SARMISTHA SUTAR
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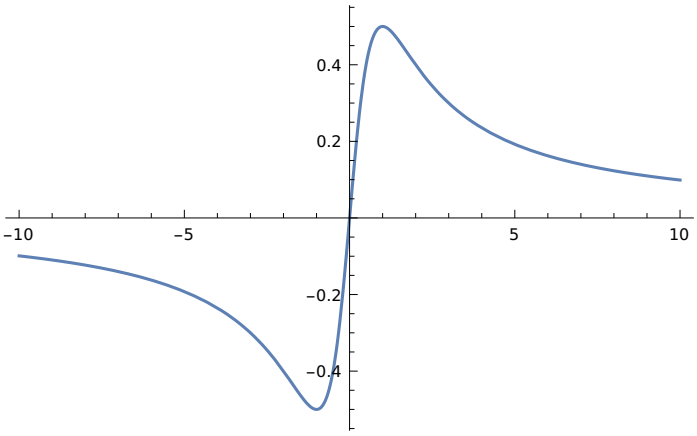
CHAPTER — 12

Q1. Graph each of the functions . Experiment with different domains or viewpoints to display the best images .

(a) f (x) = x / (1 + x ^ 2)

In[1]:=
```
Plot[x / (1 + x ^ 2), {x, -10, 10}]
```

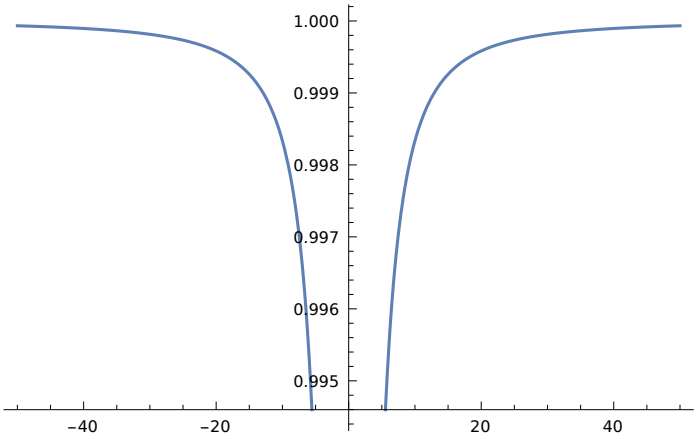
Out[1]=



(b) y = x sin (1 / x)

In[2]:=
```
Plot[x * Sin[1 / x], {x, -50, 50}]
```

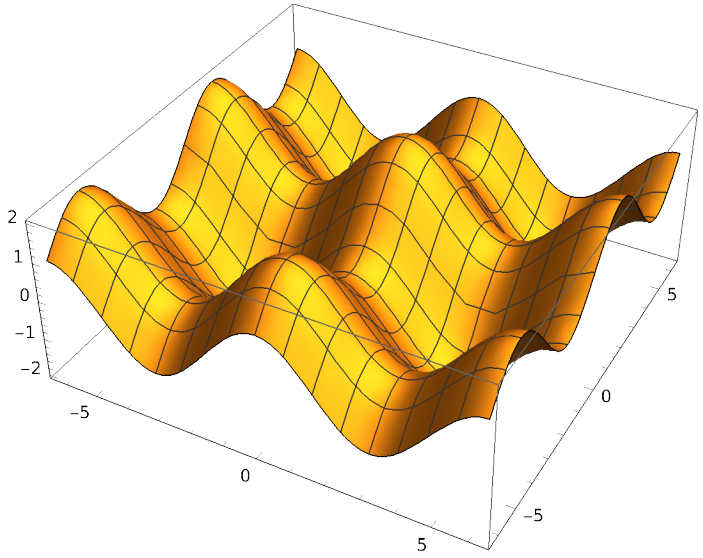
Out[2]=



(c) g (x, y) = cos (x) + sin (y)

