Assignment(Chapter-12)

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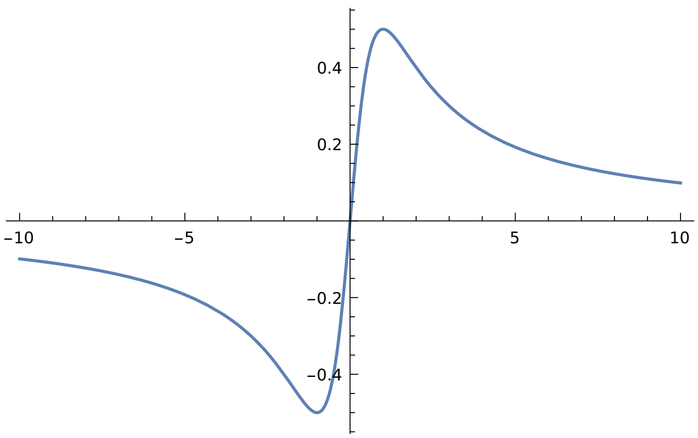
MAT/19/112

Q1). Graph each of the functions. Experiment with different domains or viewpoints to display the best images.

(a) f(x) = x/(1+x^2)

In[9]:=
```
Plot[x/(1 + x^2), {x, -10, 10}]
```

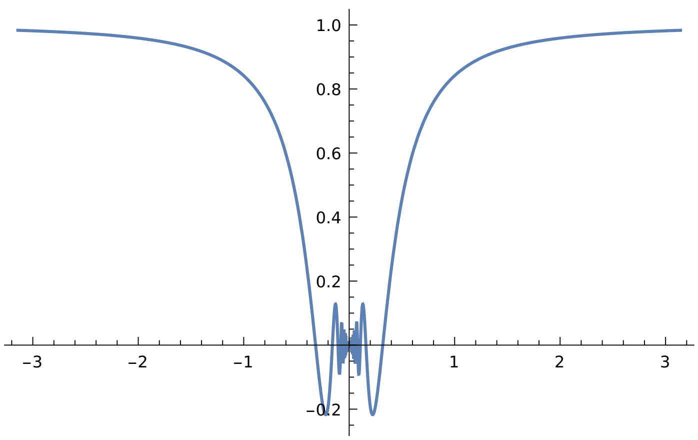
Out[9]=



(b) y = x sin(1/x)

In[10]:=
```
Plot[x Sin[1 / x], {x, -Pi, Pi}]
```

Out[10]=

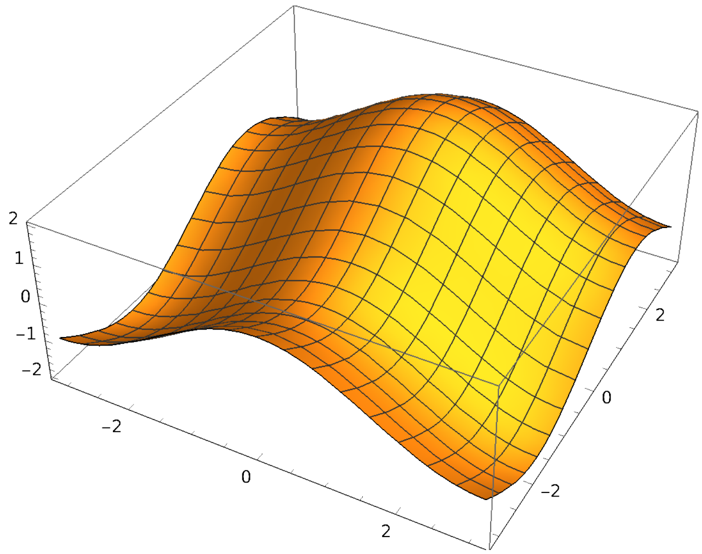


(c) g(x,y) = cos(x)+sin(y)

