Assignment

Chapter-12

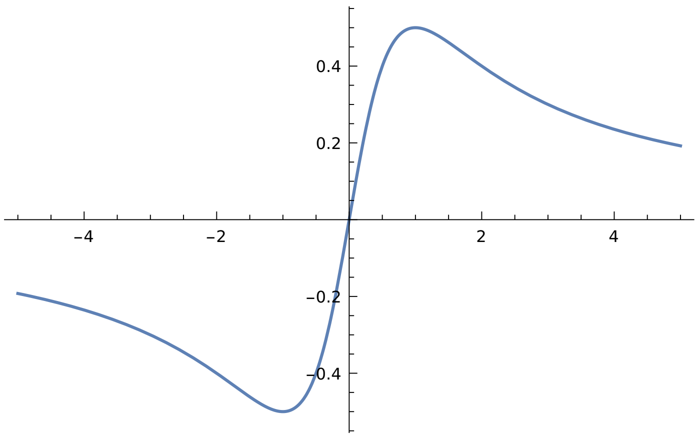
MAT/19/77

1) Group each of the function:

a) f(x)=x/(1+x^2)

In[1]:=
```
f[x_] := x / (1 + x ^ 2)
Plot[f[x], {x, -5, 5}]
```

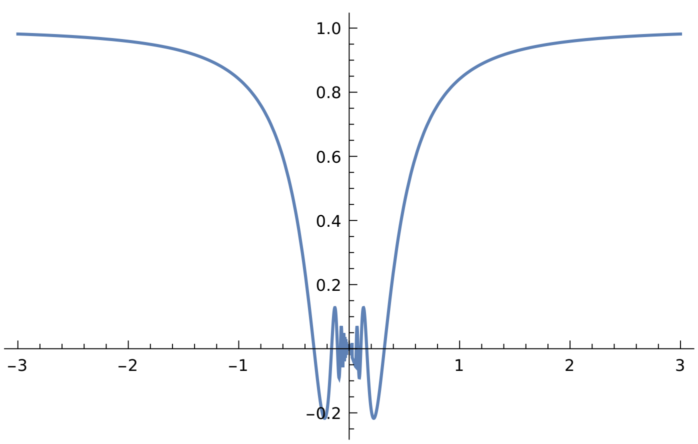
Out[2]=



b) y=xsin(1/x)

In[5]:=
```
f[x_] := x Sin[1 / x]
Plot[f[x], {x, -3, 3}]
```

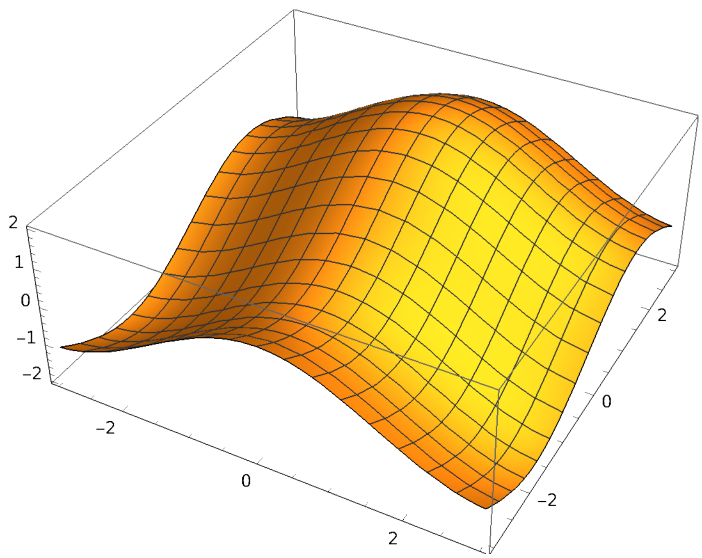
Out[6]=



c) g(x,y)=cos(x)+sin(y)

In[9]:=
```
f[x_, y_] := Cos[x] + Sin[y]
Plot3D[f[x, y], {x, -3, 3}, {y, -3, 3}]
```

