Tanya Gupta
MAT/19/73

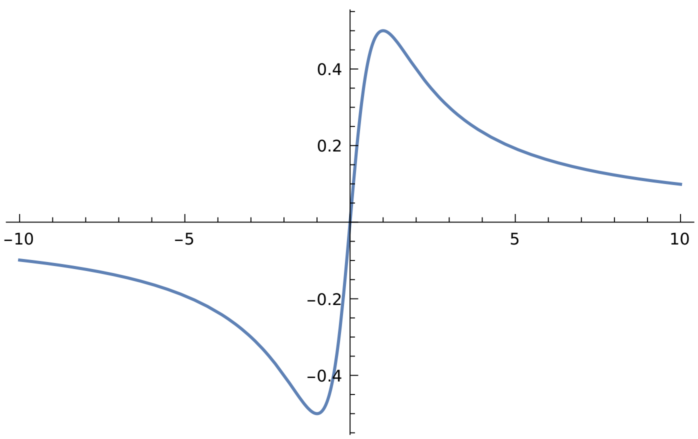
Chapter 12(Exercise Questions)

**Q1. Graph each of the functions. Experiment with different domains or view points to
display the best images.**

a)f(x)=x/(1+x^2)

In[6]:=
```
Plot[x / (1 + x ^ 2), {x, -10, 10}]
```

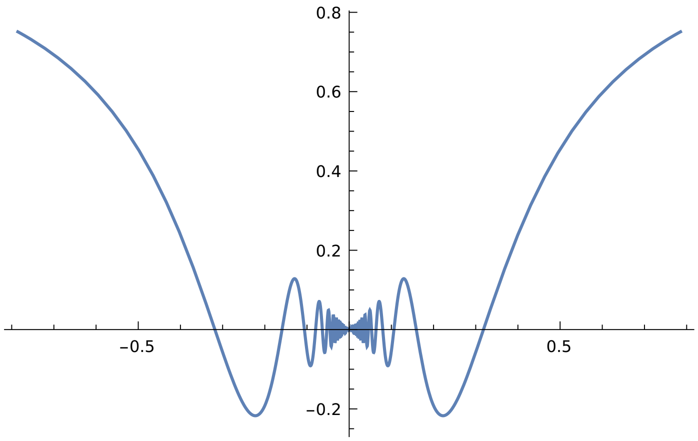
Out[6]=



b) y = x sin[1/x)]

In[7]:=
```
Plot[x Sin[1 / x], {x, -Pi / 4, Pi / 4}]
```

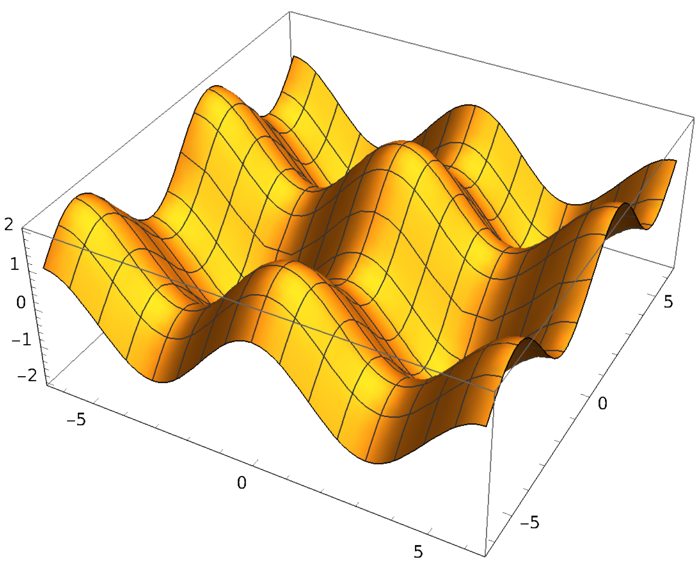
Out[7]=



c)g(x,y) = Cos[x]+Sin[y]

