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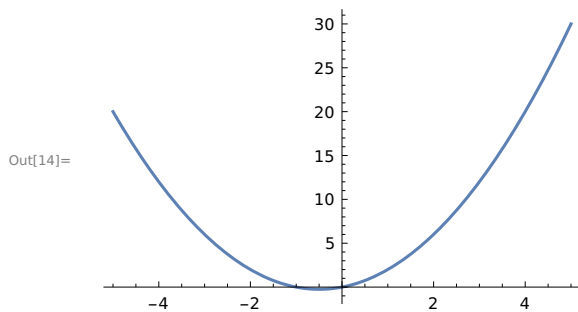
MAT/19/93

CHAPTER -12 EXERCISE

Q1. Graph each of the functions. Experiment with different domains or viewpoint to display the best images.

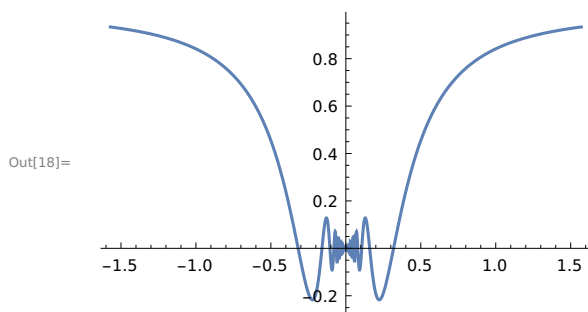
(a) $f(x) = x / 1 + x^2$

In[1]:= `Plot[x / 1 + x ^ 2, {x, -5, 5}]`



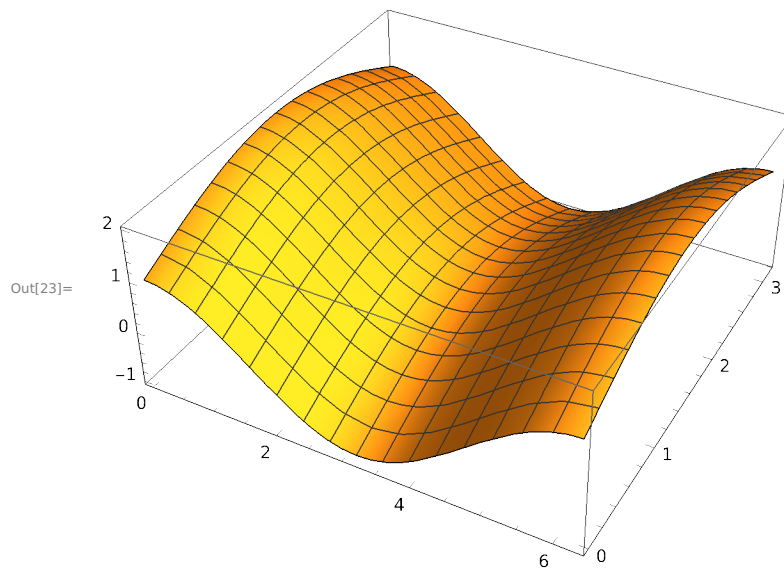
(b) $y = x \sin (1 / x)$

In[18]:= `Plot[x Sin[1 / x], {x, -Pi / 2, Pi / 2}]`



(c) $g(x, y) = \cos (x) + \sin (y)$

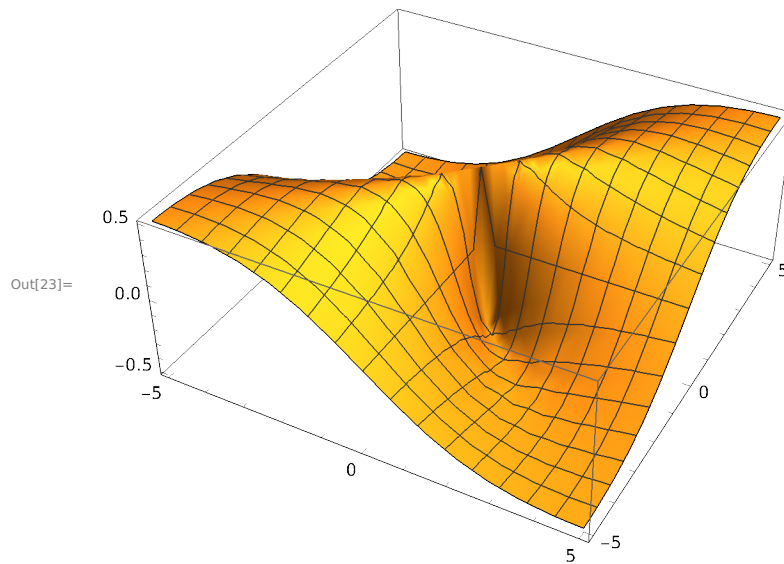
```
In[23]:= Plot3D[Cos[x] + Sin[y], {x, 0, 2 Pi}, {y, 0, Pi}]
```



