ASHWEEN KAUR(MAT/19/144)

ASSIGNMENT-1(CHAPTER-12)

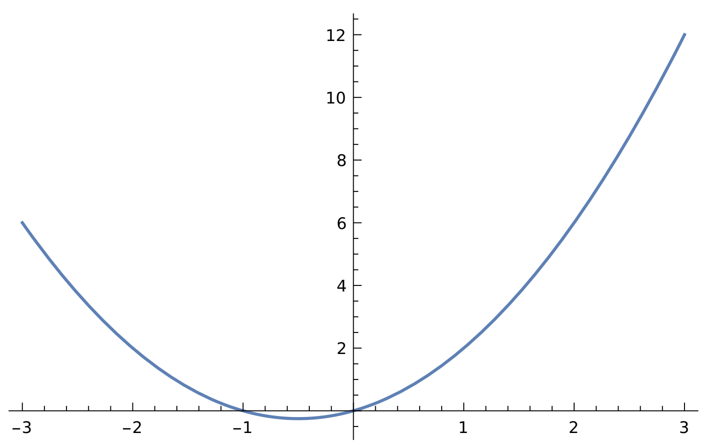
Ques 1: Graph each of the functions. Experiment with different domains or viewpoints to display the best image.

(a) f(x) = x/1+x^2

In[7]:=
```
f[x_] := x / 1 + x ^ 2
```

In[8]:=
```
Plot[f[x], {x, -3, 3}]
```

Out[8]=

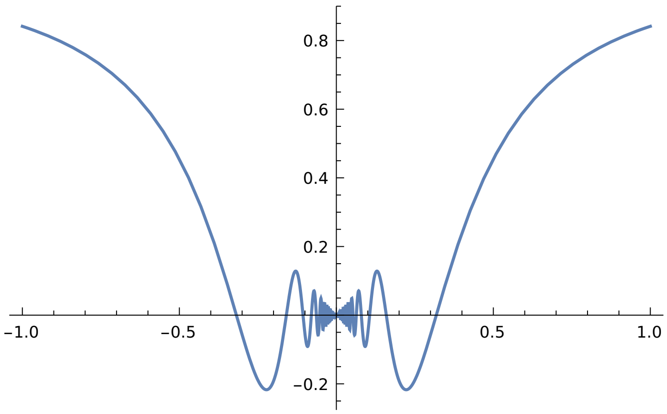


(b) y = x Sin(1/x)

In[3]:=
```
y := x * Sin[1 / x]
```

In[4]:=
```
Plot[y, {x, -1, 1}]
```

Out[4]=

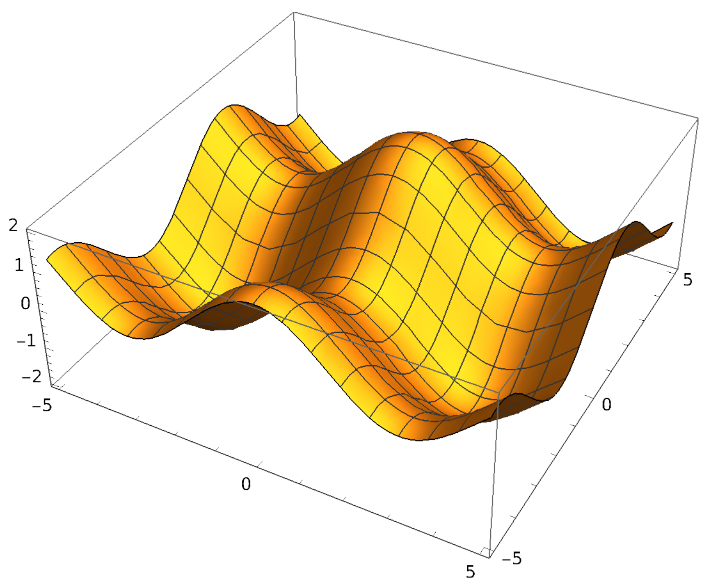


(c) g(x,y) = Cos(x) + Sin(y)

