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Chapter 12 :
Getting started with Mathematica

Exercises

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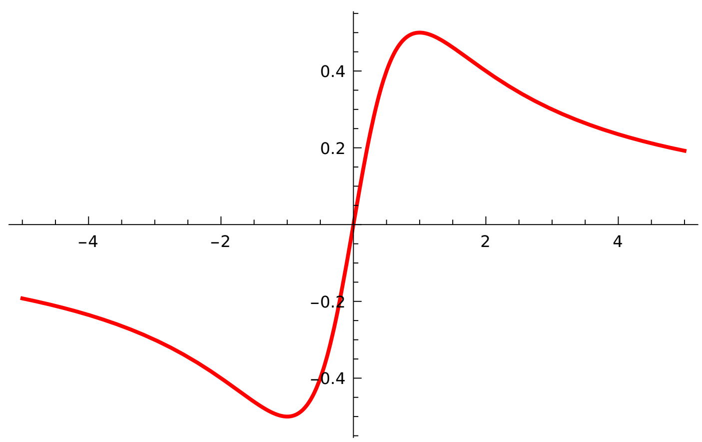
1. Graph each of the following functions.

In[2]:=
```
f[x_] := x / (1 + x ^ 2)
```

In[3]:=
```
Plot[f[x], {x, -5, 5}, PlotStyle → {Red, Thick}]
```

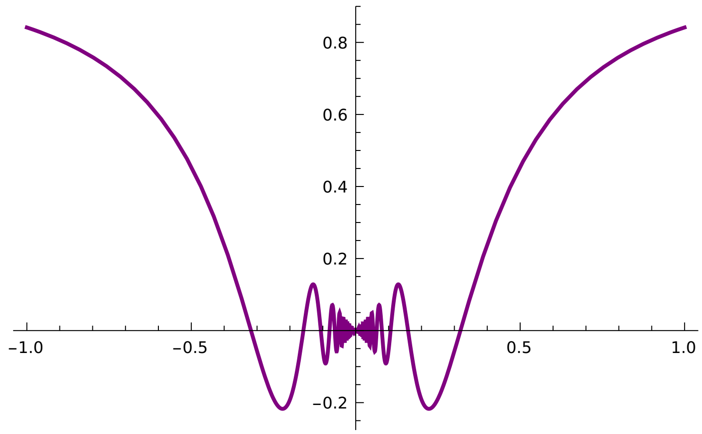
Out[3]=



In[4]:=
```
g[x_] := x Sin[1 / x]
```

In[5]:=
```
Plot[g[x], {x, -1, 1}, PlotStyle → {Purple, Thick}]
```

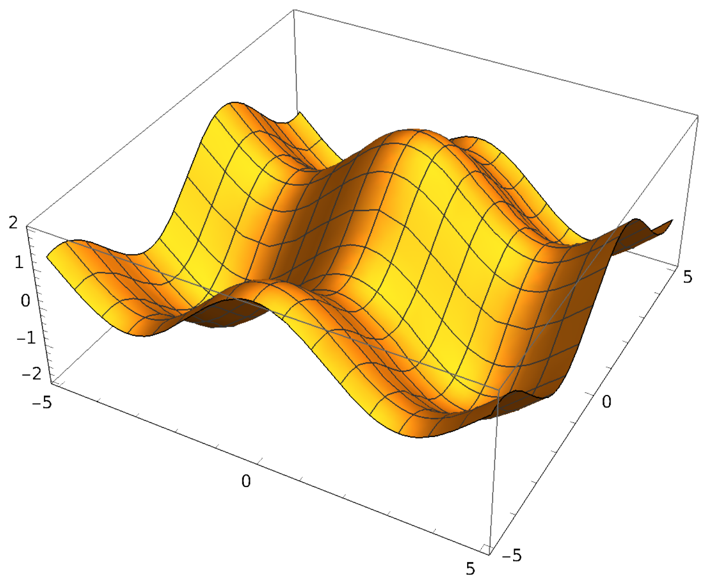
Out[5]=



In[6]:=
```
h[x_, y_] := Cos[x] + Sin[y]
```

In[7]:=
```
Plot3D[h[x, y], {x, -5, 5}, {y, -5, 5}]
```

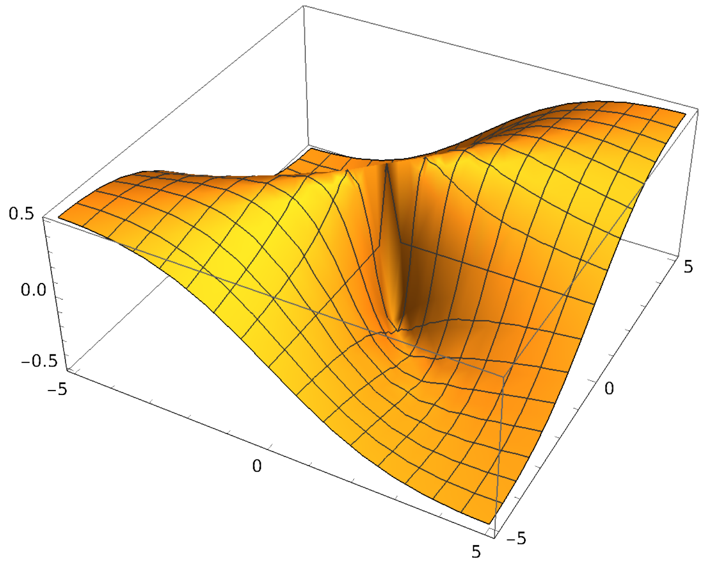
Out[7]=



In[8]:=
```
i[x_, y_] := x y / (x ^ 2 + y ^ 2)
```

In[9]:= `Plot3D[i[x, y], {x, -5, 5}, {y, -5, 5}]`

Out[9]=



In[10]:= `ClearAll[f, g, h, i]`

2. Let f(x)=x/1+x^2.

In[11]:= `f[x] := x / (1 + x ^ 2)`

a. Find f'(x) and f''(x).

In[12]:= `D[f[x], x]`

Out[12]= $$-\frac{2x^2}{(1+x^2)^2}+\frac{1}{1+x^2}$$

In[13]:= `D[f[x], {x, 2}]`

Out[13]= $$-\frac{4x}{(1+x^2)^2}+x\left(\frac{8x^2}{(1+x^2)^3}-\frac{2}{(1+x^2)^2}\right)$$

b. Find f'(-1) and f'(0).

In[14]:= `D[f[x], x] /. x → (-1)`

Out[14]= 0

In[15]:= `D[f[x], x] /. x → 0`

Out[15]= 1

c. Find f''(0) and f''(1).

In[16]:=
```
D[f[x], {x, 2}] /. x → 0
```
Out[16]= 0

In[17]:=
```
D[f[x], {x, 2}] /. x → 1
```
Out[17]= $-\frac{1}{2}$

In[18]:=
```
ClearAll[f]
```

---

