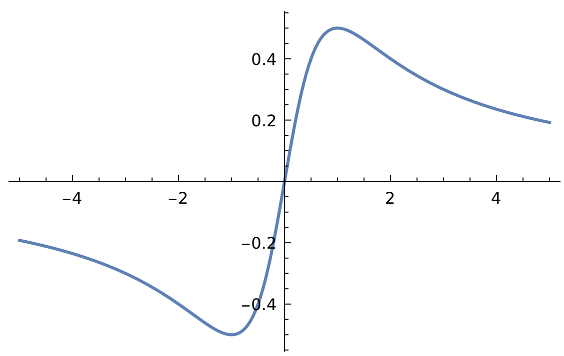
Q 1. Graph each of the following functions. Experiment with different domains or viewpoints to display the best images.

a) f(x) = x/(1+x^2)

In[2]:=
```
f[x_] := x / (1 + x ^ 2);
```

In[3]:=
```
Plot[f[x], {x, -5, 5}]
```

Out[3]=

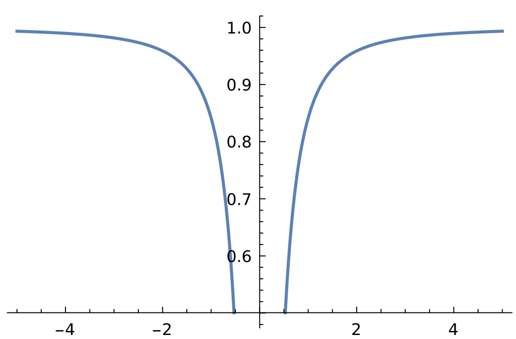


b) y = x Sin[1/x]

In[4]:=
```
f[x_] := x Sin[1 / x]
```

In[5]:=
```
Plot[f[x], {x, -5, 5}]
```

Out[5]=

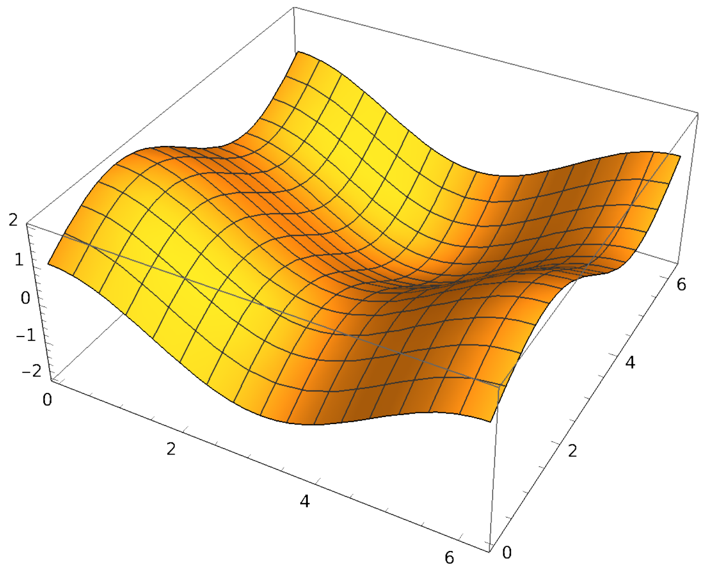


c) y(x,y) = Cos[x] + Sin[y]

In[12]:=
```
f[x_, y_] := Cos[x] + Sin[y];
```

In[13]:=
```
Plot3D[f[x, y], {x, 0, 2 Pi}, {y, 0, 2 Pi}]
```

Out[13]=

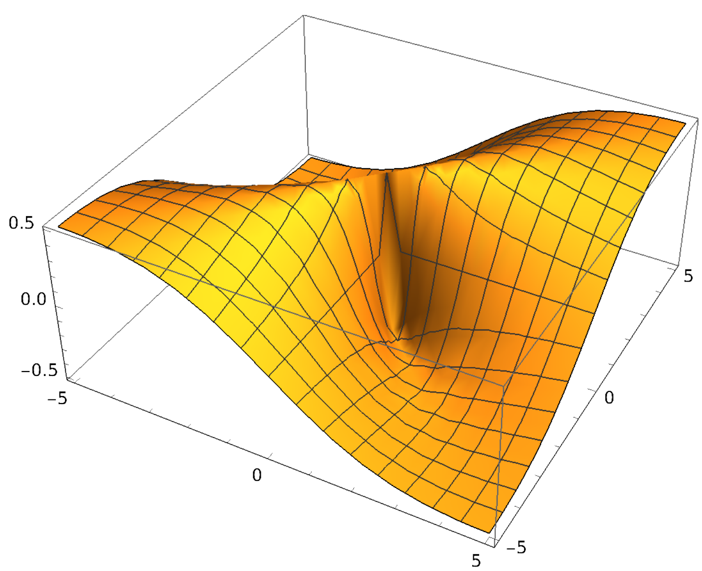


d) z = xy/(x^2 + y^2)