In[5]:=
```
Plot3D[Cos[x] + Sin[y], {x, -2 Pi, 2 Pi}, {y, -2 Pi, 2 Pi}]
```

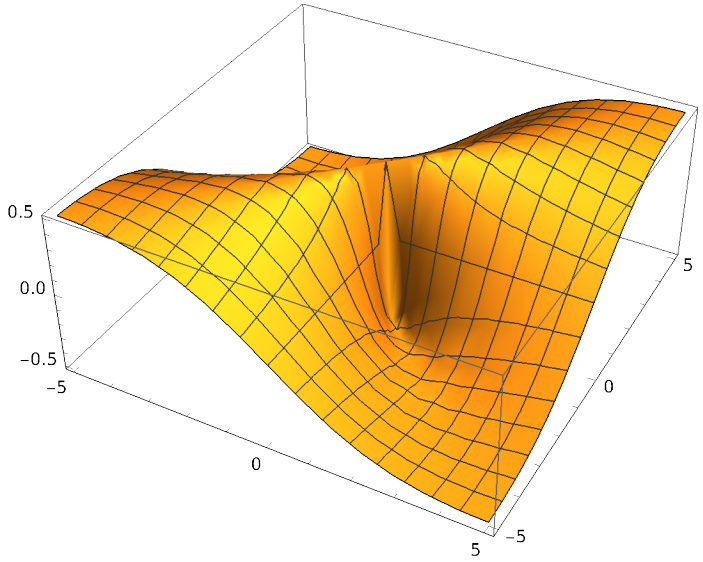
Out[5]=



(d) z = xy / x^2 + y^2

In[6]:=
```
Plot3D[x * y / (x ^ 2 + y ^ 2), {x, -5, 5}, {y, -5, 5}]
```

Out[6]=



Q2. Let f (x) = x / (1 + x^2)

In[7]:=
```
f[x_] := x / (1 + x ^ 2)
```

(a) Find f' (x) and f' (x).

In[8]:=
```
f'[x]
```

Out[8]=
$$-\frac{2x^2}{(1+x^2)^2}+\frac{1}{1+x^2}$$

In[9]:=
```
f''[x]
```

Out[9]=
$$\frac{8x^3}{(1+x^2)^3}-\frac{6x}{(1+x^2)^2}$$

(b) Find f' (– 1) and f' (0).

In[10]:=
```
f'[-1]
```

Out[10]=
```
0
```

In[11]:=
```
f'[0]
```

Out[11]=
```
1
```

(c) Find f'' (0) and f'' (1).

In[12]:=
```
f''[0]
```

Out[12]=
```
0
```

In[13]:=
```
f''[1]
```

Out[13]=
$$-\frac{1}{2}$$

In[14]:=
```
ClearAll[f]
```

Q3. Find the prime factorization of each integer .

(a) 3, 527, 218, 133, 309, 949, 276, 293

In[15]:=
```
FactorInteger [3 527 218 133 309 949 276 293 ]
```

Out[15]=
```
{{15 013 , 2}, {25 013 , 3}}
```

(b) 471, 945, 325, 930, 166, 269

In[16]:=
```
FactorInteger [ 471 945 325 930 166 269 ]
```

Out[16]=
```
{{4211 , 1}, {34 589 , 1}, {46 747 , 1}, {69 313 , 1}}
```

(c) 471, 945, 325, 930, 166, 281

In[17]:=
```
FactorInteger[471 945 325 930 166 281]
```
Out[17]= {{471 945 325 930 166 281, 1}}

Q4. Compute each expression .

(a) 3^6 mod 7

In[18]:=
```
Mod[3^6, 7]
```
Out[18]= 1

(b) 6^10 mod 11

In[19]:=
```
Mod[6^10, 11]
```
Out[19]= 1

(c) 7^20 mod 21

In[20]:=
```
Mod[7^20, 21]
```
Out[20]= 7

(d) 7^22 mod 23

In[21]:=
```
Mod[7^22, 23]
```
Out[21]= 1

Q8.