3. Find the prime factorization of each Integer.

a. 3, 527, 218, 133, 309, 949, 276, 293.

In[19]:=
```
FactorInteger[3 527 218 133 309 949 276 293 ]
```
Out[19]= {{15 013 , 2}, {25 013 , 3}}

b. 471, 945, 325, 930, 166, 269.

In[20]:=
```
FactorInteger[471 945 325 930 166 269 ]
```
Out[20]= {{4211 , 1}, {34 589 , 1}, {46 747 , 1}, {69 313 , 1}}

c. 471, 945, 325, 930, 166, 281.

In[21]:=
```
FactorInteger[471 945 325 930 166 281 ]
```
Out[21]= {{471 945 325 930 166 281 , 1}}

---

4. Compute each expression.

a. 3^6 mod 7

In[22]:=
```
Mod[3^6, 7]
```
Out[22]= 1

b. 6^10 mod 11

In[23]:=
```
Mod[6^10, 11]
```
Out[23]= 1

c. 7^20 mod 21

In[24]:=
```
Mod[7 ^ 20, 21]
```
Out[24]= 7

d. 7^22 mod 23

In[25]:=
```
Mod[7 ^ 22, 23]
```
Out[25]= 1

---

8. Let M=[{1,1},{1,0}].

In[26]:=
```
M = {{1, 1}, {1, 0}}
```
Out[26]= {{1, 1}, {1, 0}}

a. Find M^2, M^3,........,M^10.

In[27]:=
```
MatrixPower [M, 2]
```
Out[27]= {{2, 1}, {1, 1}}

In[28]:=
```
MatrixPower [M, 3]
```
Out[28]= {{3, 2}, {2, 1}}

In[29]:=
```
MatrixPower [M, 4]
```
Out[29]= {{5, 3}, {3, 2}}

In[30]:=
```
MatrixPower [M, 5]
```
Out[30]= {{8, 5}, {5, 3}}

In[31]:=
```
MatrixPower [M, 6]
```
Out[31]= {{13, 8}, {8, 5}}

In[32]:=
```
MatrixPower [M, 7]
```
Out[32]= {{21, 13}, {13, 8}}

In[33]:=
```
MatrixPower [M, 8]
```
Out[33]= {{34, 21}, {21, 13}}

In[34]:=
```
MatrixPower [M, 9]
```
Out[34]= {{55, 34}, {34, 21}}

In[35]:=
```
MatrixPower [M, 10]
```
Out[35]= {{89, 55}, {55, 34}}

b. Find the 100th Fibonacci number.