In[12]:=

```
Plot3D[Cos[x] + Sin[y], {x, -Pi, Pi}, {y, -Pi, Pi}]
```

Out[12]=

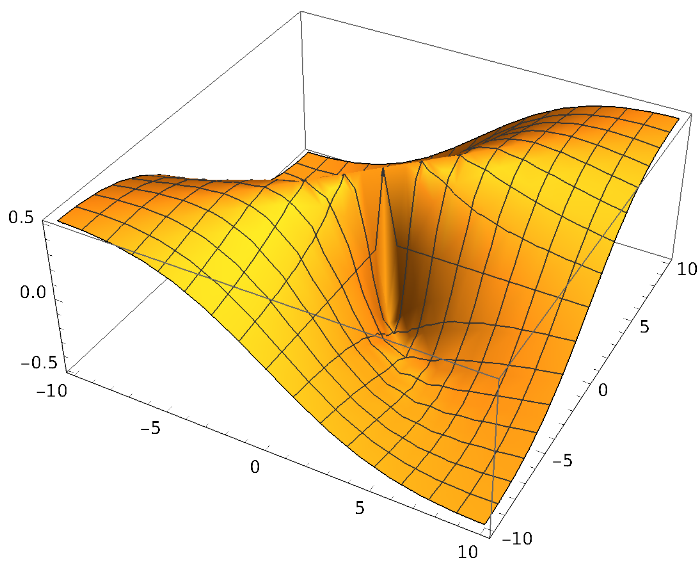


(d) z = xy/(x^2+y^2)

In[19]:=

```
Plot3D[{x y/(x^2 + y^2)}, {x, -10, 10}, {y, -10, 10}]
```

Out[19]=



Q2). Let f(x)=x/(1+x^2)

In[20]:=

```
f[x_] := x/(1 + x^2)
```

(a) Find f'(x)and f''(x)

In[28]:= `D[f[x], x]`

Out[28]= $-\frac{2x^2}{(1+x^2)^2}+\frac{1}{1+x^2}$

In[23]:= `D[f[x], {x, 2}]`

Out[23]= $-\frac{4x}{(1+x^2)^2}+x\left(\frac{8x^2}{(1+x^2)^3}-\frac{2}{(1+x^2)^2}\right)$

(b) Find f'(-1)and f'(0)

In[24]:= `f'[-1]`

Out[24]= 0

In[25]:= `f'[0]`

Out[25]= 1

(c) Find f''(0)and f''(1)

In[26]:= `f''[0]`

Out[26]= 0

In[27]:= `f''[1]`

Out[27]= $-\frac{1}{2}$

Q3). Find the prime factorization of each integer:

(a) 3527218133309949276293

In[30]:= `FactorInteger[3 527 218 133 309 949 276 293 ]`

Out[30]= {{15 013 , 2}, {25 013 , 3}}

(b) 471945325930166269

In[31]:= `FactorInteger[471 945 325 930 166 269 ]`

Out[31]= {{4211 , 1}, {34 589 , 1}, {46 747 , 1}, {69 313 , 1}}

(c) 471945325930166281

In[32]:= `FactorInteger[471 945 325 930 166 281 ]`

Out[32]= {{471 945 325 930 166 281 , 1}}

Q4). Complete each expression. Do you notice a pattern?

(a) 3^6 mod 7

In[33]:=
```
Mod[3^6, 7]
```
Out[33]= 1

(b) 6^10 mod 11

In[34]:=
```
Mod[6^10, 11]
```
Out[34]= 1

(c) 7^20 mod 21

In[35]:=
```
Mod[7^20, 21]
```
Out[35]= 7

(d) 7^22 mod 23

In[36]:=
```
Mod[7^22, 23]
```
Out[36]= 1

Q8). Let M=[{1,1},{1,0}]

In[38]:=
```
M = {{1, 1}, {1, 0}}
```
Out[38]= {{1, 1}, {1, 0}}

(a) Find M^2,M^3,.......,M^10

In[39]:=
```
MatrixPower[M, 2]
```
Out[39]= {{2, 1}, {1, 1}}

In[40]:=
```
MatrixPower[M, 3]
```
Out[40]= {{3, 2}, {2, 1}}

In[41]:=
```
MatrixPower[M, 4]
```
Out[41]= {{5, 3}, {3, 2}}

In[42]:=
```
MatrixPower[M, 5]
```
Out[42]= {{8, 5}, {5, 3}}

In[43]:=
```
MatrixPower[M, 6]
```
Out[43]= {{13, 8}, {8, 5}}

In[44]:=
```
MatrixPower[M, 7]
```
Out[44]= {{21, 13}, {13, 8}}

In[45]:=
```
MatrixPower[M, 8]
```
Out[45]= {{34, 21}, {21, 13}}

In[46]:=
```
MatrixPower[M, 9]
```
Out[46]= {{55, 34}, {34, 21}}

In[47]:=
```
MatrixPower[M, 10]
```
Out[47]= {{89, 55}, {55, 34}}

(b) Do your answers suggest a way to compute Fibonacci numbers? Find the 100th Fibonacci number.