(a)

In[22]:=
```
M = {{1, 1}, {1, 0}}
```
Out[22]= {{1, 1}, {1, 0}}

In[23]:=
```
M.M
```
Out[23]= {{2, 1}, {1, 1}}

In[24]:=
```
M.M.M
```
Out[24]= {{3, 2}, {2, 1}}

In[25]:=
```
M.M.M.M
```
Out[25]= {{5, 3}, {3, 2}}

In[26]:=
```
M.M.M.M.M
```
Out[26]= {{8, 5}, {5, 3}}

In[27]:=
```
M.M.M.M.M.M
```
Out[27]= {{13, 8}, {8, 5}}

In[28]:=
```
M.M.M.M.M.M.M
```
Out[28]= {{21, 13}, {13, 8}}

In[29]:=
```
M.M.M.M.M.M.M.M
```
Out[29]= {{34, 21}, {21, 13}}

In[30]:=
```
M.M.M.M.M.M.M.M.M
```
Out[30]= {{89, 55}, {55, 34}}

(b)

In[31]:=
```
g[0] = {{1, 1}, {1, 0}};
```

In[32]:=
```
g[1] = {{2, 1}, {1, 1}};
```

In[33]:=
```
g[n_] := g[n] = g[n - 2] + g[n - 1]
```

In[34]:=
```
g[100]
```
Out[34]= {{927 372 692 193 078 999 176 , 573 147 844 013 817 084 101 },
{573 147 844 013 817 084 101 , 354 224 848 179 261 915 075 }}

Q9.

(a)

In[35]:=
```
Solve[{x^2 + x == 1}, {x}]
```
Out[35]= $\left\{\left\{x \to \frac{1}{2}\left(-1-\sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(-1+\sqrt{5}\right)\right\}\right\}$

(b)

In[36]:=
```
Solve[{x^2 + x == -1}, {x}]
```
Out[36]= $\left\{\{x \to -(-1)^{1/3}\}, \{x \to (-1)^{2/3}\}\right\}$

(c)

In[37]:=
```
Solve[{4 * x - 3 * y == 5, 6 * x + 2 * y == 14}, {x, y}]
```
Out[37]= {{x → 2, y → 1}}

(d)

In[38]:=
```
Solve[{-2*x-2*y+3*z+t == 8, -3*x+0*y-6*z+t == -19,
  6*x-8*y+6*z+5*t == 47, x+3*y-3*z-t == -9}, {x, y, z, t}]
```

Out[38]=
```
{{x → 2, y → 1, z → 3, t → 5}}
```

Q10.

In[39]:=
```
FindRoot[250*e^(1.0*r)+300*e^(0.75*r)+350*e^(0.5*r)+400*e^(0.25*r) == 1365, {r, 1}]
```

Out[39]=
```
{r → 0.084104}
```

Q11.

In[40]:=
```
mysqrt[n_] := Module[{i = 1, g = 1}, While[i ≤ 20, g = ((g+(n/g))/2); i = i+1]; g]
```

In[41]:=
```
N[mysqrt[2], 6]
```

Out[41]=
```
1.41421
```

In[42]:=
```
N[Sqrt[2], 6]
```

Out[42]=
```
1.41421
```

In[43]:=
```
N[mysqrt[3]]
```

Out[43]=
```
1.73205
```

Q12.

(a)

In[44]:=
```
collatz[n_] := Which[n == 1, collatz[n] = 0, EvenQ[n],
  collatz[n] = 1+collatz[n/2], OddQ[n], collatz[n] = 1+collatz[3*n+1]]
```

(b)

In[45]:=
```
collatz[1]
```

Out[45]=
```
0
```

In[46]:=
```
collatz[2]
```

Out[46]=
```
1
```

In[47]:=
```
collatz[6]
```

Out[47]=
```
8
```

In[49]:= **collatz[27]**

Out[49]= 111