In[10]:=
```
g[x_, y_] := Cos[x] + Sin[y]
```

In[11]:=
```
Plot3D[g[x, y], {x, -5, 5}, {y, -5, 5}]
```

Out[11]=

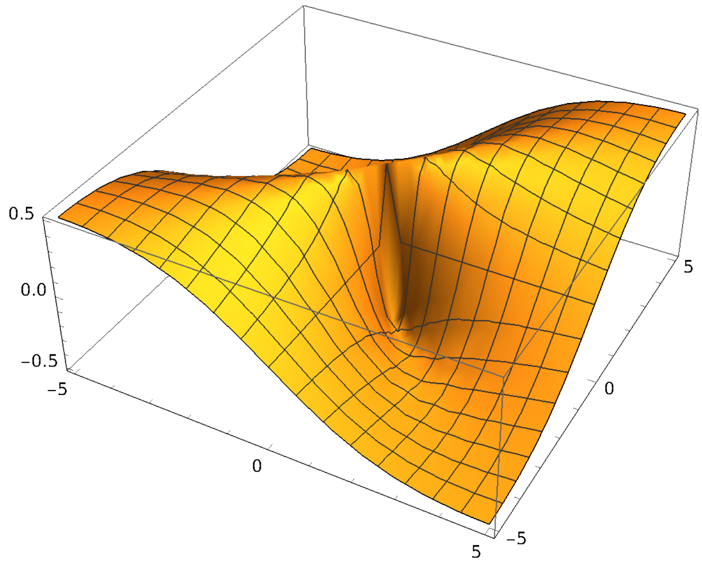


(d) z = xy/x^2 + y^2

In[13]:=
```
z := x * y / (x ^ 2 + y ^ 2)
```

In[14]:= `Plot3D[z, {x, -5, 5}, {y, -5, 5}]`

Out[14]=



Ques 2: Let f(x) = x/(1+x^2).

(a) Find f'(x) and f''(x).

In[15]:= `f[x_] := x / (1 + x ^ 2)`

In[16]:= `D[f[x], x]`

Out[16]= $-\frac{2\,x^2}{(1+x^2)^2}+\frac{1}{1+x^2}$

In[17]:= `f'[x]`

Out[17]= $-\frac{2\,x^2}{(1+x^2)^2}+\frac{1}{1+x^2}$

In[18]:= `f''[x]`

Out[18]= $\frac{8\,x^3}{(1+x^2)^3}-\frac{6\,x}{(1+x^2)^2}$

(b) Find f'(-1) and f''(0)

In[19]:= `f'[-1]`

Out[19]= 0

In[20]:= `f'[0]`

Out[20]= 1

(c) Find f"(0) and f"(1)

In[21]:=
```
f''[0]
```
Out[21]= 0

In[22]:=
```
f''[1]
```
Out[22]= $-\frac{1}{2}$

Ques 3: Find the prime factorisation of each integer.

(a) 3,527,218,133,309,949,276,293

In[17]:=
```
FactorInteger[3 527 218 133 309 949 276 293 ]
```
Out[17]= {{15 013 , 2}, {25 013 , 3}}

(b) 471,945,325,930,166,269

In[18]:=
```
FactorInteger[471 945 325 930 166 269 ]
```
Out[18]= {{4211 , 1}, {34 589 , 1}, {46 747 , 1}, {69 313 , 1}}

(c) 471,945,325,930,166,281

In[19]:=
```
FactorInteger[471 945 325 930 166 281 ]
```
Out[19]= {{471 945 325 930 166 281 , 1}}

Ques 4: Compute each of the expression. Do you notice a pattern?

(a) 3^6 mod 7

In[42]:=
```
Mod[3 ^ 6, 7]
```
Out[42]= 1

(b) 6^10 mod 11

In[43]:=
```
Mod[6 ^ 10, 11]
```
Out[43]= 1

(c) 7^20 mod 21

In[44]:=
```
Mod[7^20, 21]
```
Out[44]= 7

(d) 7^22 mod 23

In[45]:=
```
Mod[7^22, 23]
```
Out[45]= 1

Ques 8: M ={{1,1},{1,0}}

(a) Find M^2,M^3,....,M^10.

In[29]:=
```
M = {{1, 1}, {1, 0}};
```
In[30]:=
```
M // MatrixForm
```
Out[30]//MatrixForm=

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

In[31]:=
```
M.M;
```
In[32]:=
```
M.M // MatrixForm
```
Out[32]//MatrixForm=

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

In[33]:=
```
M3 = {{1, 1}, {1, 0}}.{{1, 1}, {1, 0}}.{{1, 1}, {1, 0}};
```
In[34]:=
```
M3 // MatrixForm
```
Out[34]//MatrixForm=

$$\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$$

In[35]:=
```
M4 = {{1, 1}, {1, 0}}.{{1, 1}, {1, 0}}.{{1, 1}, {1, 0}}.{{1, 1}, {1, 0}};
```
In[36]:=
```
M4 // MatrixForm
```
Out[36]//MatrixForm=

$$\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$$

In[37]:=
```
M10 = MatrixPower[{{1, 1}, {1, 0}}, 10];
```
In[38]:=
```
M10 // MatrixForm
```
Out[38]//MatrixForm=

$$\begin{pmatrix} 89 & 55 \\ 55 & 34 \end{pmatrix}$$

(b) Do your answer suggest a way to compute Fibonacci numbers? Find the 100th Fibonacci number.