In[48]:=
```
f[0] = 1;
```
In[49]:=
```
f[1] = 1;
```
In[50]:=
```
f[n_] := f[n] = f[n - 2] + f[n - 1]
```
In[51]:=
```
f[100]
```
Out[51]= 573 147 844 013 817 084 101

Q9). Find solutions to the following equations or systems of equations:

(a) Find x, if x^2+x=1

In[54]:=
```
Solve[{x ^ 2 + x == 1}, x]
```
Out[54]= $\left\{\left\{x \to \frac{1}{2}\left(-1-\sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(-1+\sqrt{5}\right)\right\}\right\}$

(b) Find x, if x^2+x=-1

In[55]:=
```
Solve[{x ^ 2 + x == -1}, x]
```
Out[55]= $\{\{x \to -(-1)^{1/3}\}, \{x \to (-1)^{2/3}\}\}$

(c) Find x and y

4x-3y=5

6x+2y=14

In[56]:=
```
Solve[{4 x - 3 y == 5 && 6 x + 2 y == 14}, {x, y}]
```
Out[56]= {{x → 2, y → 1}}

(d) Find x,y ,z znd t

-2x-2y+3z+t=8

-3x+0y-6z+t=-19

6x-8y+6z+5t=47

x+3y-3z-t=-9

In[57]:=
```
Solve[{-2 x - 2 y + 3 z + t == 8 && -3 x + 0 y - 6 z + t == -19 &&
    6 x - 8 y + 6 z + 5 t == 47 && x + 3 y - 3 z - t == -9}, {x, y, z, t}]
```
Out[57]= {{x → 2, y → 1, z → 3, t → 5}}

Q10). Assume that I invest $250 at the beginning of the year, $300 at the

beginning of the second quarter, $350 at the beginning of the third quarter, and $400 at the beginning of the fourth quarter. At the end of the year, I have $1365 (because my investments grow). To find my (continuous) rate of return,solve this equation for r :

250ⅇ^1.0r + 300ⅇ^0.75r + 350ⅇ^0.5r + 400ⅇ^0.25r = 1365

```
In[65]:= FindRoot[{250 ⅇ^(1.0 r) + 300 ⅇ^(0.75 r) + 350 ⅇ^(0.5 r) + 400 ⅇ^(0.25 r) == 1365}, {r, 0}]

Out[65]= {r → 0.084104}
```

Q11). If n is a positive number, and g>0 is any "guess" for the square root of n, then a better estimate of $\sqrt{n}$ is the average of g and n/g, i.e., (g+(n/g))/2. Write a function called mysqrt that accepts one argument, begins with an initial guess of 1.0, finds 20 new guesses, returns the answer.

```
In[66]:= mysqrt[n_] := Module[{i = 1, g = 1}, While[i ≤ 20, g = (g + (n / g)) / 2; i = i + 1]; g]

In[97]:= N[mysqrt[2], 6]

Out[97]= 1.41421

In[98]:= N[Sqrt[2], 6]

Out[98]= 1.41421

In[99]:= N[mysqrt[3]]

Out[99]= 1.73205
```

Q12). The Collatz conjecture states that if we start from any natural number a_0=n and form a sequence by the rule

a_(i+1)={a_i/2, if a_i is even & 3a_i+1, if a_i is odd ,

then the sequence eventually contains the value 1. For example, starting from a_0=6, we get the sequence 6, 3, 10, 5, 16, 8, 4, 2, 1 (we reached 1 after eight steps).

(a) Write a (recursive) function called collatz that accepts a single argument, n, and returns :

- 0 if n is equal to 1
- 1+collatz(n/2) if n is even
- 1+collatz(3*n+1) if n is odd

Thus, collatz(n) is the number of steps needed to go from n to 1

In[100]:=
```
Clear[collatz];
```

In[101]:=
```
collatz[n_] := Which[n == 1, collatz[n] = 0, EvenQ[n],
    collatz[n] = 1 + collatz[n / 2], OddQ[n], collatz[n] = 1 + collatz[3 * n + 1]];
```

(b) Verify the values :

n	collatz(n)
1	0
2	1
6	8
27	111

In[103]:=
```
collatz[1]
```

Out[103]= 0

In[104]:=
```
collatz[2]
```

Out[104]= 1

In[105]:=
```
collatz[6]
```

Out[105]= 8

In[106]:=
```
collatz[27]
```

Out[106]= 111