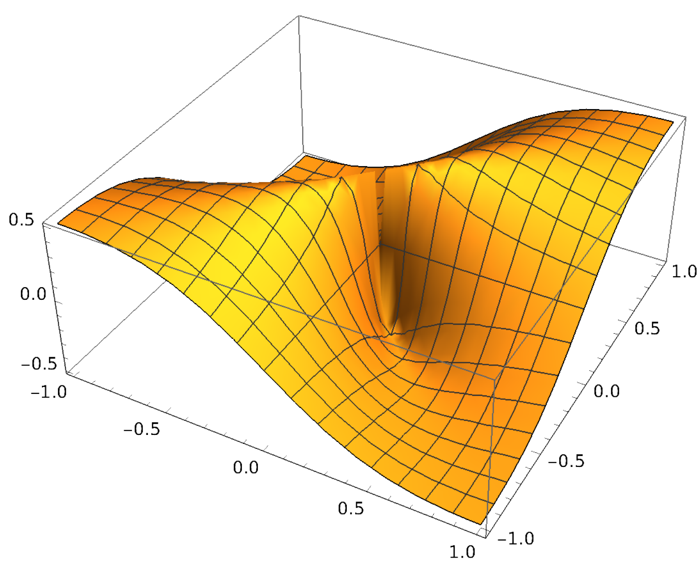
Out[10]=



d) z=xy/(x^2+y^2)

In[11]:=
```
f[x_, y_] := x y / (x ^ 2 + y ^ 2)
Plot3D[f[x, y], {x, -1, 1}, {y, -1, 1}]
```

Out[12]=



2) Let f(x)=x/(1+x^2)

a) Find f'(x) and f''(x)

In[13]:=
```
f[x_] := x / (1 + x ^ 2)
f '[x]
```

Out[14]=

$$-\frac{2\,x^2}{(1+x^2)^2}+\frac{1}{1+x^2}$$

In[15]:=
```
f''[x]
```

Out[15]= $\frac{8\ x^3}{(1+x^2)^3} - \frac{6\ x}{(1+x^2)^2}$

b) Find f'(-1) and f'(0)

In[16]:=
```
f'[x] /. {x → -1}
```

Out[16]= 0

In[17]:=
```
f'[-1]
```

Out[17]= 0

In[18]:=
```
f'[0]
```

Out[18]= 1

c) Find f''(0) and f''(1)

In[19]:=
```
f''[0]
```

Out[19]= 0

In[20]:=
```
f''[1]
```

Out[20]= $-\frac{1}{2}$

3) Find the prime factorixation of each integer:

a) 3,527,218,133,309,949,276,293

In[22]:=
```
FactorInteger [3 527 218 133 309 949 276 293  ]
```

Out[22]= {{15 013 , 2}, {25 013 , 3}}

b) 471,945,325,930,166,269

In[23]:=
```
FactorInteger [471 945 325 930 166 269 ]
```

Out[23]= {{4211 , 1}, {34 589 , 1}, {46 747 , 1}, {69 313 , 1}}

c) 471,945,325,930,166,281

In[24]:=
```
FactorInteger [471 945 325 930 166 281 ]
```

Out[24]= {{471 945 325 930 166 281 , 1}}

4) Compute each expression:

a) 3^6 mod 7

In[25]:=
```
Mod[3 ^ 6, 7]
```

Out[25]= 1

b) 6^10 mod 11

In[26]:=
```
Mod[6^10, 11]
```
Out[26]= 1

c) 7^20 mod 21

In[28]:=
```
Mod[7^20, 21]
```
Out[28]= 7

d) 7^22 mod 23

In[29]:=
```
Mod[7^22, 2]
```
Out[29]= 1

8)

In[30]:=
```
M = {{1, 1}, {1, 0}}
MatrixForm[M]
```
Out[30]= {{1, 1}, {1, 0}}

Out[31]//MatrixForm=

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

a) Find M 2, M 3,....,M 10

Now at every step after defining function F, F[n] is nothing but M^n:

In[32]:=
```
F[n_] := MatrixPower[M, (n - 1)].M
List[F[2], F[3], F[4], F[5], F[6], F[7], F[8], F[9], F[10]]
```
Out[33]= {{{2, 1}, {1, 1}}, {{3, 2}, {2, 1}}, {{5, 3}, {3, 2}}, {{8, 5}, {5, 3}}, {{13, 8}, {8, 5}}, {{21, 13}, {13, 8}}, {{34, 21}, {21, 13}}, {{55, 34}, {34, 21}}, {{89, 55}, {55, 34}}}

In[34]:=
```
MatrixForm[F[2]]
```
Out[34]//MatrixForm=

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

In[35]:=
```
MatrixForm[F[3]]
```
Out[35]//MatrixForm=

$$\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$$

In[36]:=
```
MatrixForm[F[4]]
```
Out[36]//MatrixForm=

$$\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$$

In[37]:=
```
MatrixForm[F[5]]
```
Out[37]//MatrixForm=
$$\begin{pmatrix} 8 & 5 \\ 5 & 3 \end{pmatrix}$$