In[8]:=
```
Plot3D[Cos[x] + Sin[y], {x, -2 Pi, 2 Pi}, {y, -2 Pi, 2 Pi}]
```

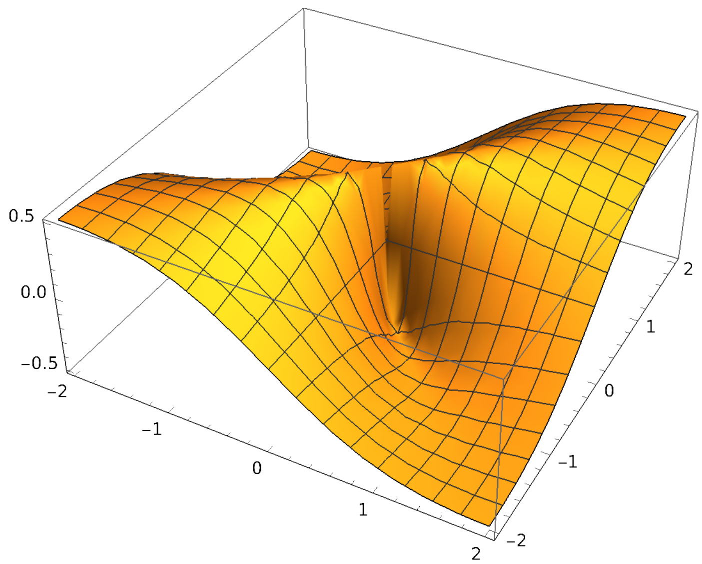
Out[8]=



d) z = x y/(x^2+y^2)

In[9]:=
```
Plot3D[x y / (x ^ 2 + y ^ 2), {x, -2, 2}, {y, -2, 2}]
```

Out[9]=



In[10]:=
```
ClearAll[x]
```

Q2. Let f(x) = x/(1+x^2)

a) Find f'(x) and f''(x)
b) Find f'(-1) and f'(0)
c) Find f''(0) and f''(1)

In[11]:=
```
f[x_] := x / (1 + x ^ 2)
```

In[12]:=
```
D[f[x], x]
```

Out[12]=

$$-\frac{2 x^2}{\left(1+x^2\right)^2}+\frac{1}{1+x^2}$$

In[13]:=
```
D[D[f[x], x], x]
```

Out[13]=

$$\frac{8 x^3}{\left(1+x^2\right)^3}-\frac{6 x}{\left(1+x^2\right)^2}$$

In[14]:=
```
D[f[x], x] /. x → -1
```

Out[14]= 0

In[15]:=
```
D[f[x], x] /. x → 0
```

Out[15]= 1

In[16]:=
```
D[D[f[x], x], x] /. x → 0
```
Out[16]= 0

In[17]:=
```
D[D[f[x], x], x] /. x → 1
```
Out[17]= $-\frac{1}{2}$

OTHER METHOD OF Q 2.

In[18]:=
```
f '[x]
```
Out[18]= $-\frac{2\,x^2}{\left(1+x^2\right)^2}+\frac{1}{1+x^2}$

In[19]:=
```
f ''[x]
```
Out[19]= $\frac{8\,x^3}{\left(1+x^2\right)^3}-\frac{6\,x}{\left(1+x^2\right)^2}$

In[20]:=
```
f '[-1]
```
Out[20]= 0

In[21]:=
```
f '[0]
```
Out[21]= 1

In[22]:=
```
f ''[0]
```
Out[22]= 0

In[23]:=
```
f ''[1]
```
Out[23]= $-\frac{1}{2}$

Q3. Find the prime factorization of each integer

a) 3,527,218,133,309,949,276,293

In[24]:=
```
FactorInteger [3 527 218 133 309 949 276 293 ]
```
Out[24]= {{15 013 , 2}, {25 013 , 3}}

b) 471,945,325,930,166,269

In[25]:=
```
FactorInteger [471 945 325 930 166 269 ]
```
Out[25]= {{4211 , 1}, {34 589 , 1}, {46 747 , 1}, {69 313 , 1}}

c) 471,945,325,930,166,281

In[26]:=
```
FactorInteger [471 945 325 930 166 281 ]
```
Out[26]=
```
{{471 945 325 930 166 281 , 1}}
```

