MAT/19/75
ASSIGNMENT - 1

CHAPTER 12 : A STUDENT'S GUIDE TO THE STUDY, PRACTICE, AND TOOLS OF MODERN MATHEMATICS BY Donald Bindner

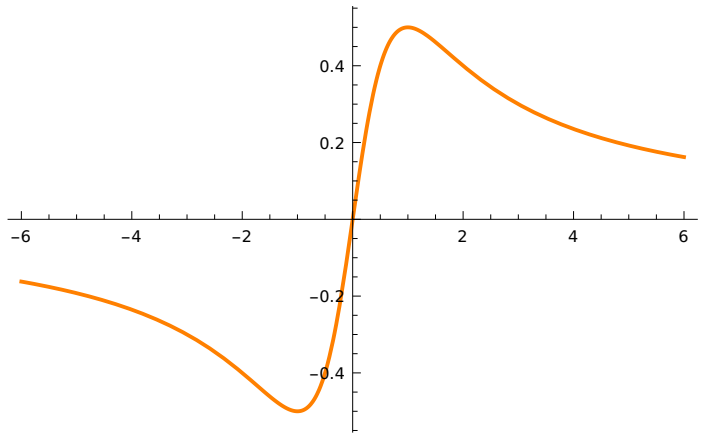
Ques 1 - Graph each of the functions.

(a) f(x)=x/(1+x^2)

In[1]:=
```
f[x_] := x / (1 + x ^ 2)
```

In[3]:=
```
Plot[f[x], {x, -6, 6}, PlotStyle → {Orange, Thick}]
```

Out[3]=

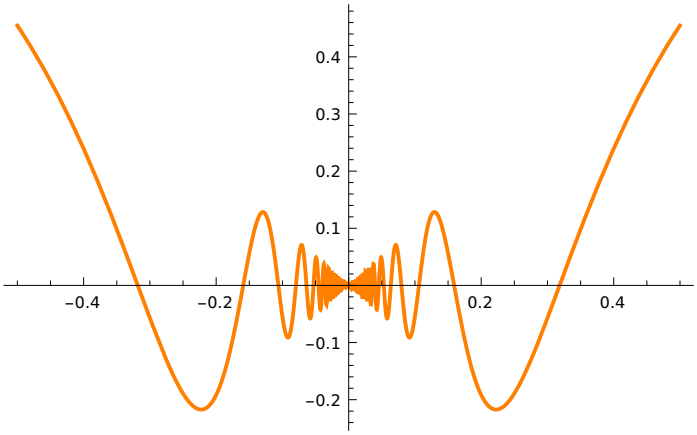


(b) y = x Sin[1 / x]

In[4]:=
```
g[x_] := x Sin[1 / x]
```

In[6]:=
```
Plot[g[x], {x, -0.5, 0.5}, PlotStyle → {Orange, Thick}]
```

Out[6]=

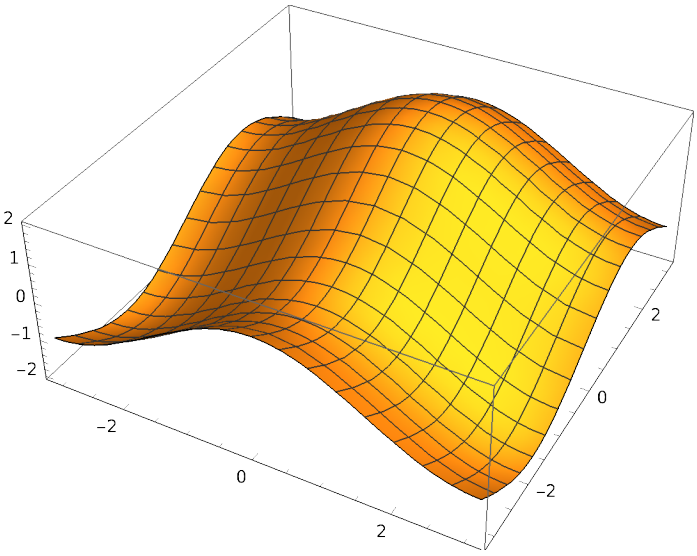


(c) g (x, y) = Cos[x] + Sin[y]