In[39]:=
```
M100 = MatrixPower[{{1, 1}, {1, 0}}, 100];
```

In[40]:=
```
M100 // MatrixForm
```

Out[40]//MatrixForm=

$$\begin{pmatrix} 573\,147\,844\,013\,817\,084\,101 & 354\,224\,848\,179\,261\,915\,075 \\ 354\,224\,848\,179\,261\,915\,075 & 218\,922\,995\,834\,555\,169\,026 \end{pmatrix}$$

yes, by finding matrix power of M , we get more efficient way to compute Fibonacci numbers . Thus the first element of M 100 matrix is 100th Fibonacci number, that is 573 147 844 013 817 084 101.

Ques 9: Find solutions to the following equations or system of equations

(a) Find x, if x^2 + x = 1.

In[41]:=
```
? Solve
```

Out[41]=

Symbol

Solve [*expr* , *vars*] attempts to solve the system *expr* of equations or inequalities for the variables *vars* .

Solve [*expr* , *vars* , *dom*] solves over the domain

dom . Common choices of *dom* are Reals , Integers , and Complexes .

In[50]:=
```
Solve[x ^ 2 + x == 1, x]
```

Out[50]=

$$\left\{\left\{x \to \frac{1}{2}\left(-1 - \sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(-1 + \sqrt{5}\right)\right\}\right\}$$

(b) Find x, if x^2 + x = -1

In[51]:=
```
Solve[x ^ 2 + x == -1, x]
```

Out[51]=

$$\{\{x \to -(-1)^{1/3}\}, \{x \to (-1)^{2/3}\}\}$$

(c) Find x and y,

4 x-3 y = 5

6 x+2 y = 4

In[52]:=
```
Solve[{4 x - 3 y == 5, 6 x + 2 y == 4}, {x, y}]
```

Out[52]= $\left\{\left\{x \to \frac{11}{13}, y \to -\frac{7}{13}\right\}\right\}$

(d) Find x , y , z and t,

-2 x -2 y+ 3 z+ t = 8

-3 x + 0 y -6 z + t= -19

6 x -8 y + 6 z + 5 t = 47

x + 3 y -3 z - t = -9

In[1]:=
```
Solve[{-2 x - 2 y + 3 z + t == 8, -3 x + 0 y - 6 z + t == -19,
   6 x - 8 y + 6 z + 5 t == 47, x + 3 y - 3 z - t == -9}, {x, y, z, t}]
```

Out[1]= {{x → 2, y → 1, z → 3, t → 5}}

Ques 10: Solve this equation for r,

250 e^(1.0 r)+300 e^(0.75 r)+350 e^(0.5 r)+400 e(0.2 r)=1365

In[42]:=
```
? FindRoot
```

Out[42]=

Symbol

FindRoot [f, {x, x_0}] searches for a numerical root of f, starting from the point $x = x_0$.

FindRoot [$lhs == rhs$, {x, x_0}] searches for a numerical solution to the equation $lhs == rhs$.

FindRoot [{f_1, f_2, …}, {{x, x_0}, {y, y_0}, …}] searches for a simultaneous numerical root of all the f_i.

FindRoot [{eqn_1, eqn_2, …}, {{x, x_0}, {y, y_0}, …}]

searches for a numerical solution to the simultaneous equations eqn_i.

In[45]:=
```
FindRoot[{250 E^{1.0 r} + 300 E^{0.75 r} + 350 E^{0.5 r} + 400 E^{0.2 r} == 1365}, {r, 0}]
```

Out[45]= {r → 0.0863129}

Ques 11: write a function that accepts one argument ,begins with initial guess of 1.0,find 20 new guesses, and returns the answer.

```
In[31]:= mysqrt[n_] := Module[{i = 1, g = 1}, While[i ≤ 20, g = (g + n / g) / 2; i = i + 1]; g]

In[22]:= N[mysqrt[2], 6]

Out[22]= 1.41421

In[32]:= N[Sqrt[2], 6]

Out[32]= 1.41421

In[24]:= N[mysqrt[3]]

Out[24]= 1.73205
```

Ques 12:

```
In[7]:= Clear[collatz];

In[30]:= Collatz[n_] := Which[n == 1, Collatz[n] = 0, EvenQ[n],
           Collatz[n] = 1 + Collatz[n / 2], OddQ[n], Collatz[n] = 1 + Collatz[3 n + 1]];

In[28]:= Collatz[27]

Out[28]= 111
```