Q4. Compute each expression.Do you notice a pattern?

a) 3^6 mod 7

In[27]:=
```
Mod[3 ^ 6, 7]
```
Out[27]=
```
1
```

b) 6^10 mod 11

In[28]:=
```
Mod[6 ^ 10, 11]
```
Out[28]=
```
1
```

c)7^20 mod 21

In[29]:=
```
Mod[7 ^ 20, 21]
```
Out[29]=
```
7
```

d)7^22 mod 23

In[30]:=
```
Mod[7 ^ 22, 23]
```
Out[30]=
```
1
```

Yes,there is a pattern in a^(n-1) mod n where a and n be any positive integers if a and n are coprime to each other then definetly a^(n-1) mod n is 1

Q8.

a)

In[31]:=
```
M = {{1, 1}, {1, 0}}
```
Out[31]=
```
{{1, 1}, {1, 0}}
```

In[32]:=
```
Table[MatrixPower [M, i], {i, 2, 10}]
```
Out[32]=
```
{{{2, 1}, {1, 1}}, {{3, 2}, {2, 1}}, {{5, 3}, {3, 2}}, {{8, 5}, {5, 3}}, {{13, 8}, {8, 5}},
 {{21, 13}, {13, 8}}, {{34, 21}, {21, 13}}, {{55, 34}, {34, 21}}, {{89, 55}, {55, 34}}}
```

b) Find the 100th Fibonacci number

In[33]:=
```
Fibonacci[100]
```
Out[33]= 354 224 848 179 261 915 075

Q9. Find solutions to the following equations or system of equations

a)

In[34]:=
```
Solve[x^2 + x == 1, x]
```
Out[34]= $\left\{\left\{x \to \frac{1}{2}\left(-1-\sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(-1+\sqrt{5}\right)\right\}\right\}$

b)

In[35]:=
```
Solve[x^2 + x == -1, x]
```
Out[35]= $\{\{x \to -(-1)^{1/3}\}, \{x \to (-1)^{2/3}\}\}$

c)

In[36]:=
```
Solve[{4 x - 3 y == 5, 6 x + 2 y == 14}, {x, y}]
```
Out[36]= $\{\{x \to 2, y \to 1\}\}$

<u>OTHER METHOD OF Q 9.</u>

In[37]:=
```
Solve[4 x - 3 y == 5 && 6 x + 2 y == 14, {x, y}]
```
Out[37]= $\{\{x \to 2, y \to 1\}\}$

d)

In[38]:=
```
Solve[{-2 x - 2 y + 3 z + t == 8, -3 x + 0 y - 6 z + t == -19,
  6 x - 8 y + 6 z + 5 t == 47, x + 3 y - 3 z - t == -9}, {x, y, z, t}]
```
Out[38]= $\{\{x \to 2, y \to 1, z \to 3, t \to 5\}\}$

Q10.

In[39]:=
```
FindRoot[250 E^(1.0 r) + 300 E^(0.75 r) + 350 E^(0.5 r) + 400 E^(0.25 r) == 1365, {r, 0}]
```
Out[39]= $\{r \to 0.084104\}$

Therefore, rate of return = $ 0.084104

Q11.

In[40]:=
```
mysqrt[n_] := Module[{i = 1, g = 1}, While[i ≤ 20, g = (g + n / g) / 2; i = i + 1]; g]
```

In[41]:=
```
N[mysqrt[3]]
```

Out[41]=
```
1.73205
```

In[42]:=
```
N[mysqrt[2], 10]
```

Out[42]=
```
1.414213562
```

Q12.

In[43]:=
```
collatz[n_] := Which[n == 1, collatz[n] = 0, EvenQ[n],
  collatz[n] = 1 + collatz[n / 2], OddQ[n], collatz[n] = 1 + collatz[3 * n + 1]]
```

In[44]:=
```
collatz[27]
```

Out[44]=
```
111
```

In[45]:=
```
collatz[26]
```

Out[45]=
```
10
```