In[7]:=
```
h[x_, y_] := Cos[x] + Sin[y]
```

In[9]:=
```
Plot3D[h[x, y], {x, -Pi, Pi}, {y, -Pi, Pi}]
```

Out[9]=

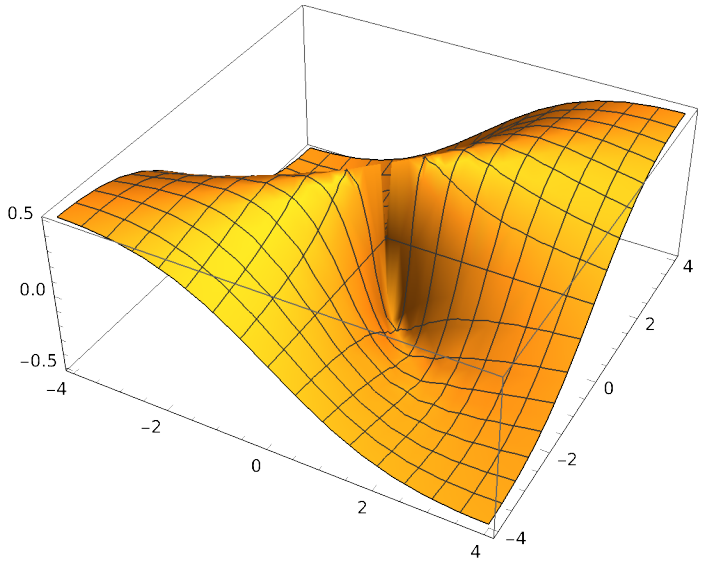


(d) z = xy / (x ^ 2 + y ^ 2)

In[12]:=
```
z[x_, y_] := x y / (x ^ 2 + y ^ 2)
```

In[13]:=
```
Plot3D[z[x, y], {x, -4, 4}, {y, -4, 4}]
```

Out[13]=



In[14]:=
```
ClearAll[f, g, h, z]
```

Ques 2 - Let f[x] = x / (1 + x ^ 2).

In[15]:=
```
f[x_] := x / (1 + x ^ 2)
```

(a) Find f'[x] and f''[x]

In[16]:=
```
D[f[x], x]
```

Out[16]=
$$-\frac{2\,x^2}{(1+x^2)^2}+\frac{1}{1+x^2}$$

In[17]:=
```
D[%, {x, 2}]
```

Out[17]=
$$\frac{40\,x^2}{(1+x^2)^3}-\frac{6}{(1+x^2)^2}-2\,x^2\left(\frac{24\,x^2}{(1+x^2)^4}-\frac{4}{(1+x^2)^3}\right)$$

(b) Find f'[-1] and f'[0]

In[18]:=
```
f'[-1]
```

Out[18]=
```
0
```

In[19]:=
```
f'[0]
```

Out[19]=
```
1
```

(c) Find f''[0] and f''[1]

In[20]:=
```
f''[0]
```

Out[20]=
```
0
```

In[21]:=
```
f''[1]
```

Out[21]=
$$-\frac{1}{2}$$

Ques 3 -

Find the prime factorization of each integer.

(a) 3, 527, 218, 133, 309, 949, 276, 293

In[22]:=
```
FactorInteger [3 527 218 133 309 949 276 293 ]
```

Out[22]=
```
{{15 013 , 2}, {25 013 , 3}}
```

(b) 471, 945, 325, 930, 166, 269

In[23]:=
```
FactorInteger [471 945 325 930 166 269 ]
```

Out[23]=
```
{{4211 , 1}, {34 589 , 1}, {46 747 , 1}, {69 313 , 1}}
```

(c) 471, 945, 325, 930, 166, 281

In[24]:=
```
FactorInteger [471 945 325 930 166 281 ]
```

Out[24]=
```
{{471 945 325 930 166 281 , 1}}
```

Ques 4 - Compute each expression.

(a) 3 ^ 6 mod 7

In[25]:=
```
Mod[3 ^ 6, 7]
```

Out[25]=
```
1
```

(b) 6 ^ 10 mod 11

In[26]:=
```
Mod[6 ^ 10, 11]
```

Out[26]=
```
1
```

(c) 7 ^ 20 mod 21

In[27]:=
```
Mod[7 ^ 20, 21]
```

Out[27]=
```
7
```

(c) 7 ^ 22 mod 23

In[28]:=
```
Mod[7^22, 23]
```

Out[28]=
```
1
```

Ques 8 – Let M = [{1, 1, {1, 0}}]

In[29]:=
```
M = {{1, 1}, {1, 0}}
```

Out[29]=
```
{{1, 1}, {1, 0}}
```

(a) Find M^2, M^3, ...M^10.