(d) $z = xy / x^2 + y^2$

```
In[22]:= f[x_, y_] := (x y) / (x^2 + y^2);
```

```
In[23]:= Plot3D[f[x, y], {x, -5, 5}, {y, -5, 5}]
```



Q2. Let $f(x) = x / 1 + x^2$

(a) Find $f'(x)$ and $f''(x)$.

(b) Find $f'(-1)$ and $f''(0)$.

(c) Find $f'(0)$ and $f''(1)$.

Sol. (a)

```
In[6]:= f[x_] = x / 1 + x ^ 2
        f '[x_]

```

```
Out[6]= x + x ^ 2

```

```
Out[7]= 1 + 2 x _

```

```
In[9]:= f ''[x_]

```

```
Out[9]= 2

```

(b)

```
In[10]:= f '[-1]

```

```
Out[10]= - 1

```

```
In[4]:= f ''[0]

```

```
Out[4]= 2

```

(c)

```
In[5]:= f '[0]

```

```
Out[5]= 1

```

```
f ''[1]

```

```
Out[13]= 2

```

Q3. Find the prime factorization of each integer .

(a) 3, 527, 218, 133, 309, 949, 276, 293

```
In[42]:= FactorInteger [ 3 527 218 133 309 949 276 293 ]

```

```
Out[42]= {{15 013 , 2}, {25 013 , 3}}

```

(b) 471, 945, 325, 930, 166, 269

```
In[43]:= FactorInteger [471 945 325 930 166 269 ]

```

```
Out[43]= {{4211 , 1}, {34 589 , 1}, {46 747 , 1}, {69 313 , 1}}

```

(c) 471, 945, 325, 930, 166, 281

```
In[44]:= FactorInteger [471 945 325 930 166 281 ]
Out[44]:= {{471 945 325 930 166 281 , 1}}
```

4. Compute each expression . Do you notice a pattern ?

(a) $3^6 \bmod 7$

```
In[46]:= Mod[3 ^ 6, 7]
Out[46]= 1
```

(b) $6^{10} \bmod 11$

```
In[47]:= Mod[6 ^ 10 , 11]
Out[47]= 1
```

(c) $7^{20} \bmod 21$

```
In[48]:= Mod[7 ^ 20 , 21]
Out[48]= 7
```

(d) $7^{22} \bmod 23$

```
In[49]:= Mod[7 ^ 22 , 23]
Out[49]= 1
```

Q8. Let $M = \begin{bmatrix} 1 & 1 \\ \times & \times \end{bmatrix}$

(a) Find $M^2, M^3, \dots, M^{10}$. $\times$

(b) Do your answers suggest a way to compute Fibonacci numbers ? Find the 100 th Fibonacci number . $\times$

Sol.

(a) $\times$

Solve $(x^2 + x = 1)$

Set : Tag Plus in $x + x^2$ is Protected .

```
Out[52]= Solve
```

```
In[70]:= m = {{1, 1}, {1, 0}}
```

```
Out[70]= {{1, 1}, {1, 0}}
```

```

In[71]:= m.m
Out[71]= {{2, 1}, {1, 1}}

In[72]:= m.m.m
Out[72]= {{3, 2}, {2, 1}}

In[73]:= %.m
Out[73]= {{5, 3}, {3, 2}}

In[74]:= %.m
Out[74]= {{8, 5}, {5, 3}}

In[75]:= %.m
Out[75]= {{13, 8}, {8, 5}}

In[76]:= %.m
Out[76]= {{21, 13}, {13, 8}}

In[77]:= %.m
Out[77]= {{34, 21}, {21, 13}}

In[78]:= %.m
Out[78]= {{55, 34}, {34, 21}}

In[79]:= %.m
Out[79]= {{89, 55}, {55, 34}}

In[80]:= %.m
Out[80]= {{144, 89}, {89, 55}}

```

(b)

```

In[24]:= f[0] = 1;
In[25]:= f[1] = 1;
In[26]:= f[n_] := f[n] = f[n - 2] + f[n - 1]
In[27]:= f[100]
Out[27]= 573 147 844 013 817 084 101

```

Q9. Find solutions to the following equations or systems of equations .