In[36]:=
```
f[0] = 1;
f[1] = 1;
f[n_] := f[n] = f[n - 2] + f[n - 1]
```

In[39]:=
```
f[100]
```

Out[39]= 573 147 844 013 817 084 101

9. Find solutions to the following equations or systems of equations.

a. Find x, if x^2 + x =1.

In[40]:=
```
Solve[x ^ 2 + x == 1, x]
```

Out[40]= $\left\{\left\{x \to \frac{1}{2}\left(-1-\sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(-1+\sqrt{5}\right)\right\}\right\}$

b. Find x, if x^2 + x =-1.

In[41]:=
```
Solve[x ^ 2 + x == -1, x]
```

Out[41]= $\{\{x \to -(-1)^{1/3}\}, \{x \to (-1)^{2/3}\}\}$

c. Find x and y.

4x - 3y = 5
6x + 2y = 14

In[42]:=
```
Solve[{4 x - 3 y == 5, 6 x + 2 y == 14}, {x, y}]
```

Out[42]= $\{\{x \to 2, y \to 1\}\}$

d. Find x, y, z and t.

-2x - 2y +3z + t = 8
-3x +0y - 6z + t = -19
6x - 8y + 6z + 5t = 47
x + 3y - 3z - t = -9

In[43]:=
```
Solve[{-2 x - 2 y + 3 z + t == 8, -3 x + 0 y - 6 z + t == -19,
  6 x - 8 y + 6 z + 5 t == 47, x + 3 y - 3 z - t == -9}, {x, y, z, t}]
```

Out[43]= $\{\{x \to 2, y \to 1, z \to 3, t \to 5\}\}$

---

10. Solve this equation in r: 250e^1.0r + 300e^0.75r + 350e^0.5r + 400e^0.25r = 1365.

```
In[44]:= FindRoot[250 Exp[1.0 r] + 300 Exp[0.75 r] + 350 Exp[0.5 r] + 400 Exp[0.25 r] == 1365, {r, 0}]
Out[44]= {r → 0.084104}
```

---

11. If n is a positive number, and g>0 is any "guess" for the square root of n, then a better estimate of Sqrt[n] is the average of g and n/g, i.e. (g+n/g)/2. Write a function called mysqrt that accepts one argument, begins with an initial guess of 1.0, finds 20 new guesses, and returns the answer.

```
In[45]:= mysqrt[n_] := Module[{i = 1, g = 1}, While[i ≤ 20, g = (g + n / g) / 2; i = i + 1]; g]

In[46]:= N[mysqrt[2], 6]
Out[46]= 1.41421

In[47]:= N[Sqrt[2], 6]
Out[47]= 1.41421

In[48]:= N[mysqrt[3]]
Out[48]= 1.73205
```

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12. The Collatz cojecture states that if we start from any natural number a_[0] = n and form a sequence by the rule : a_[i+1]={a_[i]/2 if a_[i] is even & 3a_[i]+1 if a_[i] is odd. then the sequence eventually contains the value 1. For example, starting from a_[0] = 6, we get the sequence 6,3, 10, 5,16,8,4,2,1 (we reached 1 after eight steps).

a. Write a (recursive) function called collatz that accepts a single argument, n, and returns

- ***0 if n is equal to 1***
- ***1 + collatz(n/2) if n is even***
- ***1 + collatz(3*n+1) if n is odd***

Thus, collatz(n) is the number of steps needed to go from n to 1.

In[49]:=
```
collatz[n_] := Which[n == 1, collatz[n] = 0, EvenQ[n],
   collatz[n] = 1 + collatz[n / 2], OddQ[n], collatz[n] = 1 + collatz[3 * n + 1]];
```

b. Verify the values:

n		collatz(n)
1	:	0
2	:	1
6	:	8
27	:	111

In[50]:=
```
collatz[1]
```
Out[50]= 0

In[51]:=
```
collatz[2]
```
Out[51]= 1

In[52]:=
```
collatz[6]
```
Out[52]= 8

In[53]:=
```
collatz[27]
```
Out[53]= 111