In[30]:=
```
MatrixPower[M, 2]
```

Out[30]=
```
{{2, 1}, {1, 1}}
```

In[31]:=
```
MatrixPower[M, 3]
```

Out[31]=
```
{{3, 2}, {2, 1}}
```

In[32]:=
```
MatrixPower[M, 4]
```

Out[32]=
```
{{5, 3}, {3, 2}}
```

In[33]:=
```
MatrixPower[M, 5]
```

Out[33]=
```
{{8, 5}, {5, 3}}
```

In[34]:=
```
MatrixPower[M, 6]
```

Out[34]=
```
{{13, 8}, {8, 5}}
```

In[35]:=
```
MatrixPower[M, 7]
```

Out[35]=
```
{{21, 13}, {13, 8}}
```

In[36]:=
```
MatrixPower[M, 8]
```

Out[36]=
```
{{34, 21}, {21, 13}}
```

In[37]:=
```
MatrixPower[M, 9]
```

Out[37]=
```
{{55, 34}, {34, 21}}
```

In[38]:=
```
MatrixPower[M, 10]
```

Out[38]=
```
{{89, 55}, {55, 34}}
```

(b) Find the 100 th Fibonacci number.

In[39]:=
```
f[0] = 1;
```

In[40]:=
```
f[1] = 1;
```

In[41]:=
```
f[n_] := f[n] = f[n - 2] + f[n - 1]
```

In[42]:=
```
f[100]
```

Out[42]= 573 147 844 013 817 084 101

Ques 9 – Find solutions to the following equations or systems of equations.

(a) Find x , if x^2 + x = 1.

In[43]:=
```
Solve[x^2 + x == 1, x]
```

Out[43]= $\left\{\left\{x \to \frac{1}{2}\left(-1-\sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(-1+\sqrt{5}\right)\right\}\right\}$

(b) Find x , if x^2 + x = -1

In[45]:=
```
Solve[x^2 + x == -1, x]
```

Out[45]= $\{\{x \to -(-1)^{1/3}\}, \{x \to (-1)^{2/3}\}\}$

(c) Find x and y.

$$4x - 3y = 5$$
$$6x + 2y = 14$$

In[47]:=
```
Solve[{4 x - 3 y == 5, 6 x + 2 y == 14}, {x, y}]
```

Out[47]= {{x → 2, y → 1}}

Ques 10 – Solve the equation for r :

$$250\, e^{(1.0\, r)} + 300\, e^{(0.75\, r)} + 350\, e^{(0.5\, r)} + 400\, e^{(0.25\, r)} = 1365$$

In[48]:=
```
FindRoot[{250 Exp[1.0 r] + 300 Exp[0.75 r] + 350 Exp[0.5 r] + 400 Exp[0.25 r] == 1365}, {r, 0}]
```

Out[48]= {r → 0.084104}

Ques 11 – Write a function called mysqrt that accepts one argument.

In[49]:=
```
mysqrt[n_] := Module[{i = 1, g = 1}, While[i ≤ 20, g = (g + n / g) / 2; i = i + 1]; g]
```

In[50]:=
```
N[mysqrt[2], 6]
```

Out[50]= 1.41421

In[51]:=
```
N[Sqrt[2], 6]
```

Out[51]= 1.41421

In[52]:=
```
N[mysqrt[3]]
```

Out[52]=
```
1.73205
```

Ques 12 –
Write a (recursive) function called collatz that accepts a single argument, n, and returns:

0 if n is equal to 1

1 + collatz (n / 2) if n is even

1 + collatz (3 * n + 1) if n is odd

In[54]:=
```
Clear[collatz];
```

In[55]:=
```
collatz[n_] := Which[n == 1, collatz[n] = 0, EvenQ[n],
    collatz[n] = 1 + collatz[n / 2], OddQ[n], collatz[n] = 1 + collatz[3 n + 1]];
```

In[56]:=
```
collatz[27]
```

Out[56]=
```
111
```