In[38]:=
```
MatrixForm[F[6]]
```
Out[38]//MatrixForm=
$$\begin{pmatrix} 13 & 8 \\ 8 & 5 \end{pmatrix}$$

In[39]:=
```
MatrixForm[F[7]]
```
Out[39]//MatrixForm=
$$\begin{pmatrix} 21 & 13 \\ 13 & 8 \end{pmatrix}$$

In[40]:=
```
MatrixForm[F[8]]
```
Out[40]//MatrixForm=
$$\begin{pmatrix} 34 & 21 \\ 21 & 13 \end{pmatrix}$$

In[41]:=
```
MatrixForm[F[9]]
```
Out[41]//MatrixForm=
$$\begin{pmatrix} 55 & 34 \\ 34 & 21 \end{pmatrix}$$

In[42]:=
```
MatrixForm[F[10]]
```
Out[42]//MatrixForm=
$$\begin{pmatrix} 89 & 55 \\ 55 & 34 \end{pmatrix}$$

In[67]:=
```
ClearAll
```
Out[67]=
```
ClearAll
```

b) Find the 100th Fibonacci number.

In[47]:=
```
f[0] = 1;
f[1] = 1;
f[n_] := f[n] = f[n - 2] + f[n - 1]
f[100]
```
Out[50]= 573 147 844 013 817 084 101

9) Find solutions to the following equations or system of equations:

a) Find x, if x^2+x=1

In[51]:=
```
Solve[x ^ 2 + x == 1, x]
```
Out[51]=
$$\left\{\left\{x \to \frac{1}{2}\left(-1-\sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(-1+\sqrt{5}\right)\right\}\right\}$$

b) Find x, if x^2+x=-1

In[52]:=
```
Solve[x ^ 2 + x == -1, x]
```
Out[52]= $\{\{x \to -(-1)^{1/3}\}, \{x \to (-1)^{2/3}\}\}$

c) Find x and y:

4x-3y=5

6x+2y=14

In[53]:=
```
Solve[4 x - 3 y == 5 && 6 x + 2 y == 14]
```
Out[53]= {{x → 2, y → 1}}

d) Find x,y,z and t

-2x-2y+3z+t=8

-3x+0y-6z+t=-19

6x-8y+6z+5t=47

x+3y-3z-t=-9

In[54]:=
```
Solve[-2 x - 2 y + 3 z + t == 8 &&
 -3 x + 0 y - 6 z + t == -19 && 6 x - 8 y + 6 z + 5 t == 47 && x + 3 y - 3 z - t == -9]
```
Out[54]= {{t → 5, x → 2, y → 1, z → 3}}

```
ClearAll[x, y, z, t]
```

10) Assume that I invest $250 at the beginning of the year, $300 at the beginning of the second quarter, $350 at the beginning of the third quarter and $400 at the beginning of the fourth quarter. At the end of the year, I have $1365 (because my investments grow). To find my rate of return, solve this equation for r:

250e^1.0r + 300e^0.75r + 350e^0.5r + 400e^0.25r=1365

In[55]:=
```
eqn1 = 250 * Exp[1.0 r] + 300 * Exp[0.75 r] + 350 * Exp[0.5 r] + 400 * Exp[0.25 r] == 1365
```
Out[55]= $400\, e^{0.25\, r} + 350\, e^{0.5\, r} + 300\, e^{0.75\, r} + 250\, e^{1.\, r} == 1365$

In[56]:=
```
FindRoot[eqn1, {r, 1}]
```
Out[56]= {r → 0.084104}

11)

In[57]:=
```
mysqrt[n_] := Module[{i = 1, g = 1}, While[i ≤ 20, g = (g + n / g) / 2; i = i + 1]; g]
N[mysqrt[2]]
```
Out[58]= 1.41421

In[59]:=
```
N[mysqrt[10]]
```
Out[59]= 3.16228

12) a)

In[60]:=
```
collatz[n_] := Which[n == 1, collatz[n] = 0, EvenQ[n],
collatz[n] = 1 + collatz[n / 2], OddQ[n], collatz[n] = 1 + collatz[3 * n + 1]];
collatz[15]
```
Out[61]=
```
17
```

b)

In[62]:=
```
collatz[1]
```
Out[62]=
```
0
```

In[63]:=
```
collatz[2]
```
Out[63]=
```
1
```

In[64]:=
```
collatz[6]
```
Out[64]=
```
8
```

In[65]:=
```
collatz[27]
```
Out[65]=
```
111
```