In[18]:=
```
f[x_, y_] := (x y) / (x ^ 2 + y ^ 2);
```

In[19]:=
```
Plot3D[f[x, y], {x, -5, 5}, {y, -5, 5}]
```

Out[19]=



Q 2. Let f(x) = x/(1 + x^2)

In[4]:=
```
f[x_] := x / (1 + x ^ 2);
```

a) Find f'(x) and f''(x).

In[5]:=
```
f'[x]
```

Out[5]= $-\frac{2x^2}{(1+x^2)^2}+\frac{1}{1+x^2}$

In[6]:=
```
f''[x]
```

Out[6]= $\frac{8x^3}{(1+x^2)^3}-\frac{6x}{(1+x^2)^2}$

b) Find f'(-1) and f'(0).

In[7]:=
```
f'[-1]
```

Out[7]= 0

In[8]:=
```
f'[0]
```

Out[8]= 1

c) Find f''(0) and f''(1).

In[9]:=
```
f''[0]
```

Out[9]= 0

In[10]:=
```
f''[1]
```

Out[10]= $-\frac{1}{2}$

Q 3. Find the prime factorization of each integer.

a) 3, 527, 218, 133, 309, 949, 276, 293

In[24]:=
```
FactorInteger[3 527 218 133 309 949 276 293 ]
```

Out[24]= {{15 013 , 2}, {25 013 , 3}}

b) 471, 945, 325, 930, 166, 269

In[25]:=
```
FactorInteger[471 945 325 930 166 269 ]
```

Out[25]= {{4211 , 1}, {34 589 , 1}, {46 747 , 1}, {69 313 , 1}}

c) 471, 945, 325, 930, 166, 281

```
In[26]:= FactorInteger [471 945 325 930 166 281 ]
Out[26]= {{471 945 325 930 166 281 , 1}}
```

Q 4. Compute each expression. Do you notice a pattern?

a) 3^6 mod 7

b) 6^10 mod 11

c) 7^20 mod 21

d) 7^22 mod 23

```
In[31]:= Mod[3 ^ 6, 7]
Out[31]= 1

In[32]:= Mod[6 ^ 10, 11]
Out[32]= 1

In[33]:= Mod[7 ^ 20, 21]
Out[33]= 7

In[34]:= Mod[7 ^ 22, 23]
Out[34]= 1
```

Q 8. Let *M* = [{1, 1} , {1, 0}].

```
In[35]:= M = {{1, 1}, {1, 0}}
Out[35]= {{1, 1}, {1, 0}}
```

a) Find *M*^2 , *M*^3, ..., *M*^10 .

```
In[36]:= MatrixPower [M, 2]
Out[36]= {{2, 1}, {1, 1}}

In[37]:= MatrixPower [M, 3]
Out[37]= {{3, 2}, {2, 1}}

In[38]:= MatrixPower [M, 4]
Out[38]= {{5, 3}, {3, 2}}
```

```
In[40]:= MatrixPower[M, 5]
Out[40]= {{8, 5}, {5, 3}}

In[41]:= MatrixPower[M, 6]
Out[41]= {{13, 8}, {8, 5}}

In[42]:= MatrixPower[M, 7]
Out[42]= {{21, 13}, {13, 8}}

In[43]:= MatrixPower[M, 8]
Out[43]= {{34, 21}, {21, 13}}

In[44]:= MatrixPower[M, 9]
Out[44]= {{55, 34}, {34, 21}}

In[45]:= MatrixPower[M, 10]
Out[45]= {{89, 55}, {55, 34}}
```

b) Do your answers suggest the way to compute Fibonacci numbers? Find the 100th Fibonacci numbers.

```
In[1]:= f[0] = 1;
        f[1] = 1;
        f[n_] := f[n] = f[n - 2] + f[n - 1]

In[4]:= f[100]
Out[4]= 573 147 844 013 817 084 101
```

Q 9. Find solutions to the following equations or system of equations.

a) Find x, if x^2 + x = 1.

b) Find x, if x^2 +x = -1.

c) Find x and y .