(a) Find x , if $x^2 + x = 1$.

(b) Find x , if $x^2 + x = -1$.

(c) Find x and y .
 $4x - 3y = 5$
 $6x + 2y = 14$

(d) Find x , y , z , and t .
 $-2x - 2y + 3z + t = 8$
 $-3x + 0y - 6z + t = -19$
 $6x - 8y + 6z + 5t = 47$
 $x + 3y - 3z - t = -9$

Sol.

In[1]:= `Solve[x^2 + x == 1, x]`

Out[1]= $\left\{ \left\{ x \rightarrow \frac{1}{2}(-1 - \sqrt{5}) \right\}, \left\{ x \rightarrow \frac{1}{2}(-1 + \sqrt{5}) \right\} \right\}$

In[2]:= `Solve[x^2 + x == -1, x]`

Out[2]= $\left\{ \left\{ x \rightarrow -(-1)^{1/3} \right\}, \left\{ x \rightarrow (-1)^{2/3} \right\} \right\}$

In[3]:= `Solve[4x - 3y == 5 && 6x + 2y == 14, {x, y}]`

Out[3]= $\left\{ \left\{ x \rightarrow 2, y \rightarrow 1 \right\} \right\}$

In[4]:= `Solve[-2x - 2y + 3z + t == 8 && -3x + 0y - 6z + t == -19 && 6x - 8y + 6z + 5t == 47 && x + 3y - 3z - t == -9, {x, y, z, t}]`

Out[4]= $\left\{ \left\{ x \rightarrow 2, y \rightarrow 1, z \rightarrow 3, t \rightarrow 5 \right\} \right\}$

Q 10. Some equations are difficult or impossible to solve explicitly, even with software. In such situations, we often resort to numerical methods. Mathematica uses `FindRoot`, Maple uses `fsolve()`, and Maxima uses `find_root()` to find numerical solutions to equations. Here is an example where a numerical approach works well. Assume that I invest \$250 at the beginning of the year, \$300 at the beginning of the second quarter, \$350 at the beginning of the third quarter, and \$400 at the beginning of the fourth quarter. At the end of the year, I have \$1365 (

because my investment grow). To find my (continuous) rate of return, solve this equation for r :

$$250\text{Exp}[1.0r] + 300\text{Exp}[0.75] + 350\text{Exp}[0.5r] + 400\text{Exp}[0.25r] = 1365$$

```
In[5]:= FindRoot[250 Exp[1.0 r] + 300 Exp[0.75 r] + 350 Exp[0.5 r] + 400 Exp[0.25 r] == 1365, {r, 0}]
Out[5]= {r -> 0.084104}
```

Q 11. If n is a positive number, $g > 0$ is any "guess" for the square root of n , then a better estimate of root n is the average of g and n/g , i.e., $(g + n/g)/2$. Write a function called `mysqrt` that accepts one argument, begins with an initial guess of 1.0, find 20 new guesses, and returns the answer.

```
In[6]:= mysqrt[n_] := Module[{i = 1, g = 1}, While[i ≤ 20, g = (g + (n/g))/2; i = i + 1]; g]
In[7]:= N[mysqrt[2], 6]
Out[7]= 1.41421

In[8]:= N[Sqrt[2], 6]
Out[8]= 1.41421

In[9]:= N[mysqrt[3]]
Out[9]= 1.73205
```

Q 12. The Collatz conjecture states that if we start from any natural number $a_0 = n$ and form a sequence by the rule $a_{i+1} = a_i/2$, if a_i is even; $3a_i + 1$, if a_i is odd, then the sequence eventually contains the value 1. for example, starting from $a_0 = 6$, we get the sequence 6,3,10,5,16,8,4,2,1 (we reached 1 after eight steps).

(a) Write a (recursive) function called `collatz` that accepts a single argument n , and returns:

> 0 if n is equal to 1

> $1 + \text{collatz}(n/2)$ if n is even

> $1 + \text{collatz}(3n+1)$ if n is odd

thus, `collatz(n)` is the number of steps needed to go from n to 1.

(b) verify the values:

n `collatz(n)`

1	0
2	1
6	8
27	111

```
In[17]:= Clear[collatz];
```

```
In[20]:= collatz[n_] := Which[n == 1, collatz[n] = 0, EvenQ[n],  
    collatz[n] = 1 + collatz[n/2], OddQ[n], collatz[n] = 1 + collatz[3 * n + 1]];
```

```
In[21]:= collatz[27]
```

```
Out[21]= 111
```