Chapter 12 : Exercise

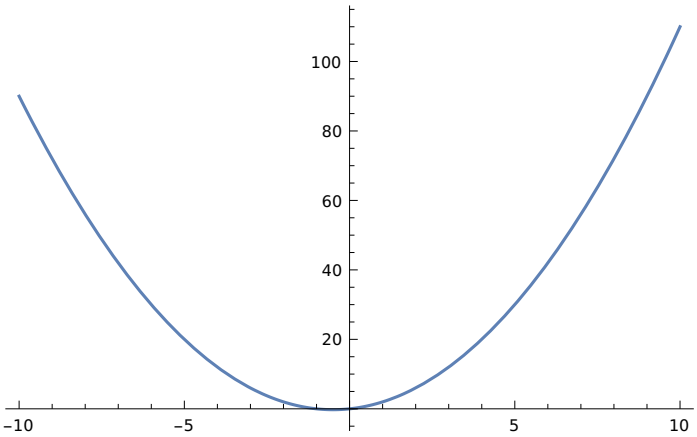
Ques 1) Graph each of these functions.

a) f(x) = x / 1+x^2

In[1]:=
```
f[x_] := x / 1 + x ^ 2
```

In[2]:=
```
Plot[f[x], {x, -10, 10}]
```

Out[2]=

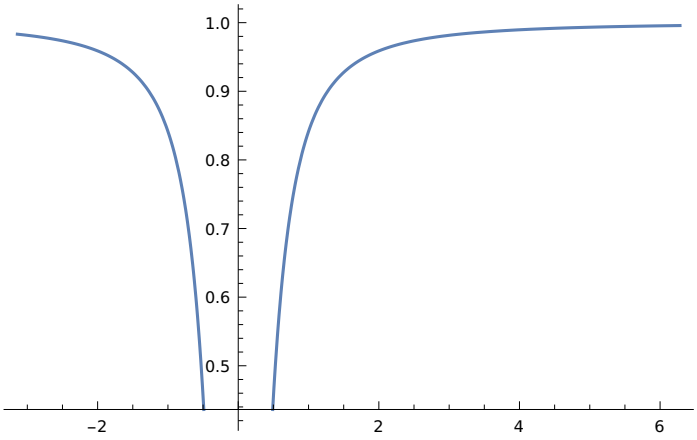


b) g(x) = x Sin[1 / x]

In[5]:=
```
g[x_] := x Sin[1 / x]
```

In[7]:=
```
Plot[g[x], {x, -Pi, 2 Pi}]
```

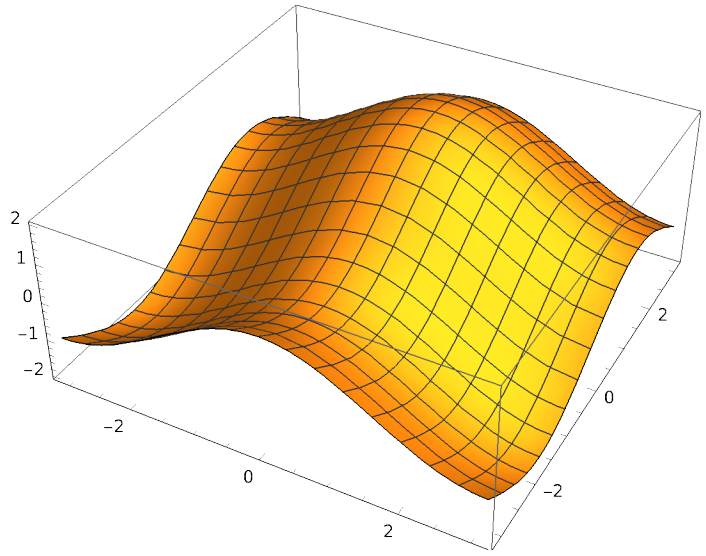
Out[7]=



c) g(x,y) = Cos[x]+Sin[y]