4x - 3y = 5

6x + 2y = 14

d) Find x, y, z and t.

-2x - 2y + 3z + t =8

-3x + 0y - 6z + t = -19

6x - 8y + 6z + 5t = 47

x + 3y - 3z - t = -9

In[46]:=
```
Solve[x ^ 2 + x == 1, x]
```

Out[46]= $\left\{\left\{x \to \frac{1}{2}\left(-1 - \sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(-1 + \sqrt{5}\right)\right\}\right\}$

In[48]:=
```
Solve[x ^ 2 + x == -1, x]
```

Out[48]= $\{\{x \to -(-1)^{1/3}\}, \{x \to (-1)^{2/3}\}\}$

In[50]:=
```
Solve[4 x - 3 y == 5 && 6 x + 2 y == 14, {x, y}]
```

Out[50]= $\{\{x \to 2, y \to 1\}\}$

In[51]:=
```
Solve[-2 x - 2 y + 3 z + t == 8 && -3 x + 0 y - 6 z + t == -19 &&
   6 x - 8 y + 6 z + 5 t == 47 && x + 3 y - 3 z - t == -9, {x, y, z, t}]
```

Out[51]= $\{\{x \to 2, y \to 1, z \to 3, t \to 5\}\}$

Q 10. Some equations are difficult or impossible to solve explicitly, even with software. In such situations, we often resort to numerical methods. Mathematica uses FindRoot, Maple uses fsolve(), and Maxima uses find_root() to find numerical solutions to equations. Here is an example where a numerical approach works well. Assume that I invest $250 at the beginning of the year, $300 at the beginning of the second quarter, $350 at the beginning of the third quarter, and $400 at the beginning of the fourth quarter. At the end of the year, I have $1365 (because my investments grow). To find my (continuous) rate of return, solve this equation for *r* :

250Exp[1.0r] + 300Exp[0.75r] + 350Exp[0.5r] + 400Exp[0.25r] = 1365

```
In[52]:= FindRoot[250 Exp[1.0 r] + 300 Exp[0.75 r] + 350 Exp[0.5 r] + 400 Exp[0.25 r] == 1365, {r, 0}]
Out[52]= {r → 0.084104}
```

Q 11. If *n* is a positive number, *g* > 0 is any "guess" for the square root of *n*, then a better estimate of $\sqrt{n}$ is the average of *g* and *n*/*g* ,i.e., (*g* + *n*/*g*)/2. Write a function called mysqrt that accepts one argument, begins with an initial guess of 1.0, finds 20 new guesses, and returns the answer.

```
In[1]:= mysqrt[n_] := Module[{i = 1, g = 1}, While[i ≤ 20, g = (g + n / g) / 2; i = i + 1]; g]

In[2]:= N[mysqrt[2], 6]
Out[2]= 1.41421

In[3]:= N[Sqrt[2], 6]
Out[3]= 1.41421

In[4]:= N[mysqrt[3]]
Out[4]= 1.73205
```

Q 12. The Collatz conjecture states that if we start from any natural number a_0= n and form a sequence by the rule

a_i+1 = a_i/2 , if a_i is even ; 3a_i + 1 , if a_i is odd,

then the sequence eventually contains the value 1. For example, starting from a_0 = 6, we get the sequence 6, 3, 10, 5, 16, 8, 4, 2, 1 (we reached 1 after eight steps).

(a) Write a (recursive) function called collatz that accepts a single argument, n, and returns:

- **0 if n is equal to 1**
- **1+collatz(n/2) if n is even**
- **1+collatz(3*n+1) if n is odd**

Thus, collatz(n) is the number of steps needed to go from n to 1.

(b) Verify the values:

n	collatz(n)
1	0
2	1
6	8
27	111

In[6]:=
```
Clear[collatz];
```

In[9]:=
```
collatz[n_] := Which[n == 1, collatz[n] = 0, EvenQ[n],
   collatz[n] = 1 + collatz[n / 2], OddQ[n], collatz[n] = 1 + collatz[3 * n + 1]];
```

In[10]:=
```
collatz[27]
```

Out[10]=
```
111
```