In[20]:=
```
Plot3D[Cos[x] + Sin[y], {x, -Pi, Pi}, {y, -Pi, Pi}]
```

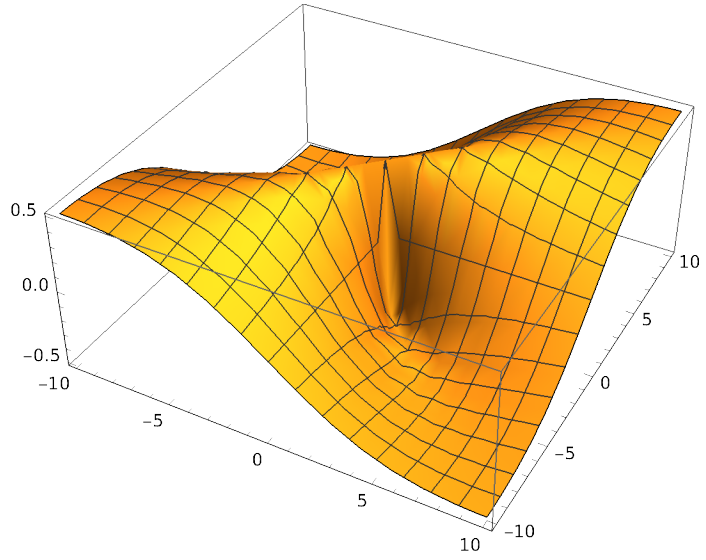
Out[20]=



d) h(x,y) = x y/(x^2+y^2)

In[26]:=
```
Plot3D[x y / (x ^ 2 + y ^ 2), {x, -10, 10}, {y, -10, 10}]
```

Out[26]=



In[10]:=
```
ClearAll[f, g, x]
```

Ques 2) f(x) = x y/(x^2+y^2)

(a) Find f ' (x) and f '' (x)

In[1]:=
```
f[x_] := x / (1 + x ^ 2)
```

In[2]:=
```
D[f[x], x]
```

Out[2]=
$$-\frac{2\,x^2}{\left(1+x^2\right)^2}+\frac{1}{1+x^2}$$

In[3]:=
```
D[%, {x, 2}]
```

Out[3]=
$$\frac{40\,x^2}{\left(1+x^2\right)^3}-\frac{6}{\left(1+x^2\right)^2}-2\,x^2\left(\frac{24\,x^2}{\left(1+x^2\right)^4}-\frac{4}{\left(1+x^2\right)^3}\right)$$

(b) Find f '(-1) and f '(0)

In[8]:=
```
f'[-1]
```

Out[8]=
```
0
```

In[5]:=
```
f'[0]
```

Out[5]=
```
1
```

(b) Find f ''(0) and f ''(1)

In[6]:=
```
f''[0]
```

Out[6]=
```
0
```

In[7]:=
```
f''[1]
```

Out[7]=
$$-\frac{1}{2}$$

Ques 3) Find prime factorization of each integer.

In[2]:=
```
FactorInteger [3 527 218 133 309 949 276 293 ]
```

Out[2]=
```
{{15 013 , 2}, {25 013 , 3}}
```

In[3]:=
```
FactorInteger [471 945 325 930 166 269 ]
```

Out[3]=
```
{{4211 , 1}, {34 589 , 1}, {46 747 , 1}, {69 313 , 1}}
```

In[4]:=
```
FactorInteger [471 945 325 930 166 281 ]
```

Out[4]=
```
{{471 945 325 930 166 281 , 1}}
```

Ques 4) Compute each expression

(a) 3^6 mod 7

In[34]:=
```
Mod[3^6, 7]
```

Out[34]=
```
1
```

(b) 6^10 mod 11

In[35]:=
```
Mod[6^10, 11]
```
Out[35]= 1

(c) 7^20 mod 21

```
Mod[7^20, 21]
```
Out[36]= 7

(d) 7^22 mod 23

In[37]:=
```
Mod[7^22, 23]
```
Out[37]= 1

Ques 8)

(a)

In[38]:=
```
m = {{1, 1}, {1, 0}}
```
Out[38]= {{1, 1}, {1, 0}}

In[45]:=
```
MatrixPower[m, 2]
```
Out[45]= {{2, 1}, {1, 1}}

In[46]:=
```
MatrixPower[m, 3]
```
Out[46]= {{3, 2}, {2, 1}}

In[47]:=
```
MatrixPower[m, 4]
```
Out[47]= {{5, 3}, {3, 2}}

In[49]:=
```
MatrixPower[m, 5]
```
Out[49]= {{8, 5}, {5, 3}}

In[50]:=
```
MatrixPower[m, 6]
```
Out[50]= {{13, 8}, {8, 5}}

In[51]:=
```
MatrixPower[m, 7]
```
Out[51]= {{21, 13}, {13, 8}}

In[52]:=
```
MatrixPower[m, 8]
```
Out[52]= {{34, 21}, {21, 13}}

In[53]:=
```
MatrixPower[m, 9]
```
Out[53]= {{55, 34}, {34, 21}}

In[54]:=
```
MatrixPower[m, 10]
```
Out[54]= {{89, 55}, {55, 34}}

(b) Find the 100th Fibonacci number.

In[55]:=
```
Fibonacci[100]
```

Out[55]= 354 224 848 179 261 915 075

Ques 9) Find solutions to the following equations or system of equations.

(a) x^2 + x = 1

In[1]:=
```
Solve[x^2 + x == 1, x]
```

Out[1]= $\left\{\left\{x \to \frac{1}{2}\left(-1-\sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(-1+\sqrt{5}\right)\right\}\right\}$

(b) x^2 + x = -1

In[2]:=
```
Solve[x^2 + x == -1, x]
```

Out[2]= $\{\{x \to -(-1)^{1/3}\}, \{x \to (-1)^{2/3}\}\}$

(c) 4 x - 3 y = 5 , 6 x + 2 y = 14

In[3]:=
```
Solve[4 x - 3 y == 5 && 6 x + 2 y == 14, {x, y}]
```

Out[3]= {{x → 2, y → 1}}

(d) -2 x-2y+ 3 z+ t = 8 , -3 x+0 y - 6 z+t = -19 , 6 x-8 y+6 z+5 t = 47 , x+3 y - 3 z-t = -9

In[18]:=
```
Solve[{-2 x - 2 y + 3 z + t == 8, -3 x + 0 y - 6 z + t == -19,
   6 x - 8 y + 6 z + 5 t == 47, x + 3 y - 3 z - t == -9}, {x, y, z, t}]
```

Out[18]= {{x → 2, y → 1, z → 3, t → 5}}

Ques 10) Solve this equation for r

In[19]:=
```
FindRoot[250 Exp[1.0 r] + 300 Exp[0.75 r] + 350 Exp[0.5 r] + 400 Exp[0.25 r] == 1365, {r, 1/4}]
```

Out[19]= {r → 0.084104}

Question 11)

In[17]:=
```
mysqrt[n_] := Module[{i = 1, g = 1}, While[i ≤ 20, g = (g + n/g)/2; i = i + 1]; g]
```

In[18]:=
```
N[mysqrt[2], 6]
```

Out[18]= 1.41421

In[19]:=
```
N[Sqrt[2], 6]
```

Out[19]= 1.41421

In[20]:=
```
N[mysqrt[3]]
```

Out[20]= 1.73205

Question 12)

In[26]:=
```
Clear[collatz];
```

In[27]:=
```
collatz[n_] := Which[n == 1, collatz[n] = 0, EvenQ[n],
    collatz[n] = 1 + collatz[n / 2], OddQ[n], collatz[n] = 1 + collatz[3 * n + 1]];
```

In[28]:=
```
collatz[27]
```

Out[28